IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received December 12, 2017, accepted January 16, 2018, date of publication January 26, 2018, date of current version March 19, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2799138

Distributed Interference-Aware Power Control in Ultra-Dense Small Cell Networks: A Robust Mean Field Game

CHUNGANG YANG[1], (Member, IEEE), HAOXIANG DAI[1], JIANDONG LI[1], (Senior Member, IEEE), YUE ZHANG[1], AND ZHU HAN[2,3], (Fellow, IEEE)

[1]State Key Laboratory on Integrated Services Networks, Xidian University, Xi'an 710071, China
[2]University of Houston, Houston, TX 77004 USA
[3]Department of Computer Science and Engineering, Kyung Hee University, Seoul 02447, South Korea

Corresponding author: Chungang Yang (cgyang@mail.xidian.edu.cn)

This work was supported in part by the National Science Foundation of China under Grant 61231008 and Grant 91638202; in part by Ministerio de Economía, Industria Competitividad under Grant TEC2017-85587-R; in part by the Special Financial Grant from the China Postdoctoral Science Foundation under Grant 2016T90894; in part by the Special Financial Grant from the Shaaxi Postdoctoral Science Foundation under Grant 154066; in part by the CETC Key Laboratory of Data Link Technology under Grant 20162309; in part by the Natural Science Basic Research Plan in Shaanxi Province of China under Grant 2017JZ021; in part by the State Key Laboratory on Integrated Services Networks under Grant ISN02080001; in part by the 111 Project under Grant B08038; in part by the Shaanxi Province Science and Technology Research and Development Program under Grant 2011KJXX-40; and in part by the U.S. NSF under Grant CNS-1717454, Grant CNS-1731424, Grant CNS-1702850, Grant CNS-1646607, Grant ECCS-1547201, Grant CMMI- 1434789, Grant CNS-1443917, and Grant ECCS-1405121.

ABSTRACT In ultra-dense small cell networks, interference mitigation is very important due to severe interference. Interference dynamics caused by time-varying environment should be aware and characterized when an interference-aware power control policy is designed to mitigate interference. Meanwhile, interference perception should not be naturally assumed to have complete information with certainty. Generally, it is known that a generic player will react to all the players actions and states in a power control game, which involves huge interference-related information exchange with dynamics and uncertainties. Therefore, to reduce requirements of complete information, we formulate a robust power control mean field game taking the uncertainties of both state dynamics and cost functions into consideration. To achieve the robust power control, we regard the power control problem as a game with players whose individual states are combined by a disturbance term and a Brownian motion. We derive the robust Fokker–Planck–Kolmogorov and Hamilton–Jacobi–Bellman equations, and based on which we propose the robust interference-aware power control algorithm. Simulation results demonstrate the improved performance and the robustness of the proposed algorithm.

INDEX TERMS Interference mitigation, mean field game, power control, ultra-dense small cell networks, robust optimization and control.

I. INTRODUCTION

Network densification is regarded as the foremost feature of overcoming energy and capacity crunches, where heterogeneous small cells densely underlay macrocells with full frequency reuse and distributed coordination. Current Heterogeneous Networks (HetNets) can be naturally seen as the origination of these very dense networks [1]–[4]. In HetNets, extreme deployment of small cells introduces opportunities and challenges. The structure of HetNets can efficiently improve the spectrum efficiency of system. Besides, by introducing small cells, UEs can achieve higher data rates because the transmission distance between small cells and UEs becomes shorter [5]. In addition to these benefits, there also emerge technical challenges, where interference mitigation already exists but deserves more research attention.

A. INTERFERENCE DYNAMICS, UNCERTAINTIES, AND OVERHEAD

Due to full frequency reuse, there exist both intra-tier and inter-tier interference. The interference level in UDN is especially severe, therefore an efficient interference management is urgently needed. Power control is an important method in interference management, it can help mitigate interference as well as reduce energy consumption. However, huge interference-related information is required to design the distributed power control policy which is always time-space dynamic in realistic network environments because of the time-varying environment and the mobility of UEs. Hence, interference dynamics should be aware and characterized when designing an interference-aware power control policy. Besides, mainly based on perfect system parameters(e.g., interference power and channel gain) have the traditional power control policies been proposed. However, due to estimation errors, quantization errors, or measurement errors in practice, those parameters are difficult to be perfectly obtained [6]. Hence, uncertainties should be taken into consideration in interference perception. Meanwhile, with the amount of small cells tending to infinity, centralized interference management meets with severe signaling overhead, flexibility, and scalability, coupling with limited backhauling capacity and random deployments. In consequence, self-organizing characteristics should be paid attention in distributed interference management.

Game theory is considered as a promising mathematical tool modeling resource competition and interference coordination between different tiers of cells in ultra-dense HetNets. Game theory has more and more been applied to mitigate interference [8]–[14]. In [8], the problem of selecting channel to mitigate interference in a canonical communication network is modeled as an exact potential game. In [9], interference management as well as resource allocation is formulated as a Stackelberg game, where information about channel state is incomplete, and [10], [11] also improved the system performance by exploring the hierarchical benefits. [12] proposed a distributed resource allocation policy for the network where a macro cellular is underlaid by small cells based on the evolutionary game theory. However, these conventional games for HetNets encounter several technical challenges, e.g., huge signaling overhead involved by the interaction between ultra-dense small cells [15]–[28].

B. DISTRIBUTED POWER CONTROL WITH INTERFERENCE PERCEPTION

In ultra-dense small cell networks, a power control policy in a distributed manner is required. First, mutual intra-tier interference between ultra-dense small cells should be well characterized, instead of being omitted or simply treated as background noise when designing an interference-aware control policy. Second, interference perception should not be naturally assumed with fully complete information with certainty. To characterize the interference interactions and design distributed interference mitigation schemes, we formulate the interference-aware power control problem as an mean field game. The mean field game is represented by coupled HJB equation and FPK equation, where HJB equation governs the optimal path of the player while the FPK equation controls the evolution of the mean field. By formulating the system as two coupled equations the mean field framework reduces the complexity to achieve the system equilibrium. Generally, there are four assumptions that the mean field games are complied with: (1) Players are rational; (2) Players are of contimuum (i.e., the mean field is continuous); (3) Players' states are interchangeable (i.e., the game's outcome will not be affected by the players' states); (4) Players' interaction of the mean field. Generally in a game, in order to make sure that the players are able to take their own logical decisions, the first assumption should be set. Because that the amount of SBSs of the system model is huge, so the second assumption is rational. To guarantee the interchangeability of the players' actions, the cost function (which will be given later) is derived. The fourth idea means that every player could interact with the mean field, rather than interact with all others [17], which significantly reduces signaling.

C. ADVANCED MEAN FIELD GAME

In a game, a generic player will react to all the players' actions and states, the process of which involves huge interference-related information exchange with dynamics and uncertainties. Here we use the stochastic game to formulate the power control problem as a game involving the dynamics and uncertainties incomplete interference information. Moreover, we apply the interference-aware stochastic game to the MFG when the amount of small cells is huge, even goes to infinite. Only a few works are available for mean field games in large-scale networks [15]–[24]. Mean field approximation of interference is applied in [15] and [16]. In [16], the mean field game theory is utilized to aid decouple a inter-cell interference management optimization problem complicated and large-scaled into a few localized optimization problems. Reference [17] is the first work to use the mean field game as a theoretic approach to solve the problem of managing interference in ultra-dense HetNets. Semasinghe and Hossain [17] formulate the mean field game with different two cost functions and analyze the performance of each cost function. More importantly, based on the Lagrange relaxation and Lax-Friedrichs scheme, [17] proposes a finite difference algorithm to settle the corresponding MFG.

The technique of mean field approximation is used by the Al-Zahrani *et al.* [20] to convert traditional game into a mean field game to make resource allocation simpler and easier. Other famous works include [18]–[24], where [18] and [19] help model different dynamics of mobility, channel, and so on, and [20] facilities the computation of the mean field. Aziz and Caines [21] consider the coverage optimization problem, and [22]–[24] are with energy efficiency as the optimization for different networking scenarios. Meanwhile, robust mean field games have found applications [25], and the computation algorithm for a specific formulated mean field game can well be solved with different finite difference methods [26]. Previously, we extend the mean field game with two-dimensional state dynamics for ultra-dense D2D communications in [27]. In [28], we find that there exist interference dominators during the computation of interference mean field, whose effects are much larger than other interferers and even close to the mean field, and thus cannot be merged into the mean field. The mean field game with dominators [28] is different from the most of the conventional games [15]–[27]. Different from the conventional MFGs [27], we propose a framework in which there exists at least one dominating player for specific generic player in ultra-dense macro-small cell networks [28]. We concentrate on the robust mean field game design in our work, to the knowledge of our best, which is the first work to solve the robust power control in the robust MFG framework.

