ESSAYS ON VOLATILITY RISK AND SECURITY RETURNS

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Abstract

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This dissertation studies the determinants of expected option returns and equilibrium determinants of variance risk and the variance risk premium. In the first essay, I analyze the relation between expected option returns and the volatility of the underlying securities. In the Black-Scholes-Merton and stochastic volatility models, the expected return from holding a call (put) option is a decreasing (increasing) function of the volatility of the underlying. These predictions are strongly supported by the data. In the cross-section of stock option returns, returns on call (put) option portfolios decrease (increase) with underlying stock volatility. This strong negative (positive) relation between call (put) option returns and volatility is not due to cross-sectional variation in expected stock returns. It holds in various option samples with different maturities and moneyness, and it is robust to alternative measures of underlying volatility and different weighting methods. Time-series evidence also supports the predictions from option pricing theory: Future returns on S&P 500 index call (put) options are negatively (positively) related to S&P 500 index volatility.

In the second essay, I show that in many consumption-based general equilibrium models with Epstein-Zin-Weil preferences, the market variance risk premium is related to the leverage effect, defined as the conditional covariance between market returns and changes in the conditional market variance. The sign of the relation between the market variance risk premium and the market leverage effect depends on the coefficient of relative risk aversion and the elasticity of intertemporal substitution. I find a statistically significant negative intertemporal relationship between the variance risk premium and the leverage effect for the S&P 500 from 1996 to 2014. This implies an elasticity of intertemporal substitution less than one and a preference for the early resolution of uncertainty. Exploiting the relation between the variance risk premium and the leverage effect also allows me to characterize the historical behavior of the variance risk premium going back to 1926.

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Chapter 1

Volatility and Expected Option Returns

1.1 Introduction

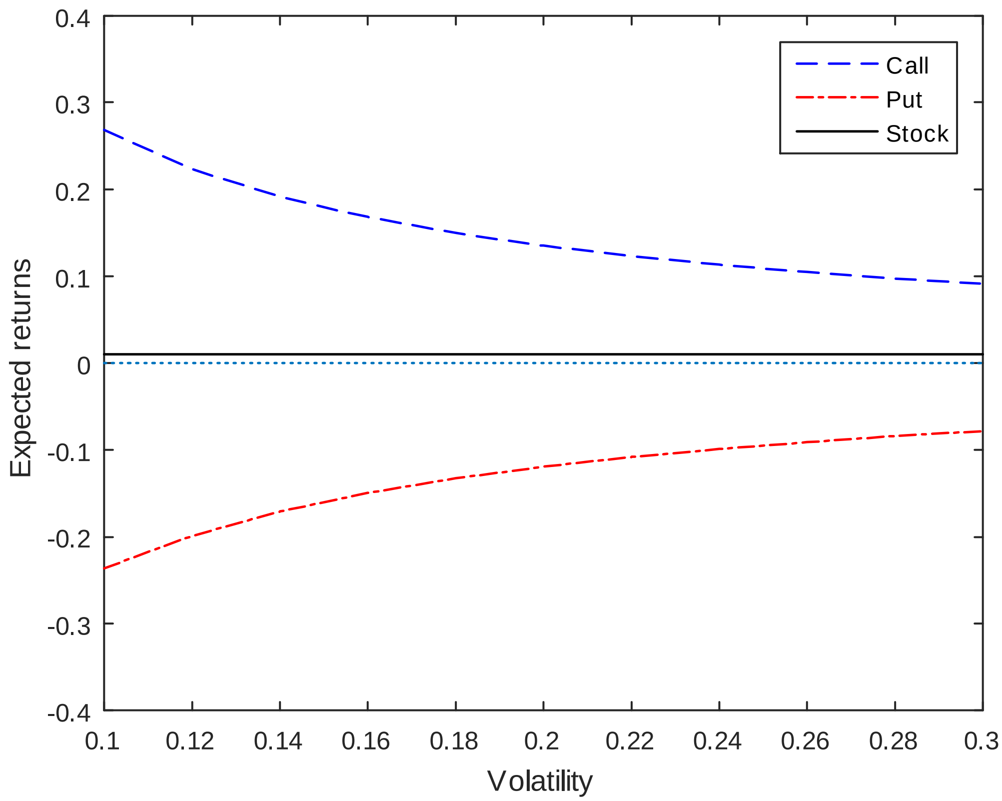
Coval and Shumway (2001) firmly integrate the study of expected option returns into mainstream asset pricing theory. They find that index option data confirm the theoretically expected relation between moneyness and expected returns on puts and calls. More recently, the empirical literature on the cross-section of equity option returns has been expanding rapidly, along with increasing liquidity and data availability. For example, Boyer and Vorkink (2014) investigate the relation between skewness and option returns. Goodman, Neamtiu, and Zhang (2013) find that fundamental accounting information is related to future option returns. A related literature documents the impact of inventory and order flow on option returns (see Muravyev, 2015 and the references therein).

Several papers control for volatility when investigating determinants of the cross-section of option returns, but the use of volatility as a control variable is usually motivated by discussing the relation between volatility and option *prices*. One argument considers the effect of an unexpected (future) change in volatility. This increases the future option price and therefore returns. From the perspective of asset pricing theory, which emphasizes the relation between ex ante risk and expected return, this argument is incomplete. An alternative argument considers a contemporaneous increase or shock in volatility. This increases the current price of the option, which leads to the hypothesis that volatility and returns are negatively related. This argument applies to a transitory shock to volatility, which ignores that changes in the volatility level may affect the future option payoff. We conclude that when discussing volatility and option returns, several arguments are used in the empirical literature that may refer to current or future volatility, to anticipated or unanticipated changes in volatility, and to the impact of volatility on current or future prices.

This paper attempts to contribute to this literature by explicitly considering the ex ante relation between volatility and expected option returns. This integrates the analysis of volatility as a determinant of expected option returns into mainstream asset pricing theory, following Coval and Shumway's (2001) analysis of moneyness. We analytically study the relation between volatility and discrete holding period returns, and empirically investigate this theoretical prediction. Building on the work of Rubinstein (1984) and Broadie, Chernov, and Johannes (2009), we first use analytical expressions for expected holding period option returns in the context of the Black-Scholes-Merton framework. The expected return on holding a call option is a decreasing function of the underlying volatility, while the expected return on holding a put option is an increasing function of the underlying volatility.

Our results can easily be understood in terms of leverage, consistent with the intuition in Coval and Shumway (2001), who analyze index call and put returns as a function of the leverage due to moneyness. The leverage embedded in an option is a function of moneyness, maturity, and volatility. Figure 1-1 plots expected returns as a function of volatility for an ATM option with one month maturity. Expected call returns are positive and expected put returns are negative, following the arguments in Coval and Shumway (2001). For both calls and puts, the absolute value of returns is higher for the low-volatility options. This reflects leverage: low volatility options are cheaper and therefore constitute a more leveraged position.

We provide several extensions of the benchmark analysis. The empirical shortcomings of the Black-Scholes-Merton model are well-documented, and we therefore investigate if realistic extensions of the Black-Scholes-Merton model lead to different theoretical predictions. We use realistic parameterizations of the Heston (1993) model to show that if volatility is time-varying and if the innovations to volatility and returns are correlated, similar predictions obtain.



We plot expected monthly returns on a call option, a put option and the underlying stock in the Black-Scholes-Merton model. We set the expected annual return on the stock μ equal to 10% and the risk-free rate r equal to 3%. Options are at-the-money (ATM) and have a maturity of 1 month.

Figure 1-1: Option Leverage as a Function of Volatility in the Black-Scholes-Merton Model

We provide cross-sectional and time-series tests of this theoretical relation between stock volatility and expected option returns. Using the cross-section of stock option returns for 1996-2013, we document that call (put) option portfolio returns exhibit a strong negative (positive) relation with underlying stock volatilities. Sorting available one-month at-the-money options into quintiles, we find a statistically significant difference of -13.8% (7.1%) per month between the average returns of the call (put) option portfolio with the highest underlying stock volatilities and the call (put) portfolio with the lowest underlying volatilities. We demonstrate that these findings are not driven by cross-sectional variation in expected stock returns. Our results are robust to using different option maturities and moneyness, alternative measures of underlying volatility and portfolio weighting methods, and relevant control variables.

We also provide time-series evidence. We find that index call (put) options tend to have lower (higher) returns in the month following high volatility periods. The findings are robust to different index volatility proxies and are not driven by illiquid option contracts. The time-series results complement our cross-sectional findings and provide empirical support for our theoretical predictions.

To the best of our knowledge the cross-sectional relation between option returns and volatility has not been documented in the empirical asset pricing literature, but some existing studies contain related results. Galai and Masulis (1976) and Johnson (2004) study very different empirical questions related to capital structure and earnings forecasts respectively, but their analytical results are related, exploiting the relation between volatility and *instantaneous* expected option returns. Galai and Masulis (1976) argue that, under the joint assumption of the CAPM and the Black-Scholes-Merton model, the expected instantaneous rate of return on firm equity, which is a call option on firm value, decreases with the variance of the rate of return on firm value under certain (realistic) additional restrictions. Johnson (2004) points out that in a levered firm, the instantaneous expected equity return decreases as a function of idiosyncratic asset risk. He uses this insight to explain the puzzling negative relation between stock returns and the dispersion of analysts' earnings forecasts. We show that in a Black-Scholes-Merton setup, the negative (positive) relation between expected call (put) option return and underlying volatility can be generalized to empirically observable holding periods, and we provide empirical evidence consistent with these theoretical predictions. It is well known (see for instance Broadie, Chernov, and Johannes, 2009) that results for instantaneous returns may not generalize to empirically observable holding periods, because the option price is a convex function of the price of underlying security. Our focus on holding period returns instead of instantaneous returns facilitates the interpretation of the empirical results. It also has certain analytical advantages, which become apparent when we analyze stochastic volatility models.

In other related work, Lyle (2014) explores the implications of the negative relation between expected call option returns and underlying volatility to study the relation between information quality and future option and stock returns. Broadie, Chernov, and Johannes (2009) use simulations to show that expected put option returns increase with underlying volatility.

Finally, recent empirical work on equity options has documented several interesting patterns in the cross-section of option returns that are related to the volatility of the underlying securities. Goyal and Saretto (2009) show that straddle returns and delta-hedged call option returns increase as a function of the volatility risk premium, the difference between historical volatility and implied volatility. Vasquez (2012) reports a positive relation between the slope of the implied volatility term structure and future option returns. Cao and Han (2013) document a negative relation between the underlying stock's idiosyncratic volatility and delta-hedged equity option returns. Duarte and Jones (2007) analyze the relation between delta-hedged equity option returns and volatility betas. These studies all focus on volatility, but they analyze its impact on delta-hedged returns and straddles. Under the null hypothesis that the Black-Scholes-Merton model is correctly specified, volatility should not affect delta-hedged returns in these studies, and therefore the main focus of these papers is by definition on the sources of model misspecification. The objective of our paper is instead to analyze the theoretical and empirical relation between volatility and raw option returns, and to integrate this analysis into the mainstream asset pricing literature. Volatility is one of the main determinants of option prices and returns. Given that the theoretical predicted relations are validated by the data, our work suggests that empirical work on option returns may want to control for the effect of volatility when identifying other determinants of option returns.

The paper proceeds as follows. Section 2 provides the analytical results on the relation between expected option returns and underlying stock volatility in the Black-Scholes-Merton model. Section 3 discusses the data. Section 4 presents our main empirical results, using data on the cross-section of stock option returns. Section 4 also presents results on straddles and investigates expected returns in a stochastic volatility model. Section 5 performs an extensive set of robustness checks. Section 6 discusses several extensions as well as related results. Section 7 presents time-series tests using index options, and Section 8 concludes the paper.

1.2 Volatility and Expected Option Returns

In this section, we derive the analytical results on the relation between option returns and the volatility of the underlying security. We first derive these results in the context of the Black-Scholes-Merton model (Black and Scholes, 1973; Merton, 1973), even though it is well known that the Black-Scholes-Merton model has some empirical shortcomings. Most importantly, more accurate valuation of options is possible by accommodating stochastic volatility as well as jumps in returns and volatility.[1] However, the Black-Scholes-Merton model has the important advantage of analytical tractability, and we therefore use it to derive a benchmark set of theoretical results. In Section 1.4.4, we investigate if these results continue to hold if other, more realistic, processes are assumed for the underlying securities.

Much of the literature on option returns uses expected instantaneous option returns. In

[1] For studies of option pricing with stochastic volatility and jumps, see for instance Bates (1996), Bakshi, Cao, and Chen (1997), Chernov and Ghysels (2000), Eraker (2004), Jones (2003), Pan (2002), and Broadie, Chernov, and Johannes (2007).

the Black-Scholes-Merton model, consider the following notation for the geometric Brownian motion dynamic of the underlying asset:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB_t \tag{1.1}$$

where S_t is the price of underlying asset at time t, σ is the volatility parameter, and μ is the drift or the expected return of the underlying asset. It can be shown that in this model, the expected instantaneous option return is linear in the expected instantaneous return on the underlying asset:

$$E(\frac{dO_t}{O_t}) = rdt + \frac{\partial O_t}{\partial S_t}\frac{S_t}{O_t}(\mu - r)dt \tag{1.2}$$

where O_t is the price of the European option, and r is the risk-free rate. This expression provides some valuable intuition regarding the determinants of expected option returns. The expected option return depends on $\frac{\partial O_t}{\partial S_t}\frac{S_t}{O_t}$, which reflects the leverage embedded in the option. The leverage itself is a function of moneyness, maturity, and the volatility of the underlying security.

Equation (1.2) provides valuable intuition on option returns, but it has some important drawbacks and limitations. Some of these drawbacks follow from the fact that for empirically observable holding periods, the linear relation between the option returns and the underlying asset returns may not hold because the option price is a convex function of the price of underlying asset. For more complex stochastic volatility models, these drawbacks are more severe, see Broadie, Chernov, and Johannes (2009). We analyze stochastic volatility models in Section 1.4.4.

These drawbacks also surface when analyzing the relation between volatility and expected option returns. We can use (1.2) to compute the derivative of expected returns with respect to volatility σ. Galai and Masulis (1976), in their analysis of the optionality of leveraged equity, characterize sufficient conditions for this derivative to be negative for a call option when the underlying dynamic is given by (1.1). Johnson (2004), using a similar setup, notes that the derivative is always negative for call options. Because these statements are somewhat contradictory, and also because this result does not seem to be sufficiently appreciated in the literature, we include it in Appendix A.

We now investigate if this result holds for empirically observable holding periods, where we have to account for the fact that the option price is a convex function of the price of underlying security. We analyze the impact of underlying volatility on expected option returns by building on the work of Rubinstein (1984) and Broadie, Chernov, and Johannes (2009), who point out that expected option returns can be computed analytically within models that allow for analytic expressions for option prices. For our benchmark results, we rely on the classical Black-Scholes-Merton option pricing model to obtain an analytical expression for the expected return of holding an option to maturity. We then compute the first derivative of the expected option return with respect to the volatility of the underlying security. We show that the expected return for holding a call option to maturity is a decreasing function of the underlying volatility, while the expected return for holding a put option to maturity is an increasing function of underlying volatility.

Denote the time t prices of European call and put options with strike price K and maturity T by $C_t(t, T, S_t, \sigma, K, r)$ and $P_t(t, T, S_t, \sigma, K, r)$ respectively. By definition, the expected gross returns for holding the options to expiration are given by:

$$R_{call} = \frac{E_t[\max(S_T - K, 0)]}{C_t(t, T, S_t, \sigma, K, r)} \tag{1.3}$$

$$R_{put} = \frac{E_t[\max(K - S_T, 0)]}{P_t(t, T, S_t, \sigma, K, r)}. \tag{1.4}$$

Propositions 1 and 2 indicate how these expected call and put option returns change with respect to the underlying volatility σ. We provide the detailed proof for the case of the call option in Proposition 1, because the proof provides valuable intuition for the result. The intuition for the case of the put option is similar and the proof is relegated to the appendix.