D. CONTRIBUTIONS

Here we concentrate on the characteristics and awareness of intra-tier interference and typical contributions in our work are summarized as follows:

(1) A robust MFG framework: we formulate a robust power control mean field game taking the uncertainties of both state dynamics and payoff function into consideration. The framework can not only help to formulate the problem as an MFG but also reduce the requirements of complete information.

(2) Interference-aware design: we formulate the power control problem as a cost minimization problem, where a generic player will determine it's own power control policy. Here we formulate the cost function as the combination of both the achieved performance and perceived interference.

(3) Upwind method solving the FPK equation: the players' mean field evolves under the government of the FPK equation. To solve the equation above, we adopt the Upwind method which has a faster convergence speed than that of the Lax-Friedrichs scheme used in previous works [17], [27].

(4) Designing a distributed iterative algorithm to gain the equilibrium of the MFG: first the corresponding FPK and HJB equations of the presented MFG framework are derived, where the optimal trajectory of the player is governed by HJB equation. Then based on finite difference algorithm, we propose a distributed iterative algorithm to obtain the equilibrium of the MFG.

The remainder is organized as below. In Section II, we give the system model and the method of interference mean field approximation. In Section III, the power control problem with interference-aware is formulated as a stochastic differential game. Following that, we formulate the robust MFG in Section IV. We propose the interference-aware power control algorithm in Section V, and we provide simulation results in Section VI. At last, in Section VII, we conclude our work.

II. SYSTEM MODEL AND INTERFERENCE MEAN FIELD APPROXIMATION

We give an introduction to the system model and the interference interaction model in this section. Moreover, we present the interference mean field to approximate the aggregate interference of a generic player.

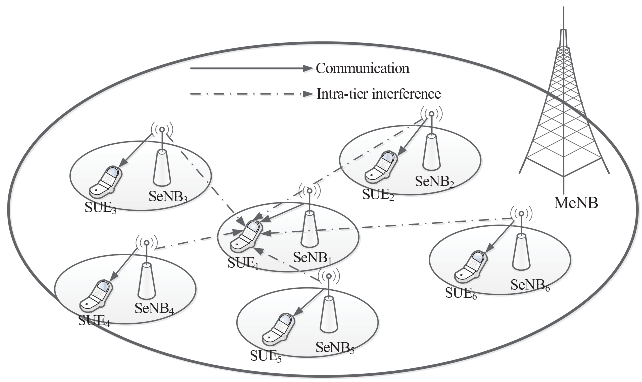


FIGURE 1. Illustration of heterogeneous ultra-dense small cell networks with densely-deployed intra-tier interference.

A. SYSTEM MODEL

We give the illustration of the ultra-dense small cell networks in Fig. 1, where N SeNBs fully share spectrum, e.g., one channel. Due to intra-tier interference, SeNBs may encounter heavy performance degradation. To be well aware and characterize the strategic behaviors among ultra-dense small cells, we concentrate on the mutual intra-tier interference.

We choose SeNB_1 as a generic player in a huge number of SeNBs, and we consider the downlink transmit case. SUE_1 serviced by SeNB_1 will receive intra-tier interference caused by the other SeNBs. The Signal to Interference plus Noise Ratio (SINR) of SUE_1 is

$$\gamma_1 = \frac{p_1(t) g_{1,1}(t)}{\sum_{j=2}^{6} p_j(t) g_{j,1}(t) + \sigma^2(t)}, \tag{1}$$

where $g_{j,1}(t)$ is the channel gain from SeNB_j to SUE_1, $p_j(t)$ represents the transmit power of SeNB_j, $\sigma^2(t)$ is background noise. Here, $\sum_{j=2}^{6} p_j(t) g_{j,1}(t) + \sigma^2(t)$ represents the total interference power plus noise perceived by SUE_1, the first term represents the interference power perceived by SeNB_1 brought by all the other SeNBs. We can see from (1) that the SINR mainly depends on transmit power and channel gain which are variational with respect to time, so that SINR is dynamic. In a game, players, here are the SeNBs, make decisions by not only their own state but also the other players'. Hence, a player will exchange information with the other players, and the amount of information exchanged will be huge when the number of players is large.

B. STOCHASTIC INTERFERENCE INTERACTION MODEL

We concentrate on the intra-tier interference mitigation among different small cells. Each small cell can be a player in the game-theoretic framework, e.g., $i \in \mathcal{N}$, where $\mathcal{N} = \{1, \ldots, N\}$, and total number N of SeNBs is huge, and even goes to infinity. We assume that SeNB_i, $i \in \mathcal{N}$, is generic player i, which is randomly chosen from N players. Here, the perceived aggregate interference of the generic player i is

$$\mu_i(t) = \sum_{j=1}^{N} p_j(t)g_{j,i}(t) - p_i(t)g_{i,i}(t), \tag{2}$$

which refers to the intra-tier interference from other players $j \in \mathcal{N}, j \neq i$. Here $p_i(t)$ is the transmit power of any player $i \in \mathcal{N}$ at time t, while $g_{j,i}(t)$ and $g_{i,i}(t)$ are the channel gains and stochastic processes respectively. To well characterize the channel dynamics, it is uniformly formulated as Ornstein-Uhlenbeck dynamics [18]:

$$dg(t) = \frac{1}{2}(\kappa_g - g(t))dt + \sigma_g^2\, d\mathscr{B}(t), \tag{3}$$

where κ_g and σ_g^2 are non-negative real values. Then the stationary distribution of $g(t)$ is Gaussian with mean κ_g and variance σ_g^2. $d\mathscr{B}(t)$ is Brownian motion's infinitesimal. Adjusting κ_g and σ_g^2 respectively, the amounts of (slow/fast) fading variance and temporal correlation are determined accordingly. In the remaining sections, assuming that all the channel gain dynamics are described as the above stochastic dynamics function, but with different values of mean κ_g and variance σ_g^2.

C. INTERFERENCE MEAN FIELD APPROXIMATION

To maximize their interference-aware preferences, we fully consider the space-time dynamics of the perceived aggregation interference. Using the perceived aggregate interference of the generic player i (2) as an example, we can see that to estimate $\mu_i(t)$ we should obtain the information of $p_j(t)$, while at the same time, the channel gains $g_{i,i}(t)$ and $g_{j,i}(t)$ should be known as well. Therefore, this estimation process involves too much signaling exchange. It will be even worse due to a huge number of players; meanwhile, the optimal control policy should be made with respect to these timely previously-estimated information.

To reduce the information exchange and signaling overhead, we present the following the mean field approximation method to approach the perceived aggregate interference of the generic player i (2). The interference mean field for the generic player i is given by

$$\mu_i(t) = \underbrace{p_j(t)\varpi_{j,i}(t)}_{\text{context information}} - \underbrace{p_i(t)g_{i,i}(t)}_{\text{local information}}, \tag{4}$$

where $\varpi_{j,i}(t)$ is derived by the mean field approximation method which will be explained by a simple practical example. Assuming a scene with N SeNBs, each SeNB transmits a test power $\hat{p}_{test}$ and the received power at the user equipment i (UE_i) served by SeNB_i would be $\hat{p}_i = \hat{p}_{test}g_{i,i} + \sum_{j=1,j\neq i}^{N} \hat{p}_{test}g_{j,i} \approx \hat{p}_{test}\varpi_{j,i}$. $\hat{p}_i$ is measured and $\hat{p}_{test}$ is previously known, then we can obtain $\varpi_{j,i} = \frac{\hat{p}_{test}}{\hat{p}_i}$. We can see that $p_j(t)\varpi_{j,i}(t)$ is the summation of effective received power and interference power received by player i, it represents the context information. $p_i(t)g_{i,i}(t)$ is the effective received power of player i, where $p_i(t)$ and $g_{i,i}(t)$ can be obtained by local information, so it represents the local information. The above equation (4) will be used to derive the state dynamics function.