Proposition 1 *Everything else equal, the expected return of holding a call option to expiration is higher if the underlying asset has lower volatility* ($\frac{\partial R_{call}}{\partial \sigma} < 0$).

Proof. We start by reviewing several well-known facts that are needed to derive the main result. If the underlying asset follows a geometric Brownian motion, the price of a European call option with maturity $\tau = T - t$ written on the asset is given by the Black-Scholes-Merton formula:

$$
\begin{aligned}
C_t(t, T, S_t, \sigma, K, r) &= S_t N(d_1) - e^{-r\tau} K N(d_2) && (1.5)\\
d_1 &= \frac{\ln \frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \\
d_2 &= \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}. && (1.6)
\end{aligned}
$$

Vega is the first-order derivative of the option price with respect to the underlying volatility. It measures the sensitivity of the option price to small changes in the underlying volatility. The Black-Scholes-Merton Vega is the same for call and put options:

$$\nu = \sqrt{\tau} S_t \psi(d_1) \tag{1.7}$$

where ψ is the probability density function of the standard normal distribution. We also have:

$$S_t \psi(d_1) = e^{-r\tau} K \psi(d_2). \tag{1.8}$$

We first write the expected call option return in (1.3) in a convenient way. This allows us to conveniently evaluate the derivative of the expected option return with respect to the underlying volatility, using the Black-Scholes-Merton Vega in (1.7).

The denominator of (1.3) is the price of the call option and is therefore given by the Black-Scholes-Merton formula in (1.5). The numerator of (1.3), the expected option payoff at expiration, can be transformed into an expression that has the same functional form as the Black-Scholes-Merton formula. We get:

$$
\begin{aligned}
E_t[\max(S_T - K, 0)] &= \int_{z^*} (S_t e^{\mu\tau - \frac{1}{2}\sigma^2\tau + \sigma\sqrt{\tau}z} - K) \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz && (1.9)\\
&= e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] && (1.10)
\end{aligned}
$$

where

$$z^* = \frac{\ln \frac{K}{S_t} - (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_1^* = \frac{\ln \frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln \frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}. \tag{1.11}$$

Combining (1.5) and (1.10), the expected return for holding a European call option to maturity is given by:

$$R_{call} = \frac{E_t[\max(S_T - K, 0)]}{C_t(t, T, S_t, \sigma, K, r)} = \frac{e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}. \tag{1.12}$$

Taking the derivative of (1.12) with respect to σ gives:

$$\begin{aligned}\frac{\partial R_{call}}{\partial \sigma} &= \frac{e^{\mu\tau}\sqrt{\tau}S_t\psi(d_1^*)[S_tN(d_1)-e^{-r\tau}KN(d_2)]-e^{\mu\tau}[S_tN(d_1^*)-e^{-\mu\tau}KN(d_2^*)]\sqrt{\tau}S_t\psi(d_1)}{[S_tN(d_1)-e^{-r\tau}KN(d_2)]^2} \\ &= \frac{e^{\mu\tau}\sqrt{\tau}S_t\{\psi(d_1^*)[S_tN(d_1)-e^{-r\tau}KN(d_2)]-\psi(d_1)[S_tN(d_1^*)-e^{-\mu\tau}KN(d_2^*)]\}}{[S_tN(d_1)-e^{-r\tau}KN(d_2)]^2}. \quad (1.13)\end{aligned}$$

Note that we use equation (1.7) to derive (1.13). From (1.13) it can be seen that $\frac{\partial R_{call}}{\partial \sigma}$ inherits the sign of $EX = \psi(d_1^*)[S_tN(d_1)-e^{-r\tau}KN(d_2)]-\psi(d_1)[S_tN(d_1^*)-e^{-\mu\tau}KN(d_2^*)]$. We now show that EX is negative. We have:

$$\frac{1}{\psi(d_1^*)\psi(d_1)}EX = \frac{S_tN(d_1)-e^{-r\tau}KN(d_2)}{\psi(d_1)} - \frac{S_tN(d_1^*)-e^{-\mu\tau}KN(d_2^*)}{\psi(d_1^*)}. \quad (1.14)$$

Using equation (1.8), it follows that

$$\begin{aligned}\frac{1}{\psi(d_1^*)\psi(d_1)}EX &= \frac{S_tN(d_1)-\frac{S_t\psi(d_1)}{\psi(d_2)}N(d_2)}{\psi(d_1)} - \frac{S_tN(d_1^*)-\frac{S_t\psi(d_1^*)}{\psi(d_2^*)}N(d_2^*)}{\psi(d_1^*)} \quad (1.15) \\ &= S_t[(\frac{N(d_1)}{\psi(d_1)}-\frac{N(d_2)}{\psi(d_2)})-(\frac{N(d_1^*)}{\psi(d_1^*)}-\frac{N(d_2^*)}{\psi(d_2^*)})]. \quad (1.16)\end{aligned}$$

According to economic theory, the expected rate of return on risky assets must exceed the risk-free rate ($\mu > r$). We therefore have $d_1^* > d_1$ and $d_2^* > d_2$. We also have $d_1^* - d_2^* = d_1 - d_2$ as well as $d_1^* > d_2^*$ and $d_1 > d_2$, from the definition of (1.6) and (1.11). Now consider $\frac{N(d)}{\psi(d)}$. It can be shown that it is an increasing and convex function of d. Evaluating $\frac{N(d)}{\psi(d)}$ at d_1, d_2, d_1^*, and d_2^*, it can be seen that the expression $(\frac{N(d_1)}{\psi(d_1)}-\frac{N(d_2)}{\psi(d_2)})-(\frac{N(d_1^*)}{\psi(d_1^*)}-\frac{N(d_2^*)}{\psi(d_2^*)})$ effectively amounts to the negative of the second difference (derivative) of an increasing and convex function. Therefore:

$$(\frac{N(d_1)}{\psi(d_1)}-\frac{N(d_2)}{\psi(d_2)})-(\frac{N(d_1^*)}{\psi(d_1^*)}-\frac{N(d_2^*)}{\psi(d_2^*)}) < 0. \quad (1.17)$$

This implies $EX < 0$ which in turn implies $\frac{\partial R_{call}}{\partial \sigma} < 0$. ■

Proposition 2 *Everything else equal, the expected return of holding a put option to expiration is higher if the underlying asset has higher volatility (*$\frac{\partial R_{put}}{\partial \sigma} > 0$*).*

Proof. See Appendix B. ■

There is a subtle but important difference compared to the proof for instantaneous returns. In the instantaneous case, one exploits the fact that $\frac{N(x)}{\psi(x)}$ is an increasing function in x. In contrast, the finite-period derivation relies on the fact that $\frac{N(x)}{\psi(x)}$ is not only an increasing but also a convex function in x. This is required because any finite holding period option return is a nonlinear function of μ, whereas the instantaneous return is a linear function of μ.

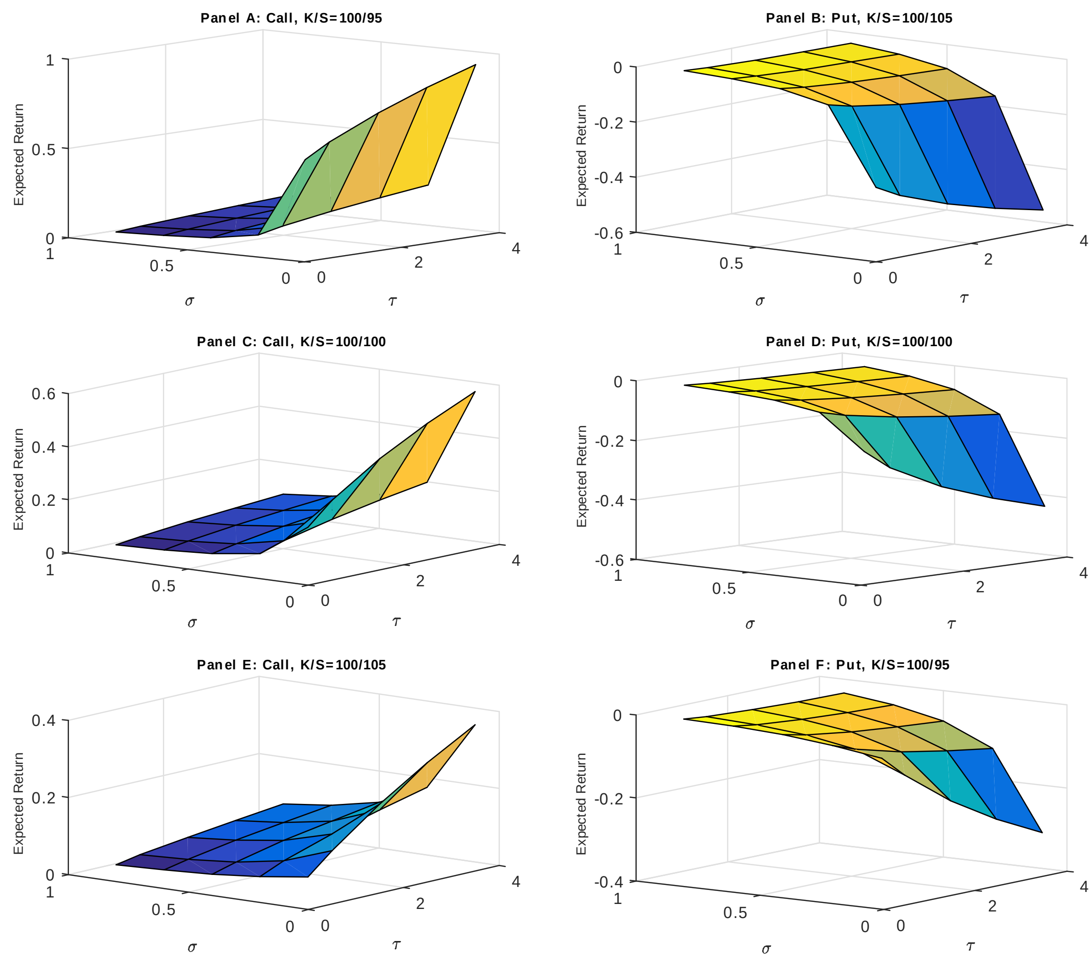
These results can be extended to compute expected option returns over any holding period h in the Black-Scholes-Merton model. Following Rubinstein (1984), the expected return on a call option is given by

$$\begin{aligned} R^h_{call} &= \frac{e^{\mu h}[S_0 N(d_1^*) - e^{-[r+(\mu-r)HP]T} K N(d_2^*)]}{S_0 N(d_1) - e^{-rT} K N(d_2)} \\ d_1^* &= \frac{\ln \frac{S_0}{K} + [HP(\mu - r) + r + \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \\ d_2^* &= \frac{\ln \frac{S_0}{K} + [HP(\mu - r) + r - \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \end{aligned} \tag{1.18}$$

where the timeline is shifted to $[0, T]$ from $[t, T]$ to ease notation, h is the holding period ($0 < h < T$), and $HP = h/T$ is the ratio of the holding period to the life of the option contract. Details are provided in Appendix C. Note that the expected holding-to-expiration option return in (1.12) is nested in (1.18), for $HP = 1$. We can use the structure of the proof of Proposition 1 to show $\frac{\partial R^h_{call}}{\partial \sigma} < 0$, by observing $r + (\mu - r)HP > r$. Thus, we conclude that expected call (put) option returns decrease (increase) with underlying volatility for any holding period in the Black-Scholes-Merton model.

Figure 1-2 graphically illustrates these results for a realistic calibration of the Black-Scholes-Merton model. We set $\mu = 10\%$ and $r = 3\%$. We present results for out-of-the-money, at-the-money, and in-the-money options. The left-side panels are for calls and the right-side panels are for puts. Figure 1-2 clearly illustrates the qualitative results in Propositions 1 and 2. We discuss the quantitative implications in more detail in Section 1.6.2.

The patterns in expected returns suggest a simple interpretation of our results in terms of leverage, consistent with the intuition in Coval and Shumway (2001). As mentioned before, in equation (1.2), $\frac{\partial O_t}{\partial S_t}\frac{S_t}{O_t}$ reflects the leverage embedded in the option, which is a function of moneyness, maturity, and volatility. Coval and Shumway (2001) analyze index call and put



Notes: We plot expected option returns in the Black-Scholes-Merton model against volatility (σ) and time-to-maturity (τ). In all computations, we set the expected return on the stock μ equal to 10% and the risk-free rate r equal to 3%. We set the strike price (K) equal to 100 and the stock price (S) equal to either 95, 100 or 105. Returns are reported as raw returns for the relevant horizons.

Figure 1-2: Expected Option Returns in the Black-Scholes-Merton Model returns as a function of the leverage due to moneyness. Figure 1-1 plots expected returns as a function of volatility for an ATM option with one month maturity. The expected return on the stock is $\mu = 10\%$. Expected call returns are positive and expected put returns are negative, following the arguments in Coval and Shumway (2001). For both calls and puts, the absolute value of returns is higher for the low-volatility options. This reflects leverage: low volatility options are cheaper and therefore constitute a more leveraged position.[2] Note that in the limit, as volatility goes to infinity, the expected call return approaches the stock return and the expected put return approaches the riskfree rate.

1.3 Data

We conduct two empirical exercises, one using the cross-section of equity option returns, and another one using the time series of index option returns. Here we discuss the two datasets used in these exercises. The sample period is from January 1996 to July 2013 for both datasets.

1.3.1 Equity Option Data

The main objective of our empirical exercise is to test Propositions 1 and 2 using the cross-section of options written on individual stocks. Propositions 1 and 2 predict a relation between expected option returns and underlying volatility, everything else equal. When studying the relation between option returns and the underlying volatility, it is therefore critical to control for other option characteristics that affect returns. Existing studies have documented that moneyness and maturity also affect option returns, see for example Coval and Shumway (2001).

To address this issue, we use option samples that are homogeneous in maturity and moneyness. For our benchmark empirical analysis, we use the cross-section of stock options

[2] Higher volatility options are sometimes incorrectly thought of as more leveraged, presumably capturing the relation between unanticipated changes in volatility and higher prices. This argument ignores the cost of the option position.

that are at-the-money and one month away from expiration, because these are the most frequently traded options, and they are subject to fewer data problems (see, among others, Goyal and Saretto, 2009). In subsequent robustness exercises, we use options with different maturities and moneyness.

We obtain stock return data from CRSP and relevant accounting information from Compustat. We obtain option data from OptionMetrics through WRDS. OptionMetrics provides historical option closing bid and ask quotes, as well as information on the underlying securities for U.S. listed index options and equity options. Every month, on the first trading day after monthly option expiration, we select equity options with $0.95 \leq K/S \leq 1.05$ that expire over the next month.[3] The expiration day for standard exchange-traded options is the Saturday immediately following the third Friday of the expiration month, so our sample consists mainly of Mondays. If Monday is an exchange holiday, we use Tuesday data.

We apply several standard filters to the option data. An option is included in the sample if it meets all of the following requirements: 1) The best bid price is positive and the best bid price is smaller than the best offer price; 2) The price does not violate no-arbitrage bounds. For call options we require that the price of the underlying exceeds the best offer, which is in turn higher than $max(0, S - K)$. For put options we require that the exercise price exceeds the best bid, which is in turn higher than $max(0, K - S)$; 3) No dividend is paid over the duration of the option contract; 4) Open interest is positive; 5) Volume is positive; 6) The bid-ask spread is higher than the minimum tick size, which is equal to \$0.05 when the option price is below \$3, and \$0.10 when the option price is higher than \$3; 7) The expiration day is standard, the Saturday following the third Friday of the month; 8) Settlement is standard; 9) Implied volatility is not missing.