III. ROBUST INTERFERENCE MITIGATION STOCHASTIC DIFFERENTIAL GAME

To achieve the robust control, we consider the players with individual states combining a disturbance term and a Brownian motion. Firstly, we give a stochastic differential game as follows:

Definition 1: Define a 5-tuple to represent the robust interference mitigation mean field game G of ultra-dense deployed SeNBs: $\mathrm{G} = \{\mathcal{N}, \{p_i\}_{i\in\mathcal{N}}, \{S_i\}_{i\in\mathcal{N}}, \{\mathcal{Q}_i\}_{i\in\mathcal{N}}, \{c_i\}_{i\in\mathcal{N}}\}$, in which

- *Player set* $\mathcal{N}$: $\mathcal{N} = \{1, \ldots, N\}$, where N represents the number of player in the game, the players here are the densely deployed SeNBs. The players make rational policy in the game, where N is huge, and even goes to infinity $N \to \infty$.
- *Power control* $\{p_i\}_{i\in\mathcal{N}}$: to jointly mitigate interference and save energy, we define the strategy set as the power control policies p_i at time t, where $t \in [0, T]$. To minimize a cost function with interference mean field dynamics as the state space, The generic player i determines powers $p_i(t)$ at any time $t \in [0, T]$.
- *State space* $\{\mathcal{S}_i\}_{i\in\mathcal{N}}$: the state space is defined as interference mean field dynamics, where we introduce the disturbance at time t.
- *Control policy* $\{\mathcal{Q}_i\}_{i\in\mathcal{N}}$: $\mathcal{Q}_i(t)$ denotes a full power control policy with $t \in [0, T]$, it aims at minimizing the average of the cost function over the time interval $[0, T]$.
- *Cost function* $\{c_i\}_{i\in\mathcal{N}}$: our work consider a novel cost function combining the achieved SINR property and consumed energy.

With the above system description and definition, we have clarified the player set $\mathcal{N}$ and power control $\{p_i\}_{i\in\mathcal{N}}$. The state space $\{S_i\}_{i\in\mathcal{N}}$ as well as the cost function $\{c_i\}_{i\in\mathcal{N}}$ will be defined later to introduce the control policy $\{\mathcal{Q}_i\}_{i\in\mathcal{N}}$ of generic player. The state space and cost function are the most important and featured ingredients of the above SeNB game model and each of them will be explained in details in the next two subsections.

A. STATE SPACE

For the interference mean field dynamics defined in (3) for generic player i, we know the term $\varpi_{j,i}(t)$ is the aggregate interference's mean field approximation, which is in the similar form of Ornstain-Uhlenbeck dynamics as (3). The rational is the independent properties of different channel dynamics. Due to the independent properties of different channel links dynamics, the aggregated interference mean field will be in line with the Ornstein-Uhlenbeck dynamics [18]. Different channel models can be assumed to the Ornstein-Uhlenbeck dynamics with different means and variance values. The dynamic function of mean field approximation $\varpi_{j,i}(t)$ is given by

$$d\varpi(t) = \frac{1}{2}(\kappa_\varpi - \varpi(t))dt + \sigma_\varpi^2 \, d\mathscr{B}(t), \qquad (5)$$

where mean κ_ϖ and variance σ_ϖ^2 are non-negative real values. In addition, the Ornstein-Uhlenbeck dynamics will finally help derive the linear-quadratic system which will facilitate the derivations of the HJB and FPK equations. As $d\mathscr{B}(t)$ is the infinitesimal Brownian motion, it is known that $\frac{d\mathscr{B}(t)}{dt} = 0$ and $d^2\mathscr{B}(t) = dt$, which are the properties of Brownian motion in the ITO's formula [18]. Meanwhile, we assume that at time t the transmit power $p_i(t)$ is constant , thus $dp_i(t) = 0$.

The state dynamics s_i of generic player i is defined as

$$s_i = d\mu_i(t) = p_j(t)d\varpi_{j,i}(t) - p_i(t)dg_{i,i}(t) + \xi_i(t)dt, \quad (6)$$

where the term $\xi_i(t)dt$ is the introduced disturbance in state dynamics with the unit variance of the stochastic process $\xi_i(t)$. It functions as an unknown parameter and denotes the unknown disturbance entering into the dynamics at time t.

Substituting (3) and (5) into (6), we have the final state dynamics:

$$\begin{aligned} s_i &= d\mu_i(t) \\ &= \left(\alpha_{j,i}(t)p_j(t) - \beta_{i,i}(t)p_i(t) + \xi_i(t)\right) dt + \sigma^2 f_i(t) d\mathscr{B}(t), \end{aligned} \qquad (7)$$

where $\alpha_{j,i}(t) = \frac{1}{2}\left(\kappa_{j,i} - \varpi_{j,i}(t)\right)$, the term $\alpha_{j,i}(t)p_j(t)$ represents the other players with $\alpha_{j,i}(t)$ as the coefficient of the drift term and specific player's power $p_j(t)$, $\beta_{i,i}(t) = \frac{1}{2}\left(\kappa_{i,i} - g_{i,i}(t)\right)$, the term $\beta_{i,i}(t)p_i(t)$ is related to the power control $p_i(t)$ of the generic player with $\beta_{i,i}(t)$ as the growth parameter function. The term $\sigma^2 f_i(t)d\mathscr{B}(t)$ represents the new stochastic process with time-varying variance $\sigma^2 f_i(t)$, where $f_i(t)$ can be pre-determined with local information and interference mean field approximation.

B. COST FUNCTION WITH DISTURBANCE ATTENUATION

With the defined system state dynamics $s(t) = [s_i(t)]$, the cost function $c(t, s, p)$ is minimized by optimal power control of $p(t) = [p_i(t)]$ for generic player i. With the state space $s_i(t)$, the optimal power control policy $\mathcal{Q}_i^\star(t)$ will be determined by generic player i to minimize the cost function. Generally, communication performance is usually related to the SINR definition, which is given as

$$\gamma_i(t) = \frac{p_i(t)g_{i,i}(t)}{\mu_i(t) + \sigma^2(t)}, \qquad (8)$$

where the aggregate interference $\mu_i(t)$ perceived by generic player i is defined in (2), and $\sigma^2(t)$ represents the power of background noise at time t. In the robust MFG we also introduce the identical SINR threshold γ^{th} for any generic player. Here, γ^{th} is the minimum SINR requirement and is predefined to meet the basic communication requirements. Therefore, the individual cost function at time t for generic player i is

$$\begin{aligned} &c_i\left(t, \mu_i(t), p_i(t)\right) \\ &\quad = \left[p_i(t)\, g_{i,i}(t) - \gamma^{th}\left(\mu_i(t) + \sigma^2(t)\right)\right]^2, \end{aligned} \qquad (9)$$

which is derived from the inequality $\gamma_i(t) \geq \gamma^{th}$. On one hand, the defined cost function is with physical meanings, which means that generic player i will determine the optimal power $p_i(t)$ to minimize the cost function, thus as far as possible to approach the SINR threshold γ^{th}. On the other hand, the defined cost function $c_i(t, \mu_i(t), p_i(t))$ is concave with constant interference $\mu_i(t)$ at any time t with respect to the power $p_i(t)$. The proof process is omitted here.

Implementation of the optimal power control over a period of $[0, T]$ should fully consider the dynamics of the perceived interference $\mu_i(t)$ at any time t. The dynamics of the perceived interference $\mu_i(t)$ is given by (7), where we introduce the disturbance $\xi_i(t)$. Therefore, the cost function of generic player i over $[0, T]$ is given by

$$L_i\left(p_i, \mu_i\right) = \int_0^T c_i\left(t, \mu_i(t), p_i(t)\right) dt + c_i(T), \qquad (10)$$

where $c_i(T)$ is the cost at the final time T. Furthermore, to make sure that the cost is finite when the disturbance is under the worst case, the integral of ξ_i^2 should be constrained. In this paper, we take the way of the ratio between L_i and the summation of ξ_i^2 and the initial cost $c_i(0)$ [25]. Thus, the introduced disturbance ξ_i is constrained as follows:

$$\frac{L_i\left(p_i, \mu_i\right)}{\xi_i^2 + c_i(0)} \leq \rho^2, \qquad (11)$$

where $c_i(0)$ is the cost at the initial time $t = 0$, and ρ measures the robustness level. Finally, the robust cost function is

$$J_i^\rho\left(p_i, \mu_i, \xi_i\right) = L_i\left(p_i, \mu_i\right) - \rho^2 \int_0^T \xi_i^2(t)\, dt, \qquad (12)$$

where we assume that $c_i(0) = 0$. By now, all the elements in the defined MFG have been clarified, which can be reformulated as the optimal control problem in the following subsection.

C. ROBUST OPTIMAL CONTROL PROBLEM

Each player i minimizing the cost function $J_i^\rho(p_i, \mu_i, \xi_i)$ which is given in (12), during the time interval $[0, T]$, is considered will choose the optimal power control policy $\mathcal{Q}^\star(t)$. Also, here substituting into the system dynamics of the state space defined in (7), the problem can be transformed into the general robust optimal control problem of

$$\mathcal{Q}^\star(t) = \arg\min_{p_i(t)} \max_{\xi_i(t)} \mathbb{E}\left[J_i^\rho(p_i, \mu_i, \xi_i)\right]. \qquad (13)$$

Definition 2: Introducing $\mathscr{B}(t)$ as a one-dimensional Brownian motion process, along with the defined robust cost function $J_i^\rho(p_i, \mu_i, \xi_i)$ in (12) and state dynamics $d\mu_i(t)$ in (7), the robust stochastic game problem is given by

$$\arg\min_{p_i(t)} \max_{\xi_i(t)} \mathbb{E}\left[J_i^\rho(p_i, \mu_i, \xi_i)\right].$$

At this time, the solution to above problem is a value function $u(t, s(t))$ which is given by:

$$u(t, s(t)) = \min_{p_i(t)} \max_{\xi_i(t)} \mathbb{E}\left[J_i^{\rho}(p_i, \mu_i, \xi_i)\right], \quad t \in [0, T]. \quad (14)$$

The value function above is supposed to meet a partial differential equation, i.e. the HJB equation, according to the Bellman's optimality principle following the optimal control theory. Namely the value function gives a solution to the HJB equation and also gives a minimum value of cost for a already given dynamic system.

IV. ROBUST INTERFERENCE MITIGATION MFG

MFG is a special differential game with mean field term. Here the system state dynamics is $s(t) = [s_i(t)]$. The following subsections show the definition of mean field and the derivative process of the FPK and HJB equations of the MFG.

A. INTERFERENCE MEAN FIELD AND EQUILIBRIUM

Here, we give the definitions of MFG.

Definition 3: Interference Mean Field: We define the mean field $m(t, s)$ as

$$m(t, s) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}_{\{s_i(t)=s\}}, \quad (15)$$

where $\mathbb{1}$ denotes an indicator function, when $s_i(t) = s$ it returns 1 and it returns 0 otherwise. $s_i(t)$ is the state at time t of player i and s is the given state. For a given time instant, the mean field is the statistics of the ratio of players in each state to the total number of player, namely the probability distribution of the states over the player set. Assuming that in a system there are 100 players and 5 different states $\{s_1, s_2, s_3, s_4, s_5\}$, each player must belong to a specific state of the given states at a given moment. Assuming by counting, we found the number of players in each state is 10, 20, 35, 15 and 20 respectively. Then we know the ratio of players in each state to the total number of player is 0.1, 0.2, 0.35, 0.15 and 0.2, also the mean field is the same as 0.1, 0.2, 0.35, 0.15 and 0.2.

Next, we formulate the MFG and whose equilibrium is introduced:

Definition 4: The derived FPK and HJB equations is combined to represent the MFG.

The player's optimal trajectory is governed by the HJB equation, while the players' mean field function evolute under the government of the FPK equation. Here, the FPK equation is forward function while the HJB equation is backward function. The meaning of forward is that we know the function's starting value, and we derive the value of the function at time $t \in [0, T]$. Hence, the FPK equation is solved from starting $t = 0$ to the end at $t = T$. The HJB equation evolves backward with time and is a condition necessary and sufficient when solved over the state space for the optimum. The interactively evolution will at last lead to the mean field equilibrium that can be obtained by utilizing the finite difference method.

Definition 5: The stable combination constructing by both the mean field $m^{\star}(t, s)$ and the control policy $u^{\star}(t, s)$ at arbitrary time t and state s is represented by Mean field equilibrium (MFE).

At arbitrary time t and state s, control policy $u(t, s)$ (i.e. value function) and mean field $m(t, s)$ interact with each other, where the control policy $u(t, s)$ is the solution of HJB equation. For a given time instant, the mean field in our work is the probability distribution of the interference state of players. The evolution of mean field $m(t, s)$ is affected by the control policy because the control policy has an effect on the interference state of player. As the mean field $m(t, s)$ determines the control policy so the control policy is determined according to the mean field. The value function and the mean field interact with each other leading to the mean field equilibrium.

B. HJB EQUATION

The value function in (14) is supposed to satisfy a HJB partial differential equation on the basis of the optimal control theory and the Bellman's optimality principle.

Theorem 1: The HJB Equation is given as

$$\partial_t u(t, s) + \frac{\sigma^2}{2} \Delta_{s_i} u(t, s) = H(c, \nabla_s u(t, s)), \quad (16)$$

where $u(t, s)$ is the value function, $\partial_t u$ and $\Delta_{s_i} u$ is respectively the differential equation and Laplacians of u with respect to t. The variance σ^2 is a non-negative real value and $H(c, \nabla_s u(t, s))$ is the Hamiltonian of the robust MFG, which is given by

$$H = \min_{p_i(t)} \max_{\xi_i(t)} \times \{c - \rho^2 \xi_i^2(t) + q(\alpha_{j,i}(t)p_j(t) - \beta_{i,i}(t)p_i(t) + \xi_i(t))\}, \quad (17)$$

where c is cost function, ρ is the robustness level, $\xi_i(t)$ is disturbance, $u(t, s)$ is value function, and $q = \Delta_{s_i} u(t, s)$ is the Laplacians of u with respect to s_i. Where $\alpha_{j,i}(t) = \frac{1}{2}\left(\kappa_{j,i} - \varpi_{j,i}(t)\right)$, $\beta_{i,i}(t) = \frac{1}{2}\left(\kappa_{i,i} - g_{i,i}(t)\right)$, where $\kappa_{j,i}$ and $\kappa_{i,i}$ are respectively means of the Ornstein-Uhlenbeck dynamics channel model from $SeNB_j$ to UE_i and from $SeNB_i$ to UE_i. $\varpi_{j,i}$ is interference mean field approximation and $g_{i,i}$ is the channel gain from $SeNB_i$ to UE_i.

Proof: We know that $u(t, s)$ represents the defined value function of the state s_i and power $p_i(t)$. Following the Richard Bellman's principle of optimality, we increase time from t to $t + dt$, then have

$$u(t, s) = \min_{p_i(t)} [c(t, s, p) + u(t + dt, s(t + dt))], \quad (18)$$

where $c(t, s, p)$ is cost function, s is the player's state , p is the transmit power of player and $u(t, s)$ is value function. We calculate the Taylor expansion of $u(t + dt, s(t + dt))$, then obtain

$$u(t+dt, s(t+dt)) = u(t, s) + \partial_t u(t, s)\, dt + \frac{\partial s_i}{\partial t} \nabla_{s_i} u(t, s) + \frac{\sigma_i^2}{2} \Delta_{s_i} u(t, s)\, dt, \quad (19)$$

where $\partial_t u$ is the differential function of u with respect to time t, $\nabla_{s_i} u$ is the gradient of u with respect to s and $\Delta_{s_i} u$ is the Laplacians of u with respect to s. $\frac{\partial s_i}{\partial t}$ is the partial differential function of s_i with respect to t, σ_i^2 is the variance of the Ornstein-Uhlenbeck dynamics channel model from $SeNB_i$ to UE_i. Meanwhile, we use the Ito's formula heuristic $d\mathscr{B}(t) = O\left(dt^{\frac{1}{2}}\right)$.

Moreover, we know the expectation of Brownian motion $d\mathscr{B}(t)$ over time dt is zero. Substituting above Taylor expansion $u(t+dt, s(t+dt))$ into $u(t,s)$ and taking the limit of the above equation as dt approaches zero, then we can get the HJB equation defined in (16). Detailed derivation of the HJB equation can be found in [26]. ■

Next, we give the method to calculate generic player's the optimal control.

Theorem 2: Since the computation of the player's optimal control trajectory is governed by the HJB equation, then the optimal control of the generic player is given as:

$$p_i^\star(t) = \frac{1}{\beta_{i,i}(t)}\{\alpha_{j,i}(t)p_j^\star(t) - \frac{\partial H}{\partial q} + \frac{1}{2\rho^2}q\}, \tag{20}$$

where $q = \triangle_{s_i} u(t,s)$, $\Delta_{s_i} u$ is the Laplacians of u with s. H is Hamiltonian, $\partial H/\partial q$ is the partial differential function of H with respect to q and ρ is the robustness level.