We compute the monthly return from holding the option to expiration using the midpoint of the bid and ask quotes as a proxy for the market price of the option contract. If an option expires in the money, the return to holding the option to maturity is the difference between the terminal payoff and the initial option price divided by the option price. If an

[3]We obtain similar results when we use options collected on the first trading day of each month.

option expires out of the money, the option return is -100%. Our equity option sample contains 247,859 call options and 188,046 put options over the time period from January 1996 to July 2013.[4]

In our benchmark results, we measure volatility using realized volatility computed using daily data for the preceding month, and we refer to this as 30-day realized volatility.[5] In the robustness analysis we use realized volatility over different horizons, which is also computed using daily data over the relevant horizon.

Table 1.1 reports summary statistics for equity options across moneyness categories. Moneyness is defined as the strike price over the underlying stock price. On average the returns to buying call (put) options are positive (negative). Put option returns increase with the strike price. Call returns increase for the first four quintiles but decrease for the fifth.[6] Also note that option-implied volatility exceeds realized volatility for all moneyness categories, but the differences are often small. Gamma and Vega are highest for at-the-money options and decrease as options move away from the money.

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[4] Stock options are American. We do not fully address the complex issue of early exercise, but attempt to reduce its impact by only including options that do not have an ex-dividend date during the life of the option contract. This of course does not address early exercise of put options (Barraclough and Whaley, 2012). However, several studies (see among others Broadie, Chernov, and Johannes, 2007; Boyer and Vorkink, 2014) argue that adjusting for early exercise has minimal empirical implications. See also the discussion in Goyal and Saretto (2009).

[5] Because this measure uses data for the previous month, it is effectively based on approximately 22 returns. For convenience, we refer to it as 30-day volatility. The same remark applies to volatility measures for other horizons used throughout the paper.

[6] We verified that returns for further out-of-the money calls continue to decrease. Returns for calls are therefore non-monotonic as a function of moneyness, consistent with the results for index returns in Bakshi, Madan, and Panayotov (2010).

Table 1.1: Summary Statistics for Equity Options

Moneyness K/S	[0.95 – 0.97]	(0.97 – 0.99]	(0.99 – 1.01]	(1.01 – 1.03]	(1.03 – 1.05]
Panel A: Call Options					
Return	0.054	0.080	0.111	0.119	0.100
30-day realized vol	47.06%	45.57%	44.70%	44.23%	44.97%
Implied vol	49.03%	46.94%	45.49%	44.90%	45.44%
Volume	232	306	385	430	396
Open interest	1846	1855	1798	1897	1885
Delta	0.68	0.61	0.53	0.45	0.38
Gamma	0.11	0.12	0.14	0.13	0.12
Vega	4.41	4.81	4.95	4.89	4.52
Panel B: Put Options					
Return	-0.137	-0.121	-0.100	-0.104	-0.087
30-day realized vol	45.86%	44.88%	45.51%	46.19%	47.62%
Implied vol	48.97%	47.29%	47.01%	47.24%	48.25%
Volume	318	359	340	278	207
Open interest	1875	1841	1672	1670	1563
Delta	-0.33	-0.39	-0.47	-0.55	-0.61
Gamma	0.10	0.11	0.13	0.12	0.11
Vega	4.69	5.15	5.27	5.25	4.87

Notes to Table: We report averages by moneyness category of monthly equity option returns (return), the underlying stock's realized volatility over the preceding month (30-day realized vol), option implied volatility (implied vol), option volume (volume) and the option Greeks. Panel A reports on call options and Panel B on put options. We compute monthly option returns using the midpoint of bid and ask quotes. Realized volatility is calculated as the standard deviation of the logarithm of daily returns over the preceding month. The sample consists of options that are at-the-money ($0.95 \leq K/S \leq 1.05$) and approximately one month from expiration. The sample period is from January 1996 to July 2013.

1.3.2 Index Option Data

We also investigate the relation between volatility and expected returns using the time series of index option returns. On the first trading day after each month's option expiration date, we collect index options with moneyness $0.9 \leq K/S \leq 1.1$ that mature in the next month. Table 1.2 provides summary statistics for SPX option data by moneyness. Index put options (especially out-of-the-money puts) generate large negative returns, consistent with the existing literature (see for example Bondarenko, 2003). For example, for the moneyness interval $0.94 < K/S \leq 0.98$, the average return is -40.6% per month. Table 1.2 also shows that in our sample, out-of-the-money SPX calls have large negative returns. This is consistent with the results in Bakshi, Madan, and Panayotov (2010).

Comparing Tables 1.2 and 1.1 highlights several important differences between index options and individual stock options. First, the volatility skew, the slope of implied volatility against moneyness, is much less pronounced for individual stock options. Second, the average realized volatility for index options is approximately 17%, and therefore the volatility risk premium for index options exceeds the volatility risk premium for stock options. This is consistent with existing findings, but note that the index variance risk premium in our paper is smaller than many existing findings due to our sample period.

1.4 Volatility and the Cross-Section of Option Returns: Empirical Results

In this section, we empirically test Propositions 1 and 2 using the cross-section of options written on individual stocks. First, we present our benchmark cross-sectional results. As mentioned before, for our benchmark empirical analysis, we use the cross-section of stock options that are at-the-money and one month away from expiration to control for option characteristics other than volatility that affect returns. Subsequently, we conduct a series of tests to control for the expected returns on the underlying stocks. We also discuss the relation between volatility and straddle returns. Finally, the Black-Scholes-Merton model

Table 1.2: Summary Statistics for S&P 500 Index Options

Moneyness K/S	$[0.90-0.94]$	$(0.94-0.98]$	$(0.98-1.02]$	$(1.02-1.06]$	$(1.06-1.10]$
Panel A: SPX Call Options					
Return	0.027	0.057	0.060	-0.112	-0.617
Implied vol	27.30%	22.75%	19.68%	17.42%	17.28%
Volume	251	306	2029	2867	2156
Open interest	9679	11770	15236	15388	14807
Delta	0.88	0.76	0.51	0.20	0.06
Gamma	0.002	0.005	0.007	0.005	0.002
Vega	60.32	93.12	119.86	80.66	32.99
Panel B: SPX Put Options					
Return	-0.540	-0.406	-0.224	-0.133	-0.171
Implied vol	26.56%	22.87%	19.66%	18.20%	22.68%
Volume	3699	2662	2619	391	338
Open interest	19604	18649	14674	8992	12322
Delta	-0.11	-0.23	-0.48	-0.75	-0.88
Gamma	0.002	0.005	0.007	0.006	0.003
Vega	55.13	90.56	119.80	93.61	53.04

Notes to Table: We report averages of monthly S&P 500 index option returns (return), implied volatility (implied vol), option volume (volume), and option Greeks by moneyness. Panel A reports on call options and Panel B reports on put options. We compute the monthly option return using the midpoint of the bid and ask quotes. The sample consists of S&P 500 index options (SPX) with moneyness $0.90 \leq K/S \leq 1.10$ and one-month maturity. The sample period is from January 1996 to July 2013.

has some well-known empirical shortcomings, and it is possible that adjusting the theoretical model for these empirical shortcomings may affect the results. The most important shortcoming is the constant volatility assumption. We therefore investigate if our findings are robust to the presence of stochastic return volatility.

1.4.1 The Cross-Section of Option Portfolio Returns

Each month, on each portfolio formation date, we sort options with moneyness $0.95 \leq K/S \leq 1.05$ into five quintile portfolios based on their realized volatility, and we compute equal-weighted returns for these option portfolios over the following month. We conduct this exercise for call and put options separately.

Panel A of Table 1.3 displays the averages of the resulting time series of returns for the five call option portfolios, as well as the return spread between the two extreme portfolios. Portfolio "Low" contains call options with the lowest realized volatility, and portfolio "High" contains call options with the highest realized volatility. Proposition 1 states that the expected call option return is a decreasing function of the underlying stock volatility. Consistent with this result, we find that call option portfolio returns decrease monotonically with the underlying stock volatility. The average returns for portfolio High and portfolio Low are 0.9% and 14.7% per month respectively. The resulting return difference between the two extreme portfolios (H-L) is −13.8% per month and highly statistically significant, with a Newey-West (1987) t-statistic of −3.42.[7]

Panel B of Table 1.3 presents the averages of the resulting time-series of returns for the five put option portfolios. Again, portfolio Low (High) contains put options with the lowest (highest) underlying stock volatilities. For put option portfolios, the average return increases from −14.6% per month for portfolio Low to −7.5% per month for portfolio High, with a positive and significant H-L return difference of 7.1%. This finding confirms Proposition 2, which states that expected put option returns are an increasing function of the underlying

[7] T-statistics computed using the i.i.d. bootstrap as in Bakshi, Madan, and Panayotov (2010) are very similar.

stock volatility.

Table 1.3 also provides results using only options with moneyness $0.975 \leq K/S \leq 1.025$. By using a tighter moneyness interval, we reduce the impact of moneyness on expected option returns. The results are very similar. The average option portfolio returns decrease (increase) with the underlying stock volatility for calls (puts). The H-L differences are -13.8% and 7.7% for call and put option portfolios respectively, and are statistically significant. This indicates that our empirical results are not due to differences in option moneyness.[8]

These results are obtained using option returns computed using the mid-point of the bid and ask quotes. To ensure that our results do not depend on this assumption, Panel C of Table 1.3 computes average option portfolio returns based on the ask price. As expected, average returns are somewhat smaller than in Panels A and B. However, we again find a strong negative (positive) relation between call (put) option portfolio returns and the underlying stock volatility. The H-L differences are both statistically significant and are of a similar order of magnitude as the ones reported in Panels A and B.

Figure 1-3 complements the average returns in Table 1.3 by plotting the cumulative returns on the long-short portfolios over time. Figure 1-3 indicates that the negative (positive) sign for the call (put) long-short returns is quite stable over time, although it of course does not obtain for every month in the sample.

1.4.2 Controlling for Expected Stock Returns

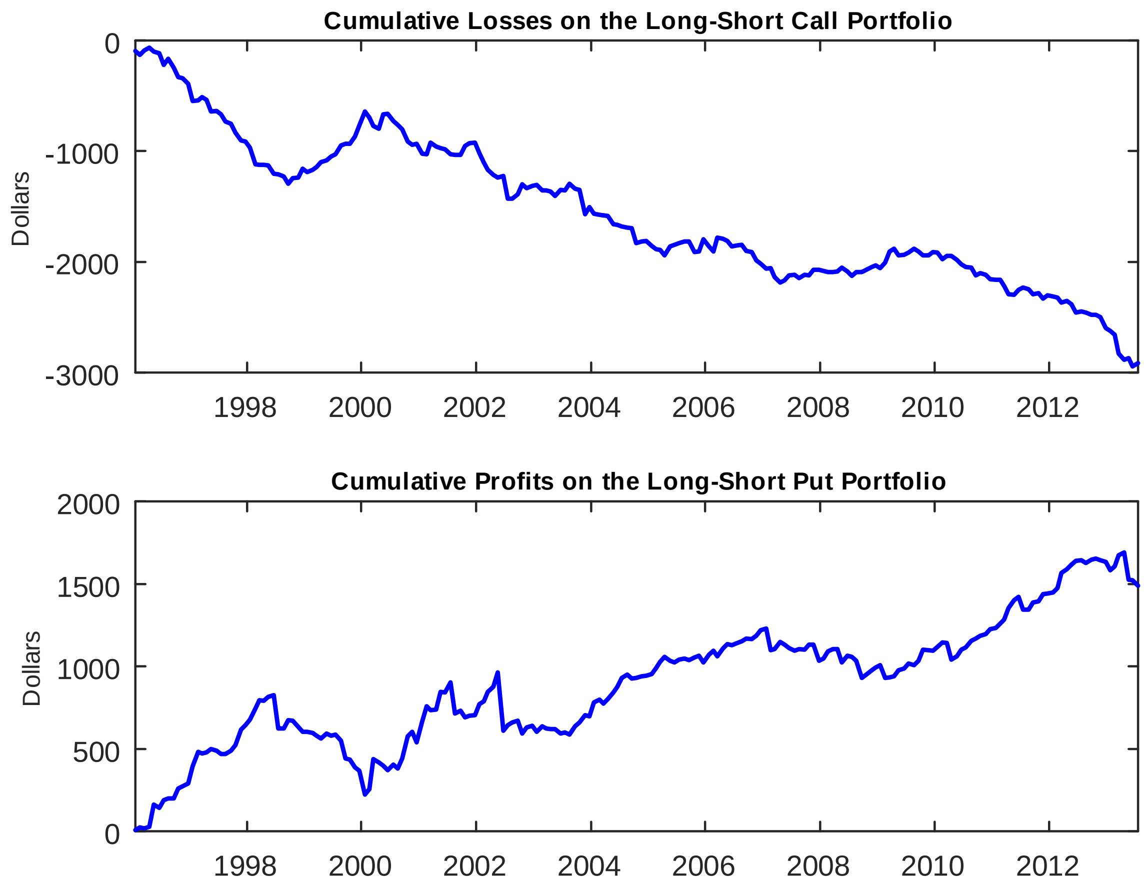
The empirical results in Table 1.3 document the relation between volatility and expected option returns. These results control for other well-known determinants of option returns such as moneyness and maturity. Now we attempt to control for other confounding factors

[8] We focus on patterns in expected returns as a function of volatility. We do not address the more complex question of the riskiness of these returns. Not surprisingly, the standard deviation of returns is lower for higher volatility quintiles, but these differences are small compared to differences in returns. The pattern in Sharpe ratios is therefore similar to the pattern in returns. However, Sharpe ratios are a poor measure of risk for option strategies.

Table 1.3: Option Portfolio Returns Sorted on Underlying Volatility

Panel A: Call Option Portfolios						
	Low	2	3	4	High	H-L
$0.95 \leq K/S \leq 1.05$	0.147	0.128	0.111	0.084	0.009	-0.138***
						(-3.422)
$0.975 \leq K/S \leq 1.025$	0.155	0.145	0.120	0.094	0.017	-0.138***
						(-3.496)
Panel B: Put Option Portfolios						
	Low	2	3	4	High	H-L
$0.95 \leq K/S \leq 1.05$	-0.146	-0.153	-0.109	-0.077	-0.075	0.071**
						(2.004)
$0.975 \leq K/S \leq 1.025$	-0.145	-0.157	-0.101	-0.065	-0.068	0.077**
						(2.081)
Panel C: Using Ask Prices						
	Low	2	3	4	High	H-L
Call Option Portfolios	0.048	0.045	0.033	0.012	-0.060	-0.108***
						(-2.942)
Put Option Portfolios	-0.209	-0.209	-0.165	-0.133	-0.133	0.076**
						(2.302)

Notes to Table: We report average equal-weighted monthly returns for option portfolios sorted on 30-day realized volatility, as well as the return differences between the two extreme portfolios. Panel A reports on call options and Panel B on put options. Panel C reports results for option returns based on ask prices rather than the midpoint of bid and ask quotes. Every month, all available one-month at-the-money options are sorted into five quintile portfolios according to their 30-day realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatilities. The sample period is from January 1996 to July 2013. Newey-West t-statistics using four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.



Notes: We plot the cumulative losses and profits from investing in the long-short portfolios documented in Table 3. We assume that an investor invests $100 in the long-short portfolio every month.

Figure 1-3: Cumulative Losses and Profits on the Long-Short Portfolios by analyzing the role of the drift μ in the law of motion for the underlying asset (1.1).

We derive Proposition 1 and 2 assuming a constant drift μ. If the drift is not constant, our empirical results in Table 1.3 may be due to patterns in the returns on the underlying stocks rather than to the mechanics of option returns studied in Propositions 1 and 2. We now discuss this in more detail. First consider a drift μ that depends on volatility. We know that $\frac{\partial R_{call}}{\partial \mu} > 0$ and $\frac{\partial R_{put}}{\partial \mu} < 0$ (see Appendix D for details). We therefore need to refer to the theoretical and empirical literature on the relation between volatility and expected stock returns. If the relation between stock returns and volatility is positive, it cannot explain the empirical relation documented in Table 1.3. If this relation is negative, on the other hand, we need to control for it in the empirical analysis.