Proof: To prove above equation, we first compute the most worst case of the disturbance, which is

$$\xi_i^\star(t) = \arg\max_{\xi_i(t)}\{-\rho^2\xi_i^2(t) + q\xi_i(t) + \varpi\}, \tag{21}$$

where

$$\varpi = c + q\left[\alpha_{j,i}(t)p_j(t) - \beta_{i,i}(t)p_i(t)\right]. \tag{22}$$

Due to the strictly concave properties of the above function as for the disturbance variable $\xi_i(t)$, it is easy to achieve the global maximizer $\xi_i^\star(t)$, which is given by

$$\xi_i^\star(t) = \frac{q}{2\rho^2}, \tag{23}$$

where $u(t,s)$ satisfies the HJB equation of (16). At this time, the robust Hamiltonian (17) of the robust MFG is given by

$$H = \min_{p_i(t)}\{\varpi + \frac{1}{4\rho^2}q^2\}, \tag{24}$$

where ϖ is (22). We know that the cost function is concave with respect to $p_i(t)$. We compute the derivative of the robust Hamiltonian H with respect to q, resulting by

$$\frac{\partial H}{\partial q} = \frac{\partial \varpi}{\partial q} + \frac{1}{2\rho^2}q, \tag{25}$$

where

$$\frac{\partial \varpi}{\partial q} = \alpha_{j,i}(t)p_j^\star(t) - \beta_{i,i}(t)p_i^\star(t).$$

Therefore, by calculating $\partial H/\partial q = 0$, we can obtain the optimal $p_i^\star(t)$ in (20). ■

C. FPK EQUATION

According to the mean field theory, the mean field $m(t,s)$ satisfies a partial differential equation, called FPK equation. Next, we will introduce the FPK equation.

Theorem 3: The evolution of the mean field is governed by the FPK equation, which is given by

$$\partial_t m(t,s) + \frac{\sigma_i^2}{2}\Delta_{s_i} m(t,s) - \frac{\partial s_i}{\partial t}\nabla_{s_i} m(t,s) = 0, \tag{26}$$

where $\partial_t m$ is the differential function of the mean field m with respect to time t. $\Delta_{s_i} m$ and $\nabla_{s_i} m$ is respectively the Laplacian and gradient of the mean field m with respect to s.

Proof: There are different ways to achieve the mean field of the interference state dynamics. We introduce a general method, and therefore one can easily achieve the specific interference mean field by introducing the detailed definitions of the state $s_i(t)$. Here, we derive the mean field $m(t,s)$ via any specific test function $g(s)$ which is smooth, compactly supported function of space.

The integral of $m(t,s)\,g(s)\,ds$ can be considered as the continuum limit of the sum $g(s(t))$, where $s(t)$ represents the player's location at time t. It is known that,

$$\int m(t,s)\,g(s)\,ds \approx \frac{1}{N}\sum_{i=1}^{N} g(s(t)).$$

At time t, the first-order differential function with respect to time t is derived to check how this integral vary in time,

$$\int \partial_t m(t,s)\,g(s)\,ds$$
$$\approx \frac{1}{N}\sum_{i=1}^{N}\left[\partial_t s(t)\,\nabla g(s(t)) + \partial_t^2 s(t)\,\Delta g(s(t))\right]$$

by using the chain rule, and therefore we can achieve the heuristic formula.

The right-hand side, in the continuum limit $N \to \infty$ and after an integration by parts, for every test function g we then have

$$\int\left[\partial_t m(t,s) + \frac{\sigma^2}{2}\Delta_s m(t,s) - \frac{\partial s}{\partial t}\nabla_s m(t,s)\right] g(s(t))\,ds = 0,$$

which leads to the advection equation

$$\partial_t m(t,s) + \frac{\sigma^2}{2}\Delta_s m(t,s) - \frac{\partial s}{\partial t}\nabla_s m(t,s) = 0.$$

With the defined states $s_i(t)$ in this work, we obtain the corresponding FPK equation defined in (26). ■

V. DISTRIBUTED POLICY BASED ON THE FINITE DIFFERENCE METHOD

In the defined MFG, to find a solution to the coupled HJB and FPK equations, we utilize the finite difference method. There are three schemes that can be used to discretize the advection equation, which are respectively Lax-Friedrichs, Lax-Wendrof, and Upwind. Each scheme has its own rate of convergence. Different from Lax-Friedrichs used in [17], we use the Upwind finite difference method that converges faster.

We discretize the investigated time interval $[0, T]$ and the interference state space $[0, I_{max}]$ into $X \times Y$ spaces in the framework of the finite difference method. First, we respectively give the definition of the iteration step of time and interference state space as below

$$\delta_t = \frac{T}{X}, \quad \delta_I = \frac{I_{max}}{Y}.$$

We consider to solve the FPK equation first. By the use of finite difference method, we introduce the operators of the Upwind method as follows:

$$\partial_t u_I^t = \frac{u_I^{t+1} - u_I^t}{\delta_t},$$
$$\nabla_{s_i} u_I^t = \frac{u_I^t - u_{I-1}^t}{\delta_I},$$
$$\Delta_{s_i} u_I^t = \frac{u_{I+1}^t - 2u_I^t + u_{I-1}^t}{\delta_I^2}.$$

A. UPWIND SCHEME TO SOLVE FPK EQUATION

By using the Upwind method, we deal with the FPK equation in (26). Then, with above operators we have

$$\frac{m_I^{t+1} - m_I^t}{\delta_t} + \frac{\sigma_i^2}{2}\frac{m_{I+1}^t - 2m_I^t + m_{I-1}^t}{\delta_I^2} - s_i \frac{m_I^t - m_{I-1}^t}{\delta_I} = 0. \tag{27}$$

Meanwhile, we have $s_i = \partial s_i / \partial t$. Therefore, we have

$$\frac{m_I^{t+1}}{\delta_t} = m_I^t\left(\frac{1}{\delta_t} + \frac{\sigma_i^2}{\delta_I^2} + \frac{s_i}{\delta_I}\right) - m_{I-1}^t\left(\frac{\sigma_i^2}{2\delta_I^2} + \frac{s_i}{\delta_I}\right) + m_{I+1}^t\frac{\sigma_i^2}{2\delta_I^2}, \tag{28}$$

where m_I^t is the mean field at time t and interference state I in the discretized grid. We can see from the above equation that m_I^t can update by iteratively using the forward function given in (28).

B. DISCRETIZED LAGRANGE RELAXATION TO HJB

Due to the Hamiltonian, we are not able to directly utilize the finite difference method to deal with the HJB equation. Therefore, the HJB equation is reformulated as a corresponding optimal control problem, which is

$$\min_{p_i} E\left[\int_0^T c(t, s, p)\,dt + c(T) - \rho^2 \int_0^T \xi_i^2(t)\,dt\right], \tag{29a}$$

$$\text{s.t.: } \partial_t m(t, s) + \frac{\sigma_i^2}{2}\Delta_{s_i} m(t, s) - \frac{\partial s_i}{\partial t}\nabla_{s_i} m(t, s) = 0. \tag{29b}$$

Then, we attain the Lagrangian $L(t, s, p, m, \lambda, \xi)$ in (30), shown at the bottom of the page, by the means of introducing a Lagrange multiplier λ, in which assuming $c(T) = 0$. Then, to solve the optimal control problem newly-defined, we discretize the Lagrangian and the result is given as (31), shown at the bottom of the page.

Besides, in the discretized grid for arbitrary point (t, I), we update the Lagrange multiplier λ_I^{t-1} by calculating $\partial L_d / \partial m_I^t = 0$, and then we have (32), shown at the bottom of the page.

Here, we can see that the Lagrangian parameter λ_I^{t-1} is determined by $\lambda_I^t, \lambda_{I+1}^t, \lambda_{I-1}^t$, also it is related to the achieved cost c_I^t and ξ_I^t at time t. Here, the involved Lagrangian parameter λ_I^t is updated by using the above backward function.

The optimal power control is derived by using $\partial L_d / \partial p_I^t$ any arbitrary point (t, I) in the discretized grid, where we further set

$$\frac{\partial L_d}{\partial p_I^t} = 0. \tag{33}$$

Then, we have (34), shown at the bottom of the page, where p_I^t is the power of the generic at time t and interference state I in the discretized grid. We can see that p_I^t is determined by $m_{I-1}^t, m_I^t, m_{I+1}^t$ and λ_I^t.

$$L(t, s, p, m, \lambda, \xi) = \int_{t=0}^{T}\int_{s=0}^{s_i^{\max}}\left\{c(t, s, p)\, m(t, s) - \rho^2 \xi_i^2(t)\, m(t, s) + \lambda\left[\partial_t m(t, s) + \frac{\sigma_i^2}{2}\Delta_{s_i} m(t, s) - \frac{\partial s_i}{\partial t}\nabla_{s_i} m(t, s)\right]\right\} dtds \tag{30}$$

$$L_d = \sum_{t=1}^{X+1}\sum_{I=1}^{Y+1}\left\{c_I^t m_I^t - \rho^2\left(\xi_I^t\right)^2 m_I^t + \lambda_I^t\left[\frac{m_I^{t+1} - m_I^t}{\delta_t} + \frac{\sigma_i^2\left(m_{i+1}^t - 2m_I^t + m_{I-1}^t\right)}{2\delta_I^2} - s_i\frac{m_I^t - m_{I-1}^t}{\delta_I}\right]\right\} \tag{31}$$

$$\frac{\lambda_I^{t-1}}{\delta_t} = \lambda_I^t\left(\frac{1}{\delta_t} + \frac{\sigma_i^2}{\delta_I^2} + \frac{s_i}{\delta_I}\right) - \lambda_{I+1}^t\left(\frac{\sigma_i^2}{2\delta_I^2} + \frac{s_i}{\delta_I}\right) - \lambda_{I-1}^t\frac{\sigma_i^2}{2\delta_I^2} - c_I^t + \rho^2\left(\xi_I^t\right)^2 \tag{32}$$

$$p_I^t = \frac{\frac{\lambda_I^t}{2\delta_I^2}\left[2\delta_I \beta_{i,i}\left(m_{I-1}^t - m_I^t\right) - \sigma^2\left(m_{I+1}^t - 2m_I^t + m_{I-1}^t\right)\right]}{2m_I^t g_{i,i}^2(t)\left(1 + \gamma^{th}\right)^2} + \frac{\gamma^{th}\left(p_{j,I}^t \varpi_{j,i}(t) + \varrho^2(t)\right)}{\left(1 + \gamma^{th}\right) g_{i,i}(t)} \tag{34}$$

Algorithm 1 Distributed Robust Interference-Aware Power Control

1 **Initialization:**
2 m_I^0:= joint interference mean field distribution;
3 λ_I^t:= initial Lagrangian parameters;
4 p_I^t:= initial power levels.
5 **Power Control:**
6 **for** $t = 1 : X$ *and* $I = 1 : Y$ **do**
7 **Update interference mean field:**
8 Update m_I^{t+1} using the update of (28)
9 **if** $m_I^{t+1} < 0$ **then**
10 $m_I^{t+1} = 0$
11 **else**
12 $m_I^{t+1} = m_I^{t+1}$
13 **end**
14 **if** $p_Y^t = p_{max}$ **then**
15 $m_Y^{t+1} = m_Y^t$
16 **else**
17 $m_Y^{t+1} = 0$
18 **end**
19 **Update Lagrangian parameter:**
20 λ_I^{t-1} using (32)
21 **Update power levels:**
22 p_I^t for generic player i using (34)
23 **if** $p_I^t > p_{max}$ **then**
24 $p_I^t = p_{max}$
25 **else**
26 $p_I^t = p_I^t$
27 **end**
28 **if** $p_I^t < 0$ **then**
29 $p_I^t = 0$
30 **else**
31 $p_I^t = p_I^t$
32 **end**
33 **end**

C. DISTRIBUTED ROBUST INTERFERENCE-AWARE POWER CONTROL

To the purpose of solving the corresponding HJB and FPK equations, a joint finite difference algorithm is proposed on the basis of the Upwind scheme and Lagrange relaxation. The algorithm is named as the distributed robust interference-aware power control policy, and is given in Algorithm 1.

Here, there exist three key steps, including the updating process of the interference mean field (from line 7-18), the Lagrangian parameter (from line 19-20), and the power levels (from line 21-32). It is noted that the interference mead field is highly dependent on the power control, while the Lagrangian parameter merges the power and the mean field in (32), and finally, both the interference mean field and Lagrangian parameter jointly determine the next power levels.

TABLE 1. Simulation parameters

Parameter	Value
Deployment scenario	Dense SBSs in $500m \times 500m$ square model
Total Bandwith	2GHz
Subcarrier Bandwidth	10MHz
Number of SBSs	36
p_i^{max}	$0.1w$
X	50
Y	10
T	0.5s
robustness level ρ	3

VI. SIMULATION RESULTS

We provide simulation results to illustrate the convergence property and effectiveness of the proposed algorithm compared to the others in this section. We generate a $500m \times 500m$ rectangular area, where multiple SeNBs are deployed with $p_{max} = 20dBm$. Here, the pass loss models we use are reffered to Table A.2.1.1.2-3 for the Femtocell in 5×5 grid model of 3GPP-TR 36.814 [17]. In our simulation, we use Femtocell as an example of SeNB. Specifically, the path loss model we use from Femtocell to UEs is $L = 127 + 30\log_{10} D$, where D in km. To show the proposed algorithm's performance, we compare it to other two algorithms. They are non-intelligent adaption algorithm and full information algorithm. Other simulation parameters we use is shown in Table I.

A. PERFORMANCE METRICS

In this paper, we mainly consider SE and EE as performance matrics. Before giving them, we show the SINR of generic player in

$$\gamma = \frac{p_i g_{i,i}}{p_j \varpi_{j,i} - p_i g_{i,i} + \sigma^2}, \tag{35}$$

where p_i and p_j represent the transmit power of player i and player j, respectively. σ^2 represents the background noise. Here we assume $p_i = p_{j,I}$. We use the spectrum and energy efficiency as the performance metrics.

B. BEHAVIOR OF MEAN FIELD EQUILIBRIUM (MFE)

We show the behavior at the equilibrium of the mean field in this subsection. The initial interference distribution is assumed to be Gaussian. In Fig. 2, we give the illustration of the distribution of the mean field with respect to time and interference state of generic player. We can see that the tendency with respect to time is rising or decreasing between different constant interference state values, which can be proved by using the iterative function of mean field. We can also see that distribution of the mean field is rising continuously and is about 1.7 at end of time when fixing interference state at about middle of the interference space. When fixing different values of time, the distribution of the mean field is approximate Gaussian with different variances.

C. PERFORMANCE RESULTS OF MEAN FIELD EQUILIBRIUM

Here, we show the performances at the MFE. We mainly focus on average spectrum efficiency and average energy efficiency.

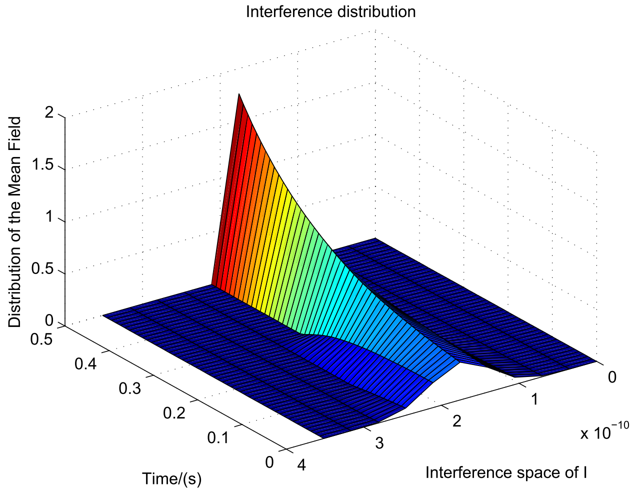


FIGURE 2. **Distribution of the mean field at equilibrium.**

1) ROBUST PERFORMANCE OF ALGORITHM

Here we show the robust performance of the algorithm proposed. We plot the figures in which the x-axis is the variance of disturbance we introduced into the dynamics state function of player and y-axis is the spectrum efficiency or energy efficiency, respectively. The distribution of disturbance we set follows Gaussian whose variance varies from 1.5 to 2.8 with 0.1 step. The results are given in Fig. 3.