Theory predicts a positive relation between stock returns and volatility, but the empirical time-series evidence is tenuous, perhaps because estimating expected returns from the time series of returns is notoriously difficult.[9] In the cross-sectional literature, Ang, Hodrick, Xing, and Zhang (2006) document a negative relation between volatility and stock returns. Their work has inspired a voluminous literature, and some studies find a positive or insignificant cross-sectional relation, but overall the literature confirms their findings.[10] We therefore need to re-visit our results while controlling for the expected return on the underlying security. It is of course well-known that controlling for the expected return on the underlying stock is difficult. To the extent that we are not able to do so, it is possible that the cross-sectional effect documented by Ang, Hodrick, Xing, and Zhang (2006) partly explains our results.

We now present empirical results that control for the expected return on the underlying

[9] See, among others, Nelson (1991), Campbell and Hentschel (1992), French, Schwert, and Stambaugh (1987), Glosten, Jagannathan and Runkle (1993), Goyal and Santa-Clara (2003), Ghysels, Santa-Clara and Valkanov (2005), Bali et al. (2005), and Bali (2008).

[10] See, among many others, Adrian and Rosenberg (2008), Ang, Hodrick, Xing, and Zhang (2009), Bali and Cakici (2008), Chen and Petkova (2012), Fu (2009), Huang, Liu, Rhee, and Zhang (2009), and Stambaugh, Yu, and Yuan (2015).

security in various ways. First we present results for double sorts on volatility and average historical stock return. Second, we specify a single-factor market model for the underlying security and control for the underlying stock's exposure to the market. Third, we use Fama-MacBeth regressions to control for a wide variety of determinants of expected stock returns. Fourth, we use the option pricing model to control for the empirical differences in stock returns between quintiles.

Controlling for Expected Stock Returns Using Historical Averages

Expected call (put) option returns increase (decrease) with the expected return on the underlying asset. If the high volatility portfolios in Table 1.3 are primarily composed of stocks that have lower expected returns than those in the low volatility portfolios, the result that average call (put) options in the high volatility portfolios earn lower (higher) returns may not be due to volatility. We therefore start by documenting if the underlying stock returns affect our results by empirically controlling for expected stock returns. This is of course challenging because unlike volatility, expected stock returns are notoriously difficult to measure.

Our first approach follows Boyer and Vorkink (2014), who estimate expected stock returns as the simple average of daily returns over the past six months. Each month we first form five quintile portfolios based on estimated expected stock returns μ, and then within each μ quintile options are further sorted into five quintile portfolios according to underlying stock volatility. We once again measure underlying stock volatility by 30-day realized volatility.

Table 1.4 presents the results of this double sort. The columns correspond to different volatility levels, and the rows correspond to different average returns. Consistent with the single sort results, in each μ quintile call (put) option portfolio returns decrease (increase) with underlying volatility. In all μ quintiles, the average return differences between the two extreme call option portfolios are negative, ranging from -24% to -11% per month, and highly significant. For put options, the high minus low differences are all positive and statistically significant in four out of five μ quintiles. These findings suggest that our results are not driven by differences between the expected returns of the underlying stocks.

A Single-Factor Market Model

Estimates of expected returns from historical averages as in Section 1.4.2 are notoriously imprecise. Our next approach controls for expected returns using the simple market or index model rather than the historical average. Panel A of Table 1.5 presents results for a double sort on market beta and volatility. Beta is estimated by the market model over the most recent 30 days preceding the portfolio formation date. The results are similar to those in Table 1.4, where we control for the expected return using lagged average returns, but the t-statistics are somewhat smaller. Average call option returns decrease with volatility for each beta quintile and the return spread between the two extreme portfolios is statistically significant across all beta quintiles. In contrast, average put option returns increase with volatility for each beta quintile and the return spread is significant for the top three beta quintiles.

Panel B of Table 1.5 uses the results from the market model in a slightly different way. We present results for sorts on idiosyncratic volatility based on the market model. Panel B indicates a negative relation between call option portfolio returns and idiosyncratic volatility, and a positive relation between put option portfolio returns and idiosyncratic volatility.[11] We obtain similar results when sorting on idiosyncratic volatility computed relative to the Fama-French three-factor model.

Fama-MacBeth Regressions

To control as comprehensively as possible for the impact of the drift of the underlying assets on option returns, we run Fama-MacBeth (1973) regressions that allow us to simul-

[11] Given the additional assumption of the market model, the results in Propositions 1 and 2 effectively establish a relation between option returns and idiosyncratic volatility. This interpretation is more in line with Johnson's (2004) analysis of the role of volatility in returns on levered equity.

Table 1.4: Option Portfolio Returns Double-Sorted on Expected Stock Return and Underlying Volatility

Panel A: Call Options		Low	3	3	4	High	H-L
	1	0.246	0.151	0.075	0.033	0.001	-0.245***
							(-5.889)
	2	0.190	0.148	0.117	0.085	-0.006	-0.195***
μ Quintiles							(-3.628)
	3	0.146	0.170	0.125	0.082	0.021	-0.125***
							(-2.769)
	4	0.131	0.122	0.136	0.094	0.018	-0.112***
							(-2.854)
	5	0.154	0.106	0.101	0.066	0.038	-0.116***
							(-2.823)
Panel B: Put Options		Low	2	3	4	High	H-L
	1	-0.113	-0.079	-0.067	-0.028	-0.044	0.069*
							(1.799)
	2	-0.162	-0.136	-0.117	-0.102	-0.056	0.107**
							(2.092)
μ Quintiles	3	-0.153	-0.187	-0.162	-0.095	-0.078	0.074*
							(1.762)
	4	-0.154	-0.158	-0.136	-0.116	-0.133	0.021
							(0.450)
	5	-0.182	-0.095	-0.132	-0.102	-0.079	0.103***
							(3.165)

Notes to Table: We report average equal-weighted monthly returns on option portfolios sorted on expected stock return (μ) and 30-day realized volatility. Panel A reports on call options and Panel B on put options.

Table 1.5: Controlling for Expected Stock Returns Using the CAPM

Panel A: Double Sorts on Beta and Volatility

Beta\Vol		Low	2	3	4	High	H-L
	1	0.156	0.126	0.094	0.067	-0.01	-0.165***
							(-3.384)
	2	0.162	0.165	0.15	0.135	0.061	-0.101**
							(-2.082)
Call	3	0.149	0.194	0.147	0.112	0.05	-0.099*
							(-1.969)
	4	0.113	0.133	0.107	0.106	0.031	-0.082**
							(-2.024)
	5	0.09	0.076	0.104	0.022	0.005	-0.085**
							(-2.115)
Beta\Vol		Low	2	3	4	High	H-L
	1	-0.15	-0.095	-0.132	-0.126	-0.121	0.029
							(0.618)
	2	-0.149	-0.165	-0.156	-0.101	-0.107	0.041
							(0.896)
Put	3	-0.147	-0.198	-0.112	-0.069	-0.065	0.082**
							(1.997)
	4	-0.14	-0.122	-0.14	-0.071	-0.047	0.093**
							(2.087)
	5	-0.133	-0.106	-0.085	-0.067	-0.065	0.068*
							(1.957)

Panel B: Sorts on Idiosyncratic Volatility

	Low	2	3	4	High	H-L
Call	0.156	0.13	0.119	0.083	0.021	-0.133***
						(-3.245)
Put	-0.157	-0.151	-0.119	-0.069	-0.077	0.080**
						(2.271)

Notes to Table: Panel A reports average equal-weighted monthly returns on option portfolios sorted on market beta and 30-day realized volatility.

taneously control for risk factors and stock characteristics that have been shown in the existing literature to be related to expected stock returns.

Every month we run the following cross-sectional regression

$$R^i_{t+1} = \gamma_{0,t} + \gamma_{1,t} VOL^i_t + \Phi_t Z^i_t + \epsilon \tag{1.19}$$

where R^i_{t+1} is the return on holding option i from month t to month $t+1$, VOL^i_t is the underlying stock volatility for option i, and Z^i_t is a vector of control variables that includes the stock's beta, firm size, book-to-market, momentum, stock return reversal, the option skew, the volatility risk premium, the slope of the implied volatility term structure, as well as option characteristics such as moneyness, Delta, Vega, Gamma and option-beta. Option beta is defined as delta times the stock price divided by the option price. Both VOL^i_t and Z^i_t are observable at time t for option i. We again use 30-day realized volatility as a proxy for the underlying stock volatility.

Table 1.6 reports the time-series averages of the cross-sectional γ and Φ estimates from equation (1.19), along with Newey-West (1987) t-statistics which adjust for autocorrelation and heteroscedasticity. Columns (1) to (3) report regression results for call options. Column (1) of Table 1.6 shows that in a univariate regression the average slope coefficient on 30-day realized volatility is -0.239 with a Newey-West t-statistic of -4.19. This estimate is consistent with the sorting results. The difference in the average underlying volatility between the two extreme call option portfolios in Table 1.3 is 0.6, which implies a decline of $-0.239 \times 0.6 = 14.34\%$ per month in average returns if a call option were to move from the bottom volatility portfolio to the top volatility portfolio, other characteristics held constant. This estimate is very similar to the result in Table 1.3.

The specification in column (2) includes several well-known determinants of cross-sectional stock returns. The loading on volatility increases in absolute value from -0.239 to -0.277 and remains highly significant. The specification in column (3) includes additional controls as well as option characteristics. The slope coefficient on volatility is even larger in absolute value and is again statistically significant. The results in columns (1)-(3) are all consistent with our theoretical conjecture in Proposition 1.

Table 1.6: Fama-MacBeth Regressions

	Calls			Puts		
	(1)	(2)	(3)	(4)	(5)	(6)
Vol	-0.239***	-0.277***	-0.389**	0.117**	0.125***	0.584***
	(-4.192)	(-5.293)	(-2.241)	(2.552)	(2.725)	(2.911)
Beta		-0.004	0.045***		0.000	-0.034*
		(-0.284)	(2.714)		(-0.002)	(-1.789)
Size		0.000	0.001		0.000	-0.002**
		(0.185)	(0.829)		(-0.234)	(-2.285)
Btm		0.002	0.057*		0.005	-0.060*
		(0.938)	(1.754)		(0.798)	(-1.832)
Mom		0.026	0.020		-0.023	-0.013
		(0.971)	(0.824)		(-1.252)	(-0.666)
Reversal			-0.188*			-0.162*
			(-1.940)			(-1.828)
Option skew			0.065			0.474**
			(0.256)			(2.133)
Vrp			-0.059			0.454**
			(-0.327)			(1.982)
Slope			0.685***			0.648**
			(3.270)			(2.580)
Moneyness			-0.090			-0.920
			(-0.090)			(-0.944)
Delta			-0.069			-0.381
			(-0.160)			(-1.000)
Vega			-0.021**			0.015
			(-2.347)			(1.379)
Gamma			0.011			0.156
			(0.022)			(0.233)
Option beta			-0.002			-0.012
			(-0.197)			(-0.847)

Notes to Table: We report results for the Fama-MacBeth regressions.

In columns (4)-(6), we provide results for put options. As expected, the average slope coefficient on underlying volatility is positive and statistically significant for all specifications, ranging from 11.7% to 58.4% per month. These findings again suggest that our results cannot be attributed to differences in expected stock returns. As we add more controls in column (6), the loading on volatility increases significantly in absolute value.

We conclude that the results in Table 1.6 are consistent with the theoretical predictions. Moreover, the empirical results get stronger when we insert more controls for expected stock returns. For call options, the slope coefficient on 30-day realized volatility is -0.389 in column (3), compared to -0.239 in column (1). For put options, the estimate in column (6) is 0.584, compared to 0.117 in column (4). This may suggest that we control more effectively for the effect of the drift of the underlying security when we include more controls. Presumably controlling for the drift using expected returns or a market model as in Sections 1.4.2 and 1.4.2 is not very effective, which explains why the results in Tables 1.4 and 1.5 are very similar to the benchmark results in Table 1.3. However, note that the t-statistic in column (3) is lower than that in column (1), and therefore some caution is advisable when interpreting these results.

In columns (3) and (6), we also include the variance risk premium studied by Goyal and Saretto (2009) and the slope of the volatility term structure studied by Vasquez (2012). The variance risk premium is significant for puts and the slope is significant for both calls and puts. However, note that Goyal and Saretto (2009) and Vasquez (2012) study delta-hedged returns, and we analyze raw returns. From our perspective, the most important conclusion is that the cross-sectional relation between volatility and returns remains in the presence of these controls.

A Controlled Experiment

Finally, we assess if Ang, Hodrick, Xing, and Zhang's (2006) finding of a negative relation between volatility and stock returns can explain our results in a more direct way using a simple computation. In Table 1.3, the difference in option returns between the fifth and the first quintiles is -13.8% for call options (0.9%-14.7%). We compute the average returns on the stocks in these portfolios, which is 10.8% for the first quintile and 4.8% in the fifth quintile. We now fix volatility σ across these quintiles to conduct a controlled experiment and compute option returns using the Black Scholes-Merton model. Consider a fixed σ of 50%, which is close to our sample average. This experiment indicates how much of the return differential in option returns is generated by the differential in returns in the underlying stocks.

For the low volatility quintile, the average return of 10.8% and the 50% volatility yield a monthly option return of 6.31%. For the high volatility quintile the average return is 4.8%, which for a 50% volatility gives a 1.63% option return. The difference between the two returns is $1.63\% - 6.31\% = -4.68\%$. In other words, this computation indicates that of the 13.8% return differential in the data, 4.68% is due to the differences in stock returns. The volatility difference accounts for the majority of the difference in option returns, empirically confirming the theoretical relation.

1.4.3 Volatility and Expected Straddle Returns

Straddle returns are not very sensitive to the expected returns on the underlying security. Therefore, several existing papers that investigate the cross-sectional relation between option returns and different aspects of volatility focus on straddle returns to separate the cross-sectional effect of volatility and volatility-related variables from that of the underlying stock returns. See for example Goyal and Saretto (2009) and Vasquez (2012).

A straddle consists of the simultaneous purchase of a call option and a put option on the same underlying asset. The call and put options have the same strike price and time to maturity. The expected gross return on a straddle is given by:

$$R_{straddle} = \frac{E_t[\max(S_T - K, 0)] + E_t[\max(K - S_T, 0)]}{C_t(\tau, S_t, \sigma, K, r) + P_t(\tau, S_t, \sigma, K, r)}$$

where $C_t(\tau, S_t, \sigma, K, r)$ and $P_t(\tau, S_t, \sigma, K, r)$ are the call and put prices that an investor has to pay to build a long position in straddle.

Because the derivative of calls and puts with respect to volatility has the opposite sign, it is impossible to obtain general results for straddles. Appendix E shows that $d_2 > 0$ is a sufficient condition for a negative relation between straddle returns and underlying volatility. Recall that $d_2 = \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$. The condition $d_2 > 0$ is thus likely to hold for straddles with strike prices below the current stock price, and we investigate if average straddle returns decrease with underlying volatility for such straddles. Table A.1 in the online appendix confirms that this relation indeed holds in the data. However, effectively the return on the straddle is dominated by the call option when $d_2 > 0$, which means that the negative sign in theory and in the data simply confirms the results above.

1.4.4 Stochastic Volatility and Expected Option Returns

The Black-Scholes-Merton model's treatment of volatility is perhaps its most important shortcoming. An extensive literature has demonstrated that volatility is time varying, and that (the innovations to) volatility and stock returns are correlated.[12] This correlation is often referred to as the leverage effect.

To address the implications of time-varying volatility and the leverage effect, we now analyze expected option returns using a stochastic volatility model instead of the Black-Scholes-Merton model. We use the Heston (1993) model, which has become the benchmark in this literature because it captures important stylized facts such as time-varying volatility and the leverage effect, while also allowing for quasi-closed form European option prices. The Heston (1993) model assumes that the asset price and its spot variance obey the following dynamics under the physical measure P:

$$\begin{aligned} dS_t &= \mu S_t dt + S_t \sqrt{V_t} dZ_1^P \\ dV_t &= \kappa(\theta - V_t)dt + \sigma \sqrt{V_t} dZ_2^P \end{aligned}$$

where μ is the drift of the stock price, θ is the long run mean of the stock variance, κ is the

[12] For seminal contributions to this literature, see Engle (1982), Bollerslev (1986), Nelson (1991), Glosten, Jagannathan and Runkle (1993), and Engle and Ng (1993).

rate of mean reversion, σ is the volatility of volatility, and Z_1 and Z_2 are two correlated Brownian motions with $E[dZ_1 dZ_2] = \rho dt$.

By focusing on discrete holding periods instead of instantaneous returns, we can express expected returns in the Heston model in quasi-closed form.[13] Appendix F shows that the expected return of holding a call option to expiration in the Heston model is given by:

$$R_{call}^{Heston}(S_t, V_t, \tau) = \frac{e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau} K P_2^*]}{S_t P_1 - e^{-r\tau} K P_2} \tag{1.20}$$

where P_1, P_2, P_1^* and P_2^* are defined in Appendix F. The expected call option return in the Heston model has the same functional form as in the Black-Scholes-Merton model, but unlike for the Black-Scholes-Merton model, the sign of $\frac{\partial R_{call}^{Heston}(S_t,V_t,\tau)}{\partial V_t}$ cannot be derived analytically. However, the expected option return in equation (1.20) can be easily calculated numerically given a set of parameter values.

In Panels B and C of Table 1.7, we compute expected option returns according to (1.20) for different parameterizations of the expected stock return μ and the conditional stock variance. For simplicity we first set the variance risk premium λ equal to zero. For all other parameters, we use the parameters from Broadie, Chernov, and Johannes (2009), which are listed in Panel A. The patterns in expected option returns in a stochastic volatility model are similar to the patterns in Black-Scholes-Merton expected option returns. In particular, expected call option returns increase (decrease) with expected stock return (current stock variance), whereas expected put option returns decrease (increase) with expected stock return (current stock variance). In unreported results, we obtain similar results using different parameterizations.

In the Black-Scholes-Merton model, volatility affects expected returns through leverage. In the Heston model, volatility affects expected returns not only through leverage, but also through the volatility risk premium λ. Figure 1-4 further explores expected returns in the Heston model. We set λ equal to 0 or -0.5. The main conclusion is that expected returns do not strongly depend on λ. The relation between volatility and expected option returns

[13] The delta and vega are not available in closed form when computing instantaneous option returns.

Table 1.7: Expected Option Returns in the Heston Model

Panel A: Parameters			r	$\sqrt{\theta}$	κ	σ	ρ	$T-t$		
			0.045	0.15	5.33	0.14	-0.52	1/12		
Panel B: Call Options						V_t				
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	8%	0.118	0.094	0.080	0.072	0.066	0.061	0.057	0.054	0.051
μ	12%	0.257	0.202	0.172	0.153	0.139	0.128	0.120	0.113	0.107
	16%	0.405	0.316	0.268	0.237	0.215	0.198	0.185	0.174	0.165
Panel C: Put Options						V_t				
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	8%	-0.104	-0.081	-0.068	-0.060	-0.054	-0.049	-0.046	-0.043	-0.040
μ	12%	-0.217	-0.172	-0.146	-0.130	-0.117	-0.108	-0.100	-0.094	-0.089
	16%	-0.320	-0.256	-0.220	-0.195	-0.178	-0.164	-0.153	-0.144	-0.136
Panel D: Straddles						λ				
		-0.5	-0.4	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3
μ	12%	0.022	0.024	0.026	0.028	0.030	0.031	0.033	0.035	0.037

Notes to Table: We report expected monthly option returns in the Heston (1993) stochastic volatility model. Panel B (C) reports expected returns on at-the-money call (put) options for different levels of the current stock variance (V_t). Panel D reports expected returns on at-the-money straddle for different levels of the volatility risk premium (λ). The computations are based on the model parameters reported in Broadie, Chernov, and Johannes (2009), which are calibrated from historical S&P 500 index return data. These parameters are reported in Panel A. For simplicity, the dividend yield is set to zero. We set λ equal to 0 in Panels B and C, and we set V_t equal to 0.0225 in Panel D.

is similar to the results in the Black-Scholes-Merton model.[14]

Finally, Panel D of Table 1.7 shows straddle returns as a function of the volatility risk premium λ. Returns increase with higher volatility risk premiums. This is consistent with the empirical findings in Goyal and Saretto (2009), who document that option returns increase as a function of the variance risk premium.

1.5 Robustness

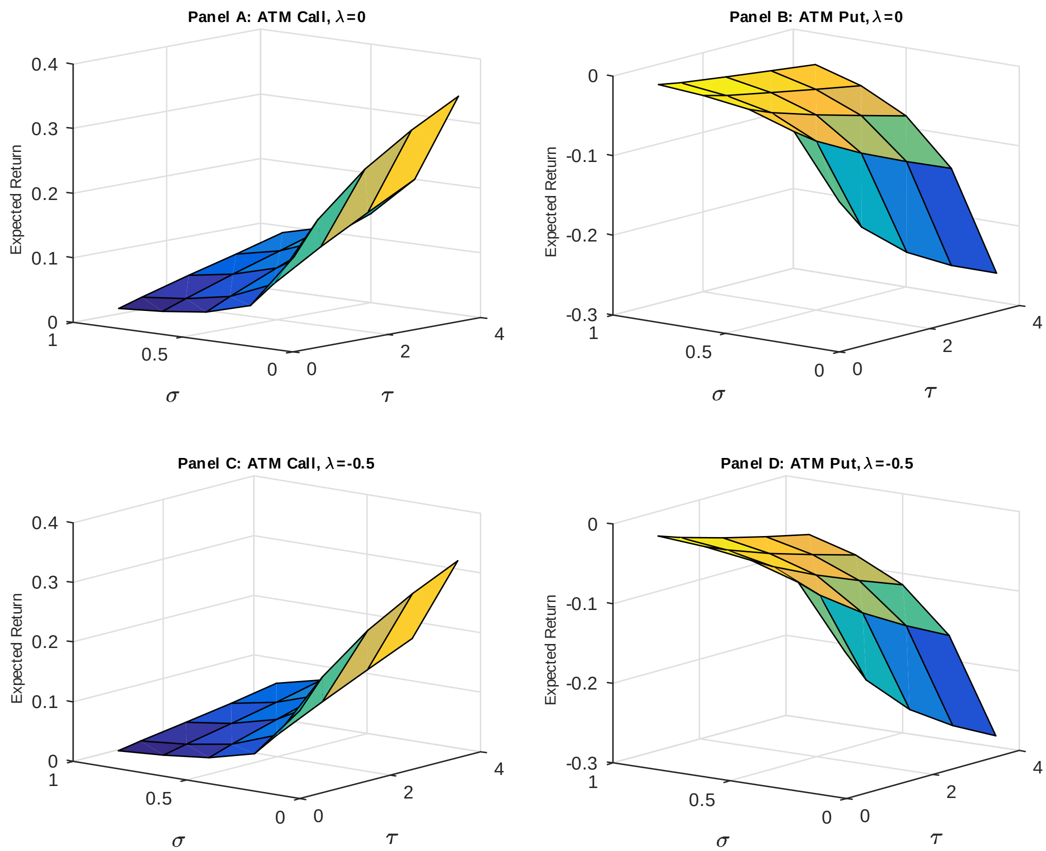
In this section we investigate the robustness of the results in Table 1.3 to a number of implementation choices. We investigate the robustness of the empirical results to the measurement of realized volatility, the composition of the option sample, and the weights used to compute portfolio returns. We also use holding-period returns rather than holding-to-maturity returns.

1.5.1 The Volatility Measure

Table 1.3 uses realized volatility computed using daily data for the preceding month as a measure of the underlying volatility. This is a standard volatility measure that is often used in the literature. Ang, Hodrick, Xing, and Zhang (2006) and Lewellen and Nagel (2006) argue that 30-day realized volatility strikes a good balance between estimating parameters with a reasonable level of precision and capturing the conditional aspect of volatility. We now consider five alternative estimators of underlying stock volatility. We proxy underlying volatility using realized volatilities computed over the past 14 days, the past 60 days, and the past 365 days, as well as option-implied volatility and a simple autoregressive AR(1) model for volatility to take into account the mean reversion in volatility.

Panel A of Table 1.8 presents average returns for the five quintile call option portfolios and Panel B reports average returns for put option portfolios. Consistent with our benchmark results in Table 1.3, we find that for all underlying volatility proxies, the returns on

[14] For (unrealistically) large negative λ, the relation is not monotone for OTM options.



Notes: We plot expected option returns in the Heston model against volatility (σ) and time-to-maturity (τ) for at-the-money (ATM) options. We use the parameter values in Panel A of Table 1.8, and set the expected stock return μ equal to 9.91% as in Broadie, Chernov and Johannes (2009). The volatility risk premium (λ) is either 0 (Panels A and B) or -0.5 (Panels C and D). Returns are reported as raw returns for the relevant horizons

Figure 1-4: Expected Option Returns in the Heston Model

Table 1.8: Option Portfolio Returns Sorted on Alternative Volatility Measures

	Low	2	3	4	High	H-L
Panel A: Call Options						
14-day realized vol	0.146	0.122	0.114	0.081	0.016	-0.130***
						(-3.539)
60-day realized vol	0.155	0.109	0.115	0.086	0.014	-0.141***
						(-3.437)
365-day realized vol	0.130	0.104	0.117	0.084	0.044	-0.086*
						(-1.805)
Implied vol	0.156	0.117	0.134	0.081	-0.010	-0.166***
						(-3.598)
AR(1) vol	0.117	0.107	0.112	0.080	0.018	-0.099**
						(-2.142)
Panel B: Put Options						
14-day realized vol	-0.146	-0.139	-0.103	-0.086	-0.087	0.059*
						(1.765)
60-day realized vol	-0.157	-0.151	-0.109	-0.084	-0.059	0.098**
						(2.488)
365-day realized vol	-0.170	-0.144	-0.120	-0.071	-0.053	0.117***
						(2.817)
Implied vol	-0.130	-0.143	-0.118	-0.087	-0.083	0.047
						(1.128)
AR(1) vol	-0.171	-0.154	-0.129	-0.077	-0.014	0.157***
						(3.868)

Notes to Table: We report equal-weighted monthly option portfolio returns sorted on different measures of underlying volatility, as well as the return differences between the two extreme portfolios.

the call option portfolios exhibit a strong negative relation with underlying stock volatilities, while put option portfolio returns display a strong positive relation with underlying stock volatilities. For example, when sorting on 60-day realized volatility, the average returns for call option portfolios with the largest and smallest underlying volatilities are 1.4% and 15.5% per month respectively. The resulting difference between the two extreme portfolios is −14.1% per month and is highly statistically significant with a Newey-West t-statistic of −3.44. For put option portfolios, the average returns monotonically increase from −15.7% per month for the lowest volatility portfolio to −5.9% per month for the highest volatility portfolio. The resulting difference is 9.8% per month and is also statistically significant.

When sorting on 14-day and 365-day realized volatility, the returns display a similar pattern. The average returns decrease (increase) with underlying volatilities for call (put) portfolios. The return differences between the two extreme call option portfolios are negative and statistically significant with a magnitude of −13% and −8.6% per month, respectively. The corresponding differences for put option portfolios are positive and statistically significant, with a magnitude of 5.9% and 11.7% per month, respectively.

We also sort options based on implied volatilities. Option-implied volatilities are attractive because they provide genuinely forward-looking estimates, but they are model-dependent and may include volatility risk premiums.[15] Again consistent with our benchmark results, we find that call (put) option portfolios with larger implied volatilities earn lower (higher) returns. Panel A of Table 1.8 reveals that returns on call option portfolios monotonically decrease with implied volatilities. The return spread is −16.6% per month and is highly statistically significant. The return spread for the two extreme put option portfolios is positive with a magnitude of 5.9% per month, but it is not statistically significant. Finally, we use an autoregressive model for volatility instead. In particular, we obtain an estimate of conditional volatility by fitting an AR (1) model on monthly realized volatilities. The results are again statistically significant. The economic magnitude is somewhat

[15] On the volatility risk premium embedded in individual stock options, see Bakshi and Kapadia (2003b), Driessen, Maenhout, and Vilkov (2009), and Carr and Wu (2009) for more details.

smaller for calls and somewhat larger for puts.

These results suggest that our empirical findings are not due to the volatility measure used in Table 1.3.

1.5.2 The Option Sample

We now investigate the relation between expected option returns and underlying volatility using five other option samples with different maturity and moneyness. We examine the following five option samples: two-month at-the-money options, one-month in-the-money options, two-month in-the-money options, one-month out-of-the-money options, and two-month out-of-the-money options. We define at-the-money as having moneyness of $0.95 \leq K/S \leq 1.05$, in-the-money calls as $0.80 \leq K/S < 0.95$, and in-the-money puts as $1.05 < K/S \leq 1.20$. Out-of-the-money calls are defined as $1.05 < K/S \leq 1.20$ and out-of-the-money puts as $0.80 \leq K/S < 0.95$.

Table 1.9 presents the results. Panel A of Table 1.9 provides average returns of call option portfolios sorted on 30-day realized volatility for the five alternative option samples. Consistent with the benchmark results in Table 1.3, we find that returns on call option portfolios decrease with underlying volatility for all option samples. The return differences between the two extreme portfolios are negative and statistically significant in all cases, with magnitudes ranging from −7.8% to −18.6% per month. For instance, for two-month at-the-money calls, the equal-weighted average option portfolio returns decrease monotonically with underlying volatility. The return spread is −17.1% per month and highly significant with a Newey-West t-statistic of −3.04.

Panel B of Table 1.9 presents average returns of put option portfolios sorted on 30-day realized volatility for the five option samples. Average put option returns exhibit a strong positive relation with underlying volatilities. The returns spreads are all positive and statistically significant, ranging from 5.7% to 17.8% per month. For instance, for two-month at-the-money puts, average returns monotonically increase from −20.7% per month for the lowest volatility portfolio to −5.6% per month for the highest volatility portfolio.

Table 1.9: Option Portfolio Returns for Alternative Option Samples

	Low	2	3	4	High	H-L
Panel A: Call Options						
Two-month ATM	0.144	0.135	0.112	0.035	-0.027	-0.171***
						(-3.037)
One-month ITM	0.053	0.060	0.042	0.026	-0.025	-0.078***
						(-3.678)
Two-month ITM	0.089	0.084	0.068	0.027	-0.067	-0.156***
						(-5.224)
One-month OTM	0.055	0.049	0.077	0.048	-0.066	-0.121**
						(-2.214)
Two-month OTM	0.132	0.088	0.098	0.022	-0.054	-0.186**
						(-2.361)
Panel B: Put Options						
Two-month ATM	-0.207	-0.149	-0.118	-0.079	-0.056	0.151***
						(3.203)
One-month ITM	-0.091	-0.069	-0.052	-0.043	-0.034	0.057***
						(2.933)
Two-month ITM	-0.127	-0.090	-0.055	-0.048	-0.023	0.105***
						(3.625)
One-month OTM	-0.309	-0.217	-0.193	-0.090	-0.131	0.178***
						(2.759)
Two-month OTM	-0.276	-0.197	-0.188	-0.118	-0.099	0.177**
						(2.037)

Notes to Table: We report equal-weighted monthly option portfolio returns sorted on 30-day realized volatility, as well as the return differences between the two extreme portfolios. Different option samples are used: two-month at-the-money (ATM) options, one-month in-the-money (ITM) options, two-month ITM options, one-month out-of-the-money (OTM) options, and two-month OTM options.

The resulting return spread is 15.1% per month and is both economically and statistically significant.

For empirical results that use index options, using out-of-the-money options is very important because this market is more liquid and has higher volume, as evidenced by Table 1.2. For equity options, the differences in liquidity and volume across moneyness are less pronounced, as evidenced by Table 1.1. Nevertheless, it is reassuring that the results are robust when we only use out-of-the-money options in Table 1.9.

These results suggest that our empirical findings are not due to the sample used in Table 1.3.