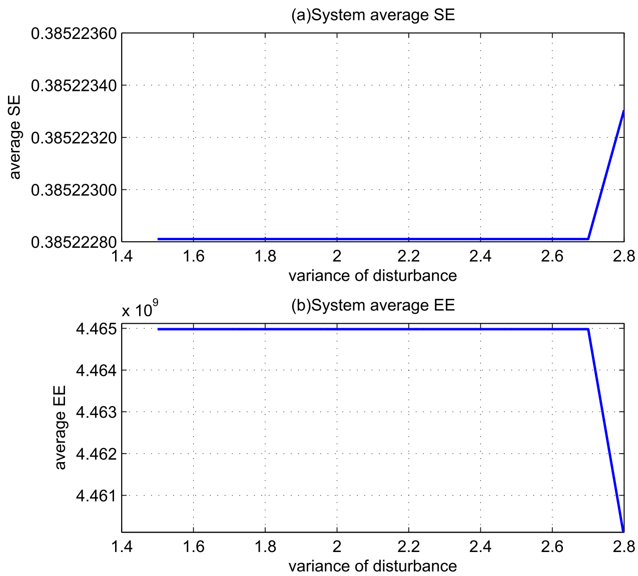


FIGURE 3. **System average performance.**

The system average spectrum efficiency and the average energy efficiency under different variances of disturbance are shown in Fig. 3. The system average SE and the system average EE are almost unchanged before the variance of disturbance reach to nearly 2.7, and then the system average SE increase and the system average EE decrease till the end in a small range. Though the system average performance is influenced by the variance of disturbance, the variation range is narrow due to the robustness of the proposed algorithm.

2) PROPOSED ALGORITHM's PERFORMANCE

We consider a scenario in which $6 \times 6 = 36$ SBSs are homogeneously deployed in a $500m \times 500m$ rectangular area and the density of SBSs is $144/km^2$. We give the illustration of the system average SE and the system average EE over time when fixing the interference state and the interchange, respectively, in Fig. 4.

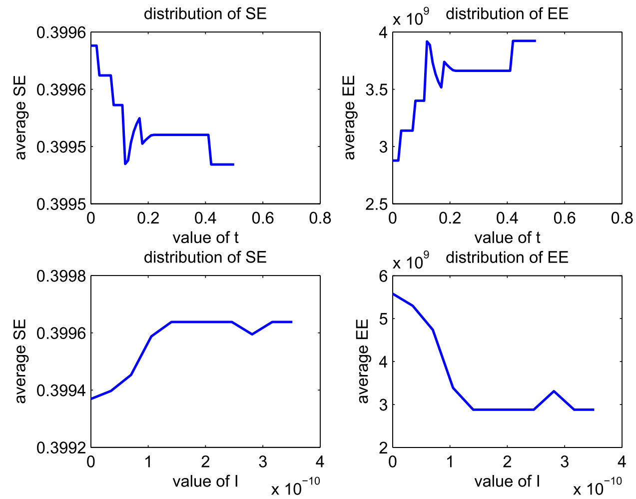


FIGURE 4. **System performance with respect to time and interference state dimensions.**

Among the four subfigures in Fig. 4, the two upper subfigures are the system performance with respect to time and two lower subfigures are with respect to interference state. In the two upper subfigures, we can see that the system average SE roughly slowly decreases while the system average EE increases in a ladder-like manner with respect to time. In the two lower subfigures, it can be seen from the figure that the system average SE increases while the system average EE decreases in initial phase and are relatively stable in the subsequent phase with respect to the interference state.

3) CUMULATIVE DENSITY FUNCTIONS OF SPECTRUM AND ENERGY EFFICIENCY

The CDF of spectrum and energy efficiency are given in Fig. 5 and Fig. 6. Here we consider two comparing algorithms, the non-intelligent adaption algorithm and the full information algorithm. In the non-intelligent adaption algorithm, we assume that the mean field is constant so the algorithm cannot adjust its parameters according to the environment. Besides, we assume that we have the full knowledge of the information of the environment, e.g. the channel state and the aggregate interference, so we can achieve the full information algorithm.

In Fig. 5, we can see that the proposed algorithm's performance is a bit worse than that of full information algorithm when the probability is lower than 0.7 and obviously a better performance when the probability is higher than 0.7. The proposed algorithm's performance is obviously better than the performance of non-intelligent adaption algorithm during

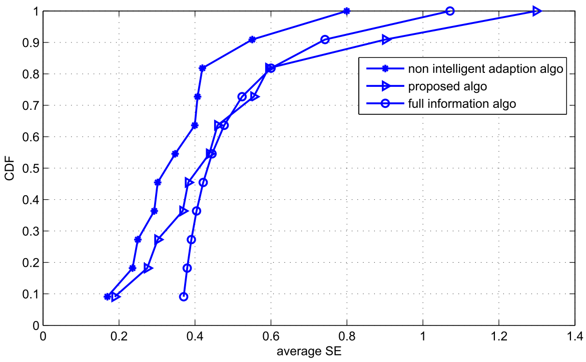


FIGURE 5. CDF over system average SE.

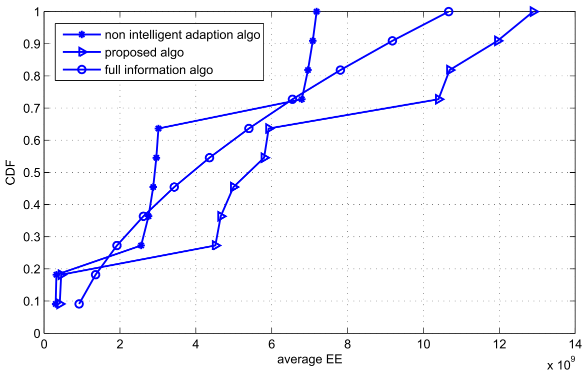


FIGURE 6. CDF over system average EE.

all the probability interval. In Fig. 6, the performance of proposed algorithm exceeds that of full information algorithm quickly at the probability about 0.2 and is always better than that of non-intelligent adaption algorithm during almost all the probability interval.

VII. CONCLUSION

To characterize the interference interactions and design distributed interference mitigation schemes, the interference-aware power control problem is formulated as a mean field game. A generic player in the proposed game will react to all the players' actions and states, which involves huge interference-related information exchange with dynamics and uncertainties. Therefore, to reduce requirements of complete information, we formulate the interference mitigation mean field game. More precisely, an interference mean field is presented in the MFG to characterize the mass interference impacts. The interference mean field can be updated autonomously by the generic player, thus leading to distributed control. To achieve the robust control, we introduce uncertainties into the state dynamics and cost function. We consider players whose individual states are combined by a disturbance term and a Brownian motion. We derive the robust FPK and HJB equations, based on which we propose the robust interference-aware power control algorithm.

REFERENCES

[1] E. Hossain, L. B. Le, and D. Niyato, *Radio Resource Management in Multi-Tier Cellular Wireless Networks*. Hoboken, NJ, USA: Wiley, 2013.

[2] C. Yang, J. Li, and M. Guizani, "Cooperation for spectral and energy efficiency in ultra-dense small cell networks," *IEEE Wireless Commun.*, vol. 23, no. 1, pp. 64–71, Feb. 2016.

[3] C. Yang, J. Li, A. Anpalagan, and M. Guizani, "Joint power coordination for spectral-and-energy efficiency in heterogeneous small cell networks: A bargaining game-theoretic perspective," *IEEE Trans. Wireless Commun.*, vol. 15, no. 2, pp. 1364–1376, Feb. 2016.

[4] B. Soret, K. I. Pedersen, N. T. K. Jørgensen, and V. Fernández-López, "Interference coordination for dense wireless networks," *IEEE Commun. Mag.*, vol. 53, no. 1, pp. 102–109, Jan. 2015.

[5] F.-S. Chu, C.-H. Lee, and K.-C. Chen, "Backhaul-constrained resource optimization for distributed femtocell interference mitigation," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Istanbul, Turkey, Apr. 2014, pp. 1485–1489.

[6] Y. Xu, X. Zhao, and Y.-C. Liang, "Robust power control and beamforming in cognitive radio networks: A survey," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 4, pp. 1834–1857, 4th Quart., 2015.

[7] J. Xu *et al.*, "Cooperative distributed optimization for the hyper-dense small cell deployment," *IEEE Commun. Mag.*, vol. 52, no. 5, pp. 61–67, May 2014.

[8] J. Zheng, Y. Cai, Y. Xu, and A. Anpalagan, "Distributed channel selection for interference mitigation in dynamic environment: A game-theoretic stochastic learning solution," *IEEE Trans. Veh. Technol.*, vol. 63, no. 9, pp. 4757–4762, Nov. 2014.

[9] S. Bu, F. R. Yu, and H. Yanikomeroglu, "Interference-aware energy-efficient resource allocation for OFDMA-based heterogeneous networks with incomplete channel state information," *IEEE Trans. Veh. Technol.*, vol. 64, no. 3, pp. 1036–1050, Mar. 2015.

[10] N. D. Duong, A. S. Madhukumar, and D. Niyato, "Stackelberg Bayesian game for power allocation in two-tier networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 4, pp. 2341–2354, Apr. 2016.