1.5.3 The Portfolio Weighting Method

In this subsection, we examine if the negative (positive) relation between call (put) option portfolio returns and underlying volatility persists if different weighting methods are used for computing option portfolio returns. We calculate option volume weighted, option open interest weighted and option value weighted average portfolio returns. Option value is defined as the product of the option's open interest and its price.[16]

Table A.2 in the online appendix contains return spreads for option portfolios sorted on 30-day realized volatility, using these alternative weighting methods. Regardless of the weighting method, the return spreads are negative (positive) for call (put) option portfolios, and they are statistically significant in most cases. These results suggest that our empirical findings are not due to the equal-weighting method used in Table 1.3.

1.5.4 Holding-Period Returns

Ni, Pearson, and Poteshman (2005) argue that holding-to-maturity option returns are affected by biases at expiration. We therefore repeat the analysis in Table 1.3 using one-month

[16] We also consider portfolio returns weighted by underlying stock capitalization and find similar results.

option returns instead of holding-to-maturity returns.[17] Table A.3 in the online appendix presents the results for ATM, ITM, and OTM call and put options. The results are again statistically significant and consistent with Propositions 1 and 2. However, the magnitudes of the long-short returns are smaller, especially for calls.

1.6 Discussion and Extensions

In this section, we further explore our results. We first discuss delta-hedged returns, which have been studied in the existing literature, and we verify if the differences in expected returns between portfolios are consistent with theoretical predictions. Subsequently we investigate if the models' quantitative implications for returns are consistent with the data and we compute option-implied average stock returns. Finally we provide a detailed discussion of the differences between our results and those in the existing literature.

1.6.1 Delta-Hedged Returns

Cao and Han (2013) document that the cross-sectional relation between idiosyncratic stock volatility and both delta-hedged call and put option returns is negative.[18] It is natural to wonder if these empirical results are consistent with our theoretical and empirical results, especially because the results seem so different.

We show that these results are mutually consistent, and simply result from the difference between raw and delta-hedged returns. Our study may seem superficially related to Cao and Han (2013) but the analysis is fundamentally different. Our study empirically investigates the theoretical relation between option returns and the underlying stock volatility, which by definition is accounted for when computing delta-hedged returns. When we investigate the robustness of our results in Section 1.4.2, we correct for the drift of the underlying stock,

---

[17] Broadie, Chernov, and Johannes (2009) argue against the use of holding period returns. Duarte and Jones (2007) argue that bid-ask bounce can bias returns, which also favors the use of holding to-maturity returns.

[18] See also Black and Scholes (1972) on the relation between delta-hedged option returns and volatility.

whereas delta hedging by definition corrects for the entire underlying return, which includes the drift as well as the diffusive part.

Panel A of Table 1.10 reports on delta-hedged returns for portfolios sorted on volatility. We repeat the analysis in Cao and Han (2013) with two differences in implementation. First, we use total volatility instead of idiosyncratic volatility because the focus of our study is on total volatility and we want to stay closer to the results in Table 1.3. Second, while Cao and Han (2013) rebalance daily, we use static hedging for reasons to be explained below. We verified that these differences in implementation do not significantly affect the results. Panel A demonstrates the robustness of the results in Cao and Han (2013). Consistent with their results, we find a statistically significant negative relation between volatility and both call and put returns.

Bollen and Whaley (2004) and Cao and Han (2013) emphasize market frictions and inventory management as plausible explanations for the negative relation between volatility and delta-hedged call and put returns. Bakshi and Kapadia (2003a, 2003b) show that delta-hedged option returns can be used to infer the market price of volatility risk if volatility is stochastic. Our implementation uses static hedging to emphasize an additional potential explanation for these results that differs from these existing explanations. In the Black-Scholes-Merton framework, delta-hedged option returns are exactly zero in the ideal case of continuous trading. In practice, however, it is impossible to rebalance the portfolio continuously. It is well understood (see for instance Branger and Schlag, 2008; Broadie, Chernov and Johannes, 2009) that empirical investigations using delta-hedged returns must be interpreted with caution, not only due to model misspecification, but also due to discretization errors and transaction costs.

Consider how underlying volatility impacts discretely delta-hedged option returns in the Black-Scholes-Merton model in the case of static hedging. We form the delta-hedged portfolio at time t and keep it unadjusted until the expiration date of the option at T. The delta-hedged return for a call option, which can also be interpreted as the hedging error, is

Table 1.10: Delta-Hedged Option Returns

Panel A: Delta-Hedged Option Returns

		Low	2	3	4	High	H-L
Calls	EW	-0.292	-0.325	-0.356	-0.231	-0.721	-0.429**
							(-2.013)
	VW	-0.180	-0.359	-0.380	-0.205	-0.863	-0.683**
							(-2.396)
Puts	EW	0.027	-0.067	0.046	0.003	-0.484	-0.511***
							(-2.790)
	VW	0.007	-0.296	0.001	0.004	-0.616	-0.623**
							(-2.260)

Panel B: Delta-Hedged Option Returns in the Black-Scholes-Merton Model

	$\mu\backslash\sigma$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
K/S=100/100	0.05	0.191	0.096	0.064	0.048	0.038	0.032	0.027	0.024	0.021
	0.07	0.764	0.384	0.256	0.192	0.154	0.128	0.109	0.096	0.085
	0.09	1.714	0.864	0.577	0.432	0.346	0.288	0.246	0.215	0.191
	0.11	3.039	1.536	1.026	0.769	0.615	0.512	0.438	0.383	0.340
	0.13	4.734	2.400	1.603	1.203	0.962	0.801	0.685	0.599	0.531
	$\mu\backslash\sigma$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
K/S=100/95	0.05	0.046	0.066	0.053	0.043	0.035	0.030	0.026	0.023	0.020
	0.07	0.192	0.267	0.215	0.172	0.142	0.120	0.104	0.091	0.081
	0.09	0.446	0.605	0.485	0.388	0.320	0.270	0.234	0.206	0.183
	0.11	0.820	1.085	0.866	0.692	0.569	0.482	0.416	0.366	0.326
	0.13	1.325	1.710	1.360	1.084	0.891	0.754	0.651	0.572	0.510
	$\mu\backslash\sigma$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
K/S=100/105	0.05	0.039	0.066	0.055	0.044	0.037	0.031	0.027	0.024	0.021
	0.07	0.152	0.261	0.219	0.178	0.147	0.125	0.108	0.095	0.085
	0.09	0.331	0.583	0.491	0.399	0.331	0.282	0.244	0.215	0.192
	0.11	0.569	1.028	0.870	0.709	0.589	0.500	0.434	0.382	0.341
	0.13	0.861	1.594	1.355	1.106	0.919	0.782	0.678	0.597	0.533

given by

$$\Pi^C_{t,T} = C_T - C_t - \Delta_t(S_T - S_t) - (C_t - \Delta_t S_t)(e^{r\tau} - 1) \tag{1.21}$$

$$= C_T - \Delta_t S_T - (C_t - \Delta_t S_t)e^{r\tau}. \tag{1.22}$$

where C_t and C_T are call option prices at time t and T, S is the stock price, and r again denotes the instantaneous risk-free rate. Using the results for the expected option payoff at maturity $E_t(C_T)$, we get

$$\begin{aligned} E_t(\Pi^C_{t,T}) &= E_t[C_T - \Delta_t S_T - (C_t - \Delta_t S_t)e^{r\tau}] \\ &= e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] - \Delta_t S_t e^{\mu\tau} - (C_t - \Delta_t S_t)e^{r\tau} \\ &= e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] - N(d_1) S_t e^{\mu\tau} + K N(d_2). \end{aligned} \tag{1.23}$$

The expected delta-hedged return (or hedging error) for a put option is given by

$$E_t(\Pi^P_{t,T}) = e^{\mu\tau}[e^{-\mu\tau} K N(-d_2^*) - S_t N(-d_1^*)] + N(-d_1) S_t e^{\mu\tau} - K N(-d_2). \tag{1.24}$$

To understand how underlying volatility affects delta-hedged option returns, we need the sign of the partial derivatives of $E_t(\Pi^C_{t,T})$ and $E_t(\Pi^P_{t,T})$ with respect to σ. First, note that equation (1.24) is equivalent to equation (1.23) by put-call parity. We therefore focus on the relation between the expected delta-hedged call option return and underlying volatility. The partial derivative $\frac{\partial E_t(\Pi^C_{t,T})}{\partial \sigma}$ is available analytically, but can either be positive or negative depending on the underlying parameters. However, we can easily evaluate expected returns numerically for a given set of parameters.

Table A.4 in the online appendix shows that this negative relationship continues to hold and is strengthened when including other control variables in a Fama-MacBeth regression. Table A.4 also shows indicates that the results of Goyal and Saretto (2009) are confirmed for our sample: delta-hedged returns on puts and calls are positively related to the variance risk premium.

Panel B of Table 1.10 considers static delta-hedging of a hypothetical call option with strike price of \$100 and a one-month maturity. We report expected delta-hedged returns for different values of S, μ, and σ. Expected delta-hedged returns are all positive, which is consistent with the finding of Branger and Schlag (2008) that discretization error induces positive expected hedging error in the Black-Scholes-Merton model. More importantly for our purpose, the expected delta-hedged return decreases with underlying volatility for all parameter combinations.

We conclude that in the context of the Black-Scholes-Merton model, static delta-hedging will result in a negative relation between delta-hedged option returns and underlying volatility for plausible parameterizations of the model. This qualitative result also obtains in any practical situation where hedging is conducted in discrete time, for instance when the hedge is rebalanced daily. Most importantly for our conclusions, the negative relation between volatility and delta-hedged call and put returns is consistent with the negative (positive) relation between volatility and call (put) returns. Both cross-sectional relations are supported by the simple analytics of the Black-Scholes-Merton model.

1.6.2 Volatility and Expected Option Returns: A Quantitative Assessment

So far we have limited ourselves to empirically verifying the qualitative predictions in Propositions 1 and 2. We now go one step further and assess the magnitude of the return difference for portfolios with different underlying volatility.

The first row of Panels A and B of Table 1.11 reports the benchmark results from Table 1.3. The call option quintile portfolio with high volatility earns 0.9% per month and the call option quintile portfolio with low volatility earns 14.7% per month. The put option quintile portfolio with high volatility earns −7.5% per month and the put option quintile portfolio with low volatility earns −14.6% per month. We assess if these return differences are consistent with theory by computing expected returns and volatility for the underlying stocks for these portfolios, and computing expected returns using the Black-Scholes-Merton model.

Using historical averages for our sample period 1996-2013, we obtain an annualized μ of 4.8 percent and volatility σ of 81.3 percent for the high volatility call option portfolio.

Table 1.11: Option Returns, Stock Returns and Option-Implied Stock Returns

Panel A: Call Options					
	Low	2	3	4	High
Average option return	0.147	0.128	0.111	0.084	0.009
Average stock return	0.108	0.132	0.132	0.108	0.048
Average stock volatility	0.209	0.308	0.403	0.525	0.813
Expected option return	0.145	0.130	0.101	0.060	0.011
Option-implied expected stock return	0.106	0.127	0.139	0.136	0.041
Panel B: Put Options					
	Low	2	3	4	High
Average option return	-0.146	-0.153	-0.109	-0.077	-0.075
Average stock return	0.108	0.132	0.132	0.096	0.048
Average stock volatility	0.214	0.313	0.408	0.532	0.827
Expected Option Return	-0.124	-0.109	-0.083	-0.040	-0.005
Option-implied expected stock return	0.120	0.172	0.162	0.153	0.225

Notes to Table: The first row of each panel repeats the benchmark results from Table 1.3. The second and third rows report the average return and volatility of stocks underlying these option portfolios. The fourth row computes expected option returns using the Black-Scholes-Merton expected option return formula given the stock data in rows 2 and 3. The last row reports the option-implied expected stock return using the data in rows 1 and 3 by inverting the Black-Scholes-Merton expected option return formula. The average option returns in the first row are monthly for consistency with Table 1.3. Stock returns, stock volatilities and option implied expected stock returns are annual. The sample period is from January 1996 to July 2013.

For the low volatility call option portfolio, we obtain an annualized μ of 10.8 percent and volatility σ of 20.9 percent. These results are reported in the second and third rows of Panel A. Assuming a 3% annual interest rate, the Black-Scholes-Merton model predicts a expected option return of 14.5% per month for the low volatility call option portfolio and 1.1% per month for the high volatility call option portfolio. These results are reported in the fourth rows of Panel A. These expected option returns are very close to the average returns in the first row.

The second and third rows of Panel B indicate that for the low volatility put option portfolio, the underlying annualized μ is 10.8% and the underlying volatility σ is 21.4%. The high volatility put option portfolio has a μ of 4.8% and a σ of 82.7%.[19] Again using the Black-Scholes-Merton model, the fourth row shows that this gives expected option returns of -12.4% and -0.5% per month. For the low volatility portfolio, the expected return is close to the sample average in the first row, but this is not the case for the high volatility put portfolio.

Overall, we conclude that the implied call option returns are close to what we observe in the data on average, despite the well-known shortcomings of the Black-Scholes-Merton model. The results for put options are less impressive than those for calls, which may be due to the well-known stylized fact that put options are expensive, possibly due to demand pressure (see Bollen and Whaley, 2004).

1.6.3 Option-Implied Returns and Volatility

The qualitative difference between the results for call and put options in Section 1.6.2 can equivalently be expressed in terms of option-implied returns. The last row of Panels A and B of Table 1.11 presents the results of this exercise. We use the average volatility in each quintile and then invert the Black-Scholes-Merton formula to obtain an estimate of

[19] Note that the average stock returns and the average stock volatilities of the five quintile portfolios are slightly different for calls and puts. This is because for some stocks we have calls but not puts and vice versa.

the implied μ.

Table 1.11 indicates that the returns implied by call options are very close to the actual average stock returns, but this is not the case for the returns implied by puts. This result is essentially the mirror image of the finding discussed in Section 1.6.2.

1.6.4 Further Discussion and Related Literature

We do not provide an overview of the entire related literature on empirical option pricing, because it is vast and our results are easily distinguished.[20] However, our results are at the intersection of several strands of empirical research on cross-sectional asset pricing. We now discuss some of these related studies in more detail in order to highlight our specific contribution.

The literature characterizing volatility in index returns and stock returns is well-known and also too vast to cite here. Our paper is most closely related to a series of papers that highlight one particular dimension of this literature, namely the cross-sectional relation between volatility and expected stock returns. Even in this cross-sectional literature, it is important to differentiate between studies that investigate (aggregate) volatility as a pricing factor in the cross-section of returns and studies that investigate stock returns as a cross-sectional function of their own idiosyncratic or total volatility. Ang, Hodrick, Xing, and Zhang (2006) investigate both issues. As discussed in Section 1.4.2, our contribution is clearly more related to their investigation of the cross-sectional relation between the stock's own (lagged) volatility and returns.

There is also a growing literature on the cross-sectional relation between option-implied information and stock returns. Once again, some papers use index options to extract marketwide information on pricing factors for the cross-section of stock returns, while other papers use equity options to extract firm-specific information that can be used as a cross-

[20] See Bates (2003) and Garcia, Ghysels, and Renault (2010) for excellent surveys on empirical option pricing.

sectional predictor of returns.[21]

Our paper differs from all of these studies because it investigates the relation between the volatility of the underlying (the stock) and the cross-section of option returns. The literature on the cross-section of equity option returns has also grown rapidly.[22] Boyer and Vorkink (2014) document a negative relation between ex-ante option total skewness and future option returns. Goodman, Neamtiu, and Zhang (2013) find that fundamental accounting information is related to future option returns. Karakaya (2014) proposes a three-factor model to explain the cross-section of equity option returns. Linn (2014) finds that index volatility is priced in the cross-section of option returns. Several recent papers use option valuation models to highlight cross-sectional differences between equity options.[23] As discussed in Section 1.6.1, our work is also related to a series of recent papers that document interesting patterns in the cross-section of delta-hedged option returns related to the volatility of the underlying securities (Goyal and Saretto, 2009; Vasquez, 2012).

We contribute to this growing literature on the cross-section of option returns by highlighting the theoretically expected relation between expected option returns and stock volatility. Stock volatility is often included in a cross-sectional study of option *returns*, because it is a well-known determinant of option *prices*. By being explicit about the relation between volatility and call and put returns, our work not only suggests that empirical

[21] Chang, Christoffersen, and Jacobs (2013) use option-implied index skewness as a pricing factor. Conrad, Dittmar and Ghysels (2013) study the relation between stock returns and volatility, skewness and kurtosis extracted from equity options. The factor used by Ang, Hodrick, Xing, and Zhang (2006) is actually the VIX, so strictly speaking it is about option-implied information as a factor. For additional work, see, for example, Bali and Hovakimian (2009), Cremers and Weinbaum (2010) and Xing, Zhang, and Zhao (2010).

[22] Another literature focus on explaining the cross-section of option prices or implied volatilities rather than option returns. See for instance Duan and Wei (2008) and Bollen and Whaley (2004). An, Ang, Bali, and Cakici (2014) document that stock returns are higher (lower) following increases in call (put) implied volatility, but also link past stock returns with future option-implied volatility.

[23] See Bakshi, Cao, and Zhong (2012) and Gourier (2015) for recent studies. Chaudhuri and Schroder (2015) study the shape of the stochastic discount factor based on equity options.

work on option returns should control for the effect of volatility when identifying other determinants of option returns, it also predicts the sign of the relation between volatility and returns. Our analysis suggests that when studying other determinants of the cross-section of option returns, it is critical to first account for total volatility, and it indicates how volatility affects expected returns. In this sense our work is most closely related to that of Coval and Shumway (2001), who analyze moneyness as a determinant of different expected option returns using returns on index options.

Finally, our work is also relevant for an important literature on the sign of the volatility risk premium embedded in equity options. The consensus in the literature is that while the negative volatility risk premium is very large for the index, it is much smaller or nonexistent for equities. Most of the literature uses parametric models to characterize this risk premium, but some studies, such as Bakshi and Kapadia (2003b) have used the cross-section of delta-hedged option returns and arrive at the same conclusion. Our analysis in Section 1.6.1 shows that these empirical findings may be partly due to hedging errors, which generate a negative relation between volatility and delta-hedged call and put returns.

1.7 Volatility and the Time Series of Index Option Returns

Thus far we have used the cross-section of equity options to provide empirical evidence supporting Propositions 1 and 2. We now turn to the implications of our results for the extensive literature on the time series properties of index option returns.[24] In this section, we explore the time-series implications of Propositions 1 and 2 by studying the relation between monthly S&P 500 index option (SPX) returns and S&P 500 index volatility. Consistent with Proposition 1 and 2, we find that SPX call (put) options tend to have lower (higher) returns in the month following a high volatility month.

[24] This literature includes the work by Jackwerth (2000), Coval and Shumway (2001), Bakshi and Kapadia (2003a), Bondarenko (2003), Jones (2006), Driessen and Maenhout (2007), Driessen, Maenhout, and Vilkov (2009), Santa-Clara and Saretto (2009), Broadie, Chernov, and Johannes (2009), Constantinides et al. (2009, 2011, 2013) and Buraschi, Trojani, and Vedolin (2014).

Propositions 1 and 2 characterize a general property of expected option returns: call (put) option returns decrease (increase) with underlying volatility. This property should hold in the time series of option returns as well as in the cross-section. We investigate the time-series implications of Propositions 1 and 2 by using index option returns to estimate the following time-series regression:

$$R_{t+1}^{i} = constant + \beta_1 VOL_t + \beta_2 Moneyness_t^{i} + \beta_3 R_t^{I} + \epsilon \tag{1.25}$$

where R_{t+1}^{i} is the return on holding index option i from month t to month $t+1$, R_t^I is the return on the S&P 500 in month t and VOL_t is the index volatility. Moneyness (K/S) is also included in the regression because previous studies (e.g., Coval and Shumway, 2001) have shown that moneyness is an important determinant of option returns. Here we consider four proxies for S&P 500 index volatility: 14-day realized volatility, 30-day realized volatility, 60-day realized volatility, and implied volatility. These volatilities are defined as in the cross-sectional analysis and are known in month t.

The slope coefficient estimate on volatility β_1 is the main object of interest. According to Propositions 1 and 2, we expect β_1 to be negative for SPX call options and positive for SPX put options.

Table 1.12 presents the coefficient estimates, t-statistics, and adjusted R-squares for the regressions in equation (1.25). Consistent with Propositions 1 and 2, the slope coefficient on index volatility is always negative (positive) for SPX call (put) options, regardless of the index volatility proxy. For example, column 2 of Panel A of Table 1.12 shows that when using 30-day realized volatility as the volatility proxy, the slope coefficient on index volatility is -0.92 for SPX calls and is highly significant with a t-statistic of -3.78. For a 1% increase in S&P 500 volatility, the return to holding an SPX call option over the next month is expected to decrease by 0.92%. In contrast, in column 2 of Panel B of Table 1.12, the slope coefficient on index volatility for SPX puts is 1.39 and it is also highly statistically significant.

These results are based on the full sample that also contains in-the-money SPX options. However, Table 1.2 indicates that in-the-money SPX options are much less traded than their

Table 1.12: Regressions of Index Option Returns on Index Volatility

Panel A: SPX Calls	Full sample				Only liquid options			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	4.091	4.178	4.285	7.113	9.273	9.211	9.128	10.849
	(5.988)	(6.360)	(6.676)	(14.025)	(9.440)	(9.311)	(9.154)	(11.931)
14 day realized vol	-0.460				-0.194			
	(-1.913)				(-0.548)			
30 day realized vol		-0.921				-0.858		
		(-3.781)				(-2.516)		
60 day realized vol			-1.456				-1.617	
			(-4.778)				(-3.767)	
implied vol				-3.697				-4.444
				(-5.571)				(-4.473)
Adjusted R-square	1.13%	1.22%	1.37%	2.01%	1.67%	1.73%	1.90%	2.54%

Panel B: SPX Puts	Full sample				Only liquid options			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	-4.243	-4.219	-4.216	-4.499	-6.324	-6.139	-6.069	-6.771
	(-8.418)	(-8.230)	(-8.155)	(-9.067)	(-8.971)	(-8.622)	(-8.444)	(-8.704)
14 day realized vol	2.106				2.664			
	(3.918)				(3.881)			
30 day realized vol		1.393				1.887		
		(2.951)				(3.140)		
60-day realized vol			1.070				1.582	
			(2.619)				(2.978)	
implied vol				0.263				0.920
				(0.652)				(1.732)
Adjusted R-square	2.70%	1.73%	1.46%	1.13%	3.18%	2.06%	1.76%	1.27%

at-the-money and out-of-the-money counterparts. To ensure our results are not driven by illiquid in-the-money options, we repeat the regressions in (1.25) using only liquid options. Specifically, we only consider SPX calls with $0.98 \leq K/S \leq 1.10$ and SPX puts with $0.90 \leq K/S \leq 1.02$.

The regression results using only liquid options are presented in columns 5 through 8 in Table 1.12. Consistent with the results using the full sample, we find that the slope coefficient estimate on index volatility is always negative (positive) and statistically significant for SPX calls (puts) regardless of the volatility proxy. For example, when using 60-day realized volatility as a proxy, we find a slope coefficient of -1.62 for SPX calls and 1.58 for SPX puts, and both are highly significant with t-statistics of -3.77 and 2.98 respectively. These results confirm that our findings are not due to illiquid index options.

1.8 Conclusion

This paper analyzes the relation between expected option returns and underlying volatility. In the Black-Scholes-Merton or stochastic volatility model, the expected return on a call is a decreasing function of underlying volatility and the expected put option return is an increasing function of underlying volatility.

Our empirical results confirm this theoretical prediction. We conduct a cross-sectional test using stock options. We find that call (put) options on high volatility stocks tend to have lower (higher) returns over the next month. We also conduct a time-series test using index option returns. Following high volatility periods, index call (put) options tend to have lower (higher) returns over the next month. Our empirical findings are robust to different empirical implementation choices, such as different option samples, weighting methods, and volatility proxies. We also discuss results for straddles and we show that our results are consistent with existing findings on the relation between volatility and delta-hedged option returns.

Our findings are important for the expanding literature on equity option returns. Theory predicts that volatility is an important determinant of expected returns, and therefore volatility should be accounted for when empirically investigating other return determinants. Our findings also have important implications for other areas of finance research. Many financial instruments, such as credit default swaps, callable bonds, and levered equity, to name just a few, have embedded option features. Our theoretical results are also applicable to these assets and we plan to address this in future research. Our analysis can also be extended in several other ways. First, a natural question is if the relation between volatility and returns can be derived without asuming a parametric model. Second, it might be interesting to study the relationships in this paper using more structural rather than reduced-form asset pricing models. Third, Bakshi, Madan, and Panayotov (2010) and Christoffersen, Heston, and Jacobs (2013) propose variance dependent pricing kernels. In future work we plan to investigate the implications of those pricing kernels for the findings in this paper. Finally, the implications of our results for portfolio allocation need to be explored in more detail.

Appendix A: Volatility and Instantaneous Option Returns

We show that expected instantaneous call option return is a decreasing function with respect to σ. We need to show that the elasticity $\frac{\partial O}{\partial S}\frac{S}{O}$, denoted by EL, is a decreasing function of σ. In the Black-Scholes-Merton model, we have

$$EL = \frac{\partial O}{\partial S}\frac{S}{O} = \frac{S_t N(d_1)}{S_t N(d_1) - e^{-r\tau} K N(d_2)}.$$

It follows that

$$\begin{aligned}
\frac{\partial EL}{\partial \sigma} &= \frac{S_t\psi(d_1)\frac{\partial d_1}{\partial \sigma}[S_t N(d_1) - e^{-r\tau}KN(d_2)] - S_t N(d_1)[S_t\psi(d_1)\frac{\partial d_1}{\partial \sigma} - e^{-r\tau}K\psi(d_2)\frac{\partial d_2}{\partial \sigma}]}{[S_t N(d_1) - e^{-r\tau}KN(d_2)]^2} \\
&= \frac{-S_t\psi(d_1)\frac{\partial d_1}{\partial \sigma}e^{-r\tau}KN(d_2) + S_t N(d_1)e^{-r\tau}K\psi(d_2)\frac{\partial d_2}{\partial \sigma}}{[S_t N(d_1) - e^{-r\tau}KN(d_2)]^2} \\
&= \frac{S_t\psi(d_1)\psi(d_2)e^{-r\tau}K[-\frac{\partial d_1}{\partial \sigma}\frac{N(d_2)}{\psi(d_2)} + \frac{N(d_1)}{\psi(d_1)}\frac{\partial d_2}{\partial \sigma}]}{[S_t N(d_1) - e^{-r\tau}KN(d_2)]^2}.
\end{aligned}$$

Clearly, the sign of $\frac{\partial EL}{\partial \sigma}$ will depend on $-\frac{N(d_2)}{\psi(d_2)}\frac{\partial d_1}{\partial \sigma} + \frac{N(d_1)}{\psi(d_1)}\frac{\partial d_2}{\partial \sigma}$, which we show below is always negative. To see this, using the fact that

$$\begin{aligned}
\frac{\partial d_1}{\partial \sigma} &= \sqrt{\tau} - \frac{d_1}{\sigma} \\
\frac{\partial d_2}{\partial \sigma} &= -\sqrt{\tau} - \frac{d_2}{\sigma},
\end{aligned}$$

we have

$$\begin{aligned}
-\frac{\partial d_1}{\partial \sigma}\frac{N(d_2)}{\psi(d_2)} + \frac{N(d_1)}{\psi(d_1)}\frac{\partial d_2}{\partial \sigma} &= -(\sqrt{\tau} - \frac{d_1}{\sigma})\frac{N(d_2)}{\psi(d_2)} + \frac{N(d_1)}{\psi(d_1)}(-\sqrt{\tau} - \frac{d_2}{\sigma}) \\
&= \frac{1}{\sigma}\{-(\sigma\sqrt{\tau} - d_1)\frac{N(d_2)}{\psi(d_2)} + \frac{N(d_1)}{\psi(d_1)}(-\sigma\sqrt{\tau} - d_2)\} \\
&= \frac{1}{\sigma}\{d_2\frac{N(d_2)}{\psi(d_2)} - d_1\frac{N(d_1)}{\psi(d_1)}\}.
\end{aligned}$$

Note that in the last step, we use the fact that $d_2 = d_1 - \sigma\sqrt{\tau}$. Finally, it can be shown that $x\frac{N(x)}{\psi(x)}$ is an increasing function in x, and therefore

$$d_1 > d_2 \Rightarrow d_2\frac{N(d_2)}{\psi(d_2)} - d_1\frac{N(d_1)}{\psi(d_1)} < 0.$$

Appendix B: Proof of Proposition 2

The expected gross return of holding a put option to expiration in (1.4) can be rewritten using the Black-Scholes-Merton formula.

$$\begin{aligned}
R_{put} &= \frac{E_t[\max(K - S_T, 0)]}{P_t(\tau, S_t, \sigma, K, r)} \\
&= \frac{\int^{z^*}(K - S_t e^{\mu\tau - \frac{1}{2}\sigma^2\tau + \sigma\sqrt{\tau}z})\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz}{P_t(\tau, S_t, \sigma, K, r)} \\
&= \frac{e^{\mu\tau}[e^{-\mu\tau}KN(-d_2^*) - S_t N(-d_1^*)]}{e^{-r\tau}KN(-d_2) - S_t N(-d_1)}
\end{aligned} \tag{B.1}$$

$$d_1^* = \frac{\ln\frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln\frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln\frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2 = \frac{\ln\frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}.$$

Taking the derivative with respect to σ in (B.1) yields:

$$\begin{aligned}
\frac{\partial R_{put}}{\partial \sigma} &= \frac{e^{\mu\tau}\sqrt{\tau}S_t\psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - e^{\mu\tau}[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]\sqrt{\tau}S_t\psi(-d_1)}{[e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2} \\
&= \frac{e^{\mu\tau}\sqrt{\tau}S_t\{\psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - \psi(-d_1)[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]\}}{[e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2}
\end{aligned}$$

where we use the fact that the Vega of a put option is $\sqrt{\tau}S_t\psi(-d_1)$. Clearly, the sign of $\frac{\partial R_{put}}{\partial \sigma}$ depends on $\psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - \psi(-d_1)[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]$, which we denote by B. Next we show B is positive. To see this,

$$\begin{aligned}
B &= \psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - \psi(-d_1)[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)] \\
\frac{B}{\psi(-d_1^*)\psi(-d_1)} &= \frac{e^{-r\tau}KN(-d_2) - S_tN(-d_1)}{\psi(-d_1)} - \frac{e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)}{\psi(-d_1^*)}.
\end{aligned}$$

Using the fact that $e^{-r\tau}K\psi(-d_2) = S_t\psi(-d_1)$,

$$\begin{aligned}
\frac{B}{\psi(-d_1^*)\psi(-d_1)} &= \frac{\frac{S_t\psi(-d_1)}{\psi(-d_2)}N(-d_2) - S_tN(-d_1)}{\psi(-d_1)} - \frac{\frac{S_t\psi(-d_1^*)}{\psi(-d_2^*)}N(-d_2^*) - S_tN(-d_1^*)}{\psi(-d_1^*)} \\
&= S_t\{[\frac{N(-d_2)}{\psi(-d_2)} - \frac{N(-d_1)}{\psi(-d_1)}] - [\frac{N(-d_2^*)}{\psi(-d_2^*)} - \frac{N(-d_1^*)}{\psi(-d_1^*)}]\} \\
&= S_t\{[\frac{N(-d_1^*)}{\psi(-d_1^*)} - \frac{N(-d_2^*)}{\psi(-d_2^*)}] - [\frac{N(-d_1)}{\psi(-d_1)} - \frac{N(-d_2)}{\psi(-d_2)}]\}.
\end{aligned}$$