[11] R. Yin, G. Yu, H. Zhang, Z. Zhang, and G. Y. Li, "Pricing-based interference coordination for D2D communications in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 3, pp. 1519–1532, Mar. 2015.

[12] P. Semasinghe, E. Hossain, and K. Zhu, "An evolutionary game for distributed resource allocation in self-organizing small cells," *IEEE Trans. Mobile Comput.*, vol. 14, no. 2, pp. 274–287, Feb. 2015.

[13] Z. Zhang, L. Song, Z. Han, and W. Saad, "Coalitional games with overlapping coalitions for interference management in small cell networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 5, pp. 2659–2669, May 2014.

[14] H. Zhang, C. Jiang, N. C. Beaulieu, X. Chu, X. Wang, and T. Q. S. Quek, "Resource allocation for cognitive small cell networks: A cooperative bargaining game theoretic approach," *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3481–3493, Jun. 2015.

[15] S. Samarakoon, M. Bennis, W. Saad, M. Debbah, and M. Latva-Aho, "Energy-efficient resource management in ultra dense small cell networks: A mean-field approach," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, San Diego, CA, USA, Dec. 2015, pp. 1–6.

[16] A. Y. Al-Zahrani, F. R. Yu, and M. Huang, "A joint cross-layer and colayer interference management scheme in hyperdense heterogeneous networks using mean-field game theory," *IEEE Trans. Veh. Technol.*, vol. 65, no. 3, pp. 1522–1535, Mar. 2016.

[17] P. Semasinghe and E. Hossain, "Downlink power control in self-organizing dense small cells underlaying macrocells: A mean field game," *IEEE Trans. Mobile Comput.*, vol. 15, no. 2, pp. 350–363, Feb. 2016.

[18] T. Alpcan, H. Boche, M. L. Honig, and H. V. Poor, *Mechanisms and Games for Dynamic Spectrum Allocation-Reacting to the Interference Field*. Cambridge, U.K.: Cambridge Univ. Press, 2014.

[19] J. Park, S. Y. Jung, S.-L. Kim, M. Bennis, and M. Debbah, "User-centric mobility management in ultra-dense cellular networks under spatio-temporal dynamics," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Washington, DC, USA, Dec. 2016, pp. 1–6.

[20] A. Y. Al-Zahrani, F. R. Yu, and M. Huang, "A mean-field game approach for distributed interference and resource management in heterogeneous cellular networks," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Atlanta, GA, USA, Dec. 2013, pp. 4964–4969.

[21] M. Aziz and P. E. Caines, "A mean field game computational methodology for decentralized cellular network optimization," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 2, pp. 563–576, Mar. 2017.

[22] J. Park, S.-L. Kim, M. Bennis, and M. Debbah, "Spatio-temporal network dynamics framework for energy-efficient ultra-dense cellular networks," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Washington, DC, USA, Dec. 2016, pp. 1–7.
[23] S. Samarakoon, M. Bennis, W. Saad, M. Debbah, and M. Latva-Aho, "Ultra dense small cell networks: Turning density into energy efficiency," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 5, pp. 1267–1280, May 2016.
[24] J. Li, R. Bhattacharyya, S. Paul, S. Shakkottai, and V. Subramanian, "Incentivizing sharing in realtime D2D streaming networks: A mean field game perspective," *IEEE/ACM Trans. Netw.*, vol. 25, no. 1, pp. 3–17, Feb. 2017.
[25] D. Bauso, H. Tembine, and T. Başar, "Robust mean field games with application to production of an exhaustible resource," *IFAC Proc. Volumes*, vol. 45, no. 13, pp. 454–459, 2012.
[26] M. Burger and J. M. Schulte, "Adjoint methods for hamilton-jacobibellman equations," M.S. thesis, Nov. 2010.
[27] C. Yang, J. Li, P. Semasinghe, E. Hossain, S. M. Perlaza, and Z. Han, "Distributed interference and energy-aware power control for ultra-dense D2D networks: A mean field game," *IEEE Trans. Wireless Commun.*, vol. 16, no. 2, pp. 1205–1217, Feb. 2017.
[28] C. Yang, Y. Zhang, J. Li, and Z. Han, "Power control mean field game with dominator in ultra-dense small cell networks," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Singapore, Dec. 2017, pp. 1–6.
[29] *3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Further Advancements for E-UTRA Physical Layer Aspects (Release 9)*, document TR 36.814, 3GPP, 2010.

CHUNGANG YANG (S'09–M'12) received the B.S. and Ph.D. degrees from Xidian University, Xi'an, China, in 2006 and 2011, respectively.

From 2010 to 2011, he held the visiting scholar position at the Department of Electrical and Computer Engineering, Michigan Technological University. From 2015 to 2016, he held the visiting scholar position at the Department of Electrical and Computer Engineering, University of Houston. He is currently an Associate Professor at Xidian University, where he leads the Game, Utility, Intelligent Computing Design for Emerging Communications Research Team (GUIDE). He has edited two books *Game Theory Framework Applied to Wireless Communication Networks* (IGI Global, 2016) and *Interference Mitigation and Energy Management in 5G Heterogeneous Cellular Networks* (IGI Global, 2017). His research interests include resource and interference management, network optimization, and mechanism design for cognitive radio networks, heterogeneous cellular networks, and game theory for wireless communication and computing networks.

Dr. Yang serves as an Editor for the *KSII Transactions on Internet and Information Systems*, and an Editor of special issue of Green and Energy Harvesting Wireless Networks, Wireless Communications, and Mobile Computing.

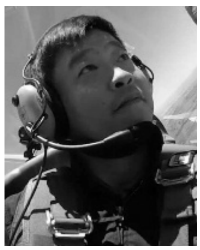
HAOXIANG DAI received the B.S. degree in communication engineering in 2011. He is currently working toward the M.S. degree at Xidian University. He is with the Game, Utility, Intelligent Computing Design for Emerging Communications Research Team, which is guided by Dr. C. Yang. His research interests include robust resource management for *ad hoc* networks and robust mean field game.

JIANDONG LI (SM'05) received the B.S., M.S., and Ph.D. degrees in communications and electronic system from Xidian University in 1982, 1985, and 1991, respectively.

He has been with Xidian University since 1985, was an Associate Professor from 1990 to 1994, has been a Professor since 1994, has been the Ph.D. Student Supervisor since 1995, and has been the Dean of the School of Telecommunication Engineering with Xidian University since 1997. He is currently the Executive Vice Dean of the Graduate School, Xidian University.

Dr. Li is a Senior Member of the China Institute of Electronics and a fellow of the China Institute of Communication. He was a member of the PCN Specialist Group for China 863 Communication High Technology Program from 1993 to 1994 and from 1999 to 2000. He is also a member of the Communication Specialist Group, Ministry of Information Industry. His current research interest and projects are funded by the 863 High Tech Project, NSFC, National Science Fund for Distinguished Young Scholars, TRAPOYT, MOE, and MOI.

YUE ZHANG received the B.S. degree in communication engineering from Xidian University in 2012, where he is currently working toward the M.S. degree. He is with the Game, Utility, Intelligent Computing Design for Emerging Communications Research Team, which is guided by Dr. C. Yang. His research interests include cross-layer design for ad hoc networks and mean field game with dominators.



ZHU HAN (S'01–M'04–SM'09–F'14) received the B.S. degree in electronic engineering from Tsinghua University in 1997 and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Maryland, College Park, MD, USA, in 1999 and 2003, respectively.

From 2000 to 2002, he was a Research and Development Engineer at JDSU, Germantown, MD, USA. From 2003 to 2006, he was a Research Associate at the University of Maryland. From 2006 to 2008, he was an Assistant Professor at Boise State University, Boise, ID, USA. He is currently a Professor at the Electrical and Computer Engineering Department and also at the Computer Science Department, University of Houston, Houston, TX, USA. His research interests include wireless resource allocation and management, wireless communications and networking, game theory, big data analysis, security, and smart grid. He has received an NSF Career Award in 2010, the Fred W. Ellersick Prize from the IEEE Communication Society in 2011, the EURASIP Best Paper Award for the *Journal on Advances in Signal Processing* in 2015, the IEEE Leonard G. Abraham Prize in the field of Communications Systems (Best Paper Award in IEEE JSAC) in 2016, and several best paper awards at the IEEE conferences. He is an IEEE Communications Society Distinguished Lecturer. He is 1% highly cited researcher 2017 according to Web of Science.