Because the expected rate of return on a risky asset exceeds the risk-free rate ($\mu > r$), we have $d_1^* > d_1$ and $d_2^* > d_2$. One can easily verify that $\frac{N(-d)}{\psi(-d)}$ is a decreasing and convex function in d. It follows that[25]

$$[\frac{N(-d_1^*)}{\psi(-d_1^*)} - \frac{N(-d_2^*)}{\psi(-d_2^*)}] - [\frac{N(-d_1)}{\psi(-d_1)} - \frac{N(-d_2)}{\psi(-d_2)}] > 0.$$

Therefore,

$$B > 0 \Rightarrow \frac{\partial R_{put}}{\partial \sigma} > 0.$$

Appendix C: Holding-Period Expected Option Returns

We derive expected holding-period option returns in the Black-Scholes-Merton model. To save space, we only focus on call options. The analysis of put options proceeds along the same lines. To facilitate the notation, we consider an European call option at time 0 that matures at time T. By definition, the expected return of holding the call option from time 0 to time h ($h < T$) is:

$$R_{call}^h = \frac{E_0\{S_h N(d_1') - e^{-r(T-h)} K N(d_2')\}}{S_0 N(d_1) - e^{-rT} K N(d_2)}$$

where $S_h N(d_1') - e^{-r(T-h)} K N(d_2')$ is the future value of the option at time h, and

$$d_1' = \frac{\ln \frac{S_h}{K} + (r + \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}} \qquad d_2' = \frac{\ln \frac{S_h}{K} + (r - \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}}$$

$$d_1 = \frac{\ln \frac{S_0}{K} + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{\ln \frac{S_0}{K} + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

The expected future value of the option at time h can be split into two pieces:

$$\begin{aligned} E_0\{S_h N(d_1') - e^{-r(T-h)} K N(d_2')\} &= \int_{-\infty}^{\infty} [S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h} z} N(d_1') - e^{-r(T-h)} K N(d_2')] \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\ &= \int_{-\infty}^{\infty} S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h} z} N(d_1') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\ &\quad + \int_{-\infty}^{\infty} -e^{-r(T-h)} K N(d_2') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz. \end{aligned}$$

[25] The second-order derivative of a decreasing and convex function is positive. Effectively $[\frac{N(-d_1^*)}{\psi(-d_1^*)} - \frac{N(-d_2^*)}{\psi(-d_2^*)}] - [\frac{N(-d_1)}{\psi(-d_1)} - \frac{N(-d_2)}{\psi(-d_2)}]$ is the second order derivative of $\frac{N(-d)}{\psi(-d)}$ with respect to d and therefore it is positive.

For the first integral, it can be shown that

$$\begin{aligned}
& \int_{-\infty}^{\infty} S_0 e^{\mu h-\frac{1}{2}\sigma^2 h+\sigma\sqrt{h}z} N(d_1') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\
= \quad & S_0 e^{\mu h} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-\sigma\sqrt{h})^2}{2}} N(\frac{\ln\frac{S_0}{K}+\mu h-\frac{1}{2}\sigma^2 h+\sigma\sqrt{h}z+(r+\frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}}) dz
\end{aligned} \tag{C.1}$$

Define a new variable $z^* = z - \sigma\sqrt{h}$. (C.1) becomes

$$S_0 e^{\mu h} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^{*2}}{2}} N(\frac{\ln\frac{S_0}{K}+(\mu-r)h+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T-h}} + \sqrt{\frac{h}{T-h}} z^*) dz^*. \tag{C.2}$$

Using (see Rubinstein 1984)

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^{*2}}{2}} N(A+Bz^*) = N(\frac{A}{\sqrt{1+B^2}}),$$

(C.2) can be further simplified as

$$S_0 e^{\mu h} N(\frac{\ln\frac{S_0}{K}+(\mu-r)h+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}). \tag{C.3}$$

Following the same steps, the second integral can be rewritten as

$$\int_{-\infty}^{\infty} -e^{-r(T-h)} K N(d_2') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = -e^{-r(T-h)} K N(\frac{\ln\frac{S_0}{K}+(\mu-r)h+(r-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}). \tag{C.4}$$

Putting (C.3) and (C.4) together, we obtain

$$R_{call}^h = \frac{S_0 e^{\mu h} N(\frac{\ln\frac{S_0}{K}+(\mu-r)h+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}) - e^{-r(T-h)} K N(\frac{\ln\frac{S_0}{K}+(\mu-r)h+(r-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}})}{S_0 N(d_1) - e^{-rT} K N(d_2)}.$$

This can be further simplified to

$$\begin{aligned}
R_{call}^h &= \frac{e^{\mu h}[S_0 N(d_1^*) - e^{-[r+(\mu-r)HP]T} K N(d_2^*)]}{S_0 N(d_1) - e^{-rT} K N(d_2)} \qquad \text{(C.5)} \\
d_1^* &= \frac{\ln\frac{S_0}{K}+[HP(\mu-r)+r+\frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \\
d_2^* &= \frac{\ln\frac{S_0}{K}+[HP(\mu-r)+r-\frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}}
\end{aligned}$$

where $HP = h/T$.

Appendix D: Expected Stock Returns and Expected Option Returns

We show that expected call (put) option returns increase (decrease) with expected stock returns: $\frac{\partial R_{call}}{\partial \mu} > 0$ and $\frac{\partial R_{put}}{\partial \mu} < 0$. First, recall from (1.12):

$$R_{call} = \frac{e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}$$

$$d_1^* = \frac{\ln \frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln \frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2 = \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}.$$

Taking the derivative with respect to μ

$$\frac{\partial R_{call}}{\partial \mu} = \frac{\tau e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] + e^{\mu\tau}[\tau e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}$$

where ψ is the probability density function of standard normal distribution. Note that we apply the fact that the Rho of a call option is $\tau e^{-\mu\tau} K N(d_2^*)$ in deriving the above equation. $\frac{\partial R_{call}}{\partial \mu}$ can be further simplified:

$$\begin{aligned}\frac{\partial R_{call}}{\partial \mu} &= \frac{\tau e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] + \tau K N(d_2^*)}{S_t N(d_1) - e^{-r\tau} K N(d_2)} \\ &= \frac{\tau e^{\mu\tau} S_t N(d_1^*)}{S_t N(d_1) - e^{-r\tau} K N(d_2)} > 0.\end{aligned}$$

To see that the derivative is positive, notice that the denominator is just the price of call option which is always positive, and the numerator is obviously greater than zero.

Next we show that the expected put option return is a decreasing function of the expected stock return. Recall that the expected put option return is:

$$R_{put} = \frac{e^{\mu\tau}[e^{-\mu\tau} K N(-d_2^*) - S_t N(-d_1^*)]}{e^{-r\tau} K N(-d_2) - S_t N(-d_1)}$$

where d_1^*, d_2^*, d_1, and d_2 are defined the same as the above. Taking the derivative with respect to μ yields:

$$\begin{aligned}\frac{\partial R_{put}}{\partial \mu} &= \frac{\tau e^{\mu\tau}[e^{-\mu\tau} K N(-d_2^*) - S_t N(-d_1^*)] + e^{\mu\tau}[-\tau e^{-\mu\tau} K N(-d_2^*)]}{e^{-r\tau} K N(-d_2) - S_t N(-d_1)} \\ &= \frac{-\tau e^{\mu\tau} S_t N(-d_1^*)}{e^{-r\tau} K N(-d_2) - S_t N(-d_1)} < 0.\end{aligned}$$

Note the denominator is the price of put option which is always positive, and therefore the ratio itself is negative.

Appendix E: Expected Straddle Returns

We study the relation between expected straddle returns and the underlying volatility. The expected gross return on a straddle is defined as

$$\begin{aligned} R_{straddle} &= \frac{E_t[\max(S_T - K, 0)] + E_t[\max(K - S_T, 0)]}{C_t(\tau, S_t, \sigma, K, r) + P_t(\tau, S_t, \sigma, K, r)} \\ &= \frac{[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]e^{\mu\tau} + [e^{-\mu\tau} K N(-d_2^*) - S_t N(-d_1^*)]e^{\mu\tau}}{S_t N(d_1) - e^{-r\tau} K N(d_2) + e^{-r\tau} K N(-d_2) - S_t N(-d_1)}. \end{aligned}$$

We investigate the impact of volatility on expected straddle returns by taking the derivative of $R_{straddle}$ with respect to σ. It follows that

$$\begin{aligned} \frac{\partial R_{straddle}}{\partial \sigma} &= \frac{2e^{\mu\tau}\sqrt{\tau} S_t \psi(d_1^*) A - 2e^{\mu\tau}\sqrt{\tau} S_t \psi(d_1) B}{[S_t N(d_1) - e^{-r\tau} K N(d_2) + e^{-r\tau} K N(-d_2) - S_t N(-d_1)]^2} \\ &= \frac{2e^{\mu\tau}\sqrt{\tau} S_t \{\psi(d_1^*) A - \psi(d_1) B\}}{[S_t N(d_1) - e^{-r\tau} K N(d_2) + e^{-r\tau} K N(-d_2) - S_t N(-d_1)]^2} \end{aligned}$$

where $A = S_t N(d_1) - e^{-r\tau} K N(d_2) + e^{-r\tau} K N(-d_2) - S_t N(-d_1)$ and $B = S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*) + e^{-\mu\tau} K N(-d_2^*) - S_t N(-d_1^*)$. It is clear that the sign of $\frac{\partial R_{straddle}}{\partial \sigma}$ is determined by $\psi(d_1^*)A - \psi(d_1)B$. This term can be positive or negative depending on underlying parameters.

We now show that $d_2 > 0$ implies $\psi(d_1^*)A - \psi(d_1)B < 0$ and therefore $\frac{\partial R_{straddle}}{\partial \sigma} < 0$. First recall from previous analysis $d_1^* > d_1 > d_2$. We then have

$$d_2 > 0 \Rightarrow 0 < \psi(d_1^*) < \psi(d_1). \tag{E.1}$$

Moreover, note that

$$\frac{\partial A}{\partial r} = \tau e^{-r\tau} K[N(d_2) - N(-d_2)]$$

and therefore,

$$d_2 > 0 \Rightarrow \frac{\partial A}{\partial r} > 0$$

which further implies

$$0 < A < B \tag{E.2}$$

by noting that B is obtained by replacing r with μ in A. Putting together (E.1) and (E.2),

$$d_2 > 0 \Rightarrow \psi(d_1^*)A - \psi(d_1)B < 0 \Rightarrow \frac{\partial R_{straddle}}{\partial \sigma} < 0.$$

Appendix F: Expected Option Returns in the Heston Model

We derive the expected return of holding a call option to expiration in the Heston (1993) stochastic volatility model. The Heston (1993) model assumes that the asset price and its spot variance obey the following dynamics under the physical measure P

$$\begin{aligned} dS_t &= \mu S_t dt + S_t \sqrt{V_t} dZ_1^P \\ dV_t &= \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dZ_2^P \end{aligned}$$

where μ is the drift of the stock price, θ is the long run mean of the stock variance, κ is the rate of mean reversion, σ is the volatility of volatility, and Z_1 and Z_2 are two correlated Brownian motions with $E[dZ_1 dZ_2] = \rho dt$. The dynamics under the risk-neutral measure Q are

$$\begin{aligned} dS_t &= r S_t dt + S_t \sqrt{V_t} dZ_1^Q \\ dV_t &= [\kappa(\theta - V_t) - \lambda V_t]dt + \sigma\sqrt{V_t} dZ_2^Q \end{aligned}$$

where r is the risk-free rate and λ is the market price of volatility risk. Again we consider the expected return of holding a call option to expiration:

$$R_{Call}^{Heston}(S_t, V_t, \tau) = \frac{E_t[\max(S_T - K, 0)]}{C_t(t, T, S_t, V_t))} = \frac{E_t^P[\max(S_T - K, 0)]}{E_t^Q[e^{-r\tau}\max(S_T - K, 0)]}.$$

Heston (1993) provides a closed-form solution to an European call option, up to a univariate numerical integral:

$$C(t, T, S_t, V_t) = E_t^Q[e^{-r\tau}\max(S_T - K, 0)] = S_t P_1 - e^{-r\tau} K P_2 \quad \text{(F.1)}$$

where P_1 and P_2 are given by[26]

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\Big(\frac{e^{-i\phi \ln K} f_j(x, V, \tau; \phi)}{i\phi}\Big)d\phi \quad \text{(F.2)}$$

[26] Note that $x = \ln S$.

$$f_j(x,V,\tau;\phi) = e^{C(\tau;\phi)+D(\tau;\phi)V+i\phi x}$$

$$C(\tau;\phi) = r\phi i\tau + \frac{a}{\sigma^2}\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[\frac{1-ge^{d\tau}}{1-g}]\}$$

$$D(\tau;\phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1-e^{d\tau}}{1-ge^{d\tau}}]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = \kappa\theta, b_1 = \kappa + \lambda - \rho\sigma, b_2 = \kappa + \lambda.$$

By analogy, it can be shown that expected call option payoff at expiration is

$$E_t^P[\max(S_T - K), 0] = e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau}KP_2^*] \tag{F.3}$$

where

$$P_j^* = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}(\frac{e^{-i\phi\ln K}f_j^*(x,V,\tau;\phi)}{i\phi})d\phi \tag{F.4}$$

$$f_j^*(x,V,\tau;\phi) = e^{C(\tau;\phi)+D(\tau;\phi)V+i\phi x}$$

$$C(\tau;\phi) = \mu\phi i\tau + \frac{a}{\sigma^2}\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[\frac{1-ge^{d\tau}}{1-g}]\}$$

$$D(\tau;\phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1-e^{d\tau}}{1-ge^{d\tau}}]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = \kappa\theta, b_1 = \kappa - \rho\sigma, b_2 = \kappa.$$

Putting (F.1) and (F.3) together, the analytical expected holding-to-maturity call option return in Heston model is

$$R_{Call}^{Heston}(S_t, V_t, \tau) = \frac{e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau}KP_2^*]}{S_t P_1 - e^{-r\tau}KP_2}. \tag{F.5}$$

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Chapter 2

The Leverage Effect and the Variance Risk Premium

2.1 Introduction

Understanding risk premium is the central subject of both theoretical and empirical research in financial economics. The literature on risk premium has largely been concerned with the first moment of asset returns.[1] With the development of derivatives markets, investors now can buy and sell variance of asset returns just like the underlying asset. While a long position in the stock market is rewarding, a long position in stock market return variance is associated with large losses on average. For example, in the index option market, long variance strategies such as delta-hedged option portfolio and a zero-beta straddle have negative average returns (Coval and Shumway, 2001; Bakshi and Kapadia, 2003). Recent studies on volatility claims (e.g., variance swaps and VIX futures) which offer investors pure exposure to market variance risk also report a large negative variance premium (Carr and Wu, 2009; Ait-Sahalia, Karaman and Mancini, 2015; Eraker and Wu, 2016).

While the empirical evidence for the presence of a variance risk premium is overwhelming, the economic source of the variance risk premium is less clear. The variance risk premium is formally defined as the difference between physical and risk-neutral expectations of future market return variance. A number of important studies in the literature investigate the equilibrium determinants of the variance risk premium in consumption-based models with Epstein-Zin-Weil preferences (Epstein and Zin, 1989 and 1991; Weil, 1989). Depending on the specification of consumption dynamics, the variance risk premium is attributed

[1] One prominent example is the equity risk premium puzzle literature. There are also numerous studies that focus on cross-sectional differences in average rates of return. For excellent reviews, see Campbell (2003) and Nagel (2012).

to the stochastic volatility (Bansal and Yaron, 2004), the stochastic volatility of volatility (Bollerslev, Tauchen and Zhou, 2009), jumps (Drechsler and Yaron, 2011), or multiple volatility components (Zhou and Zhu, 2015) in the underlying consumption process.

Instead of relating the variance risk premium to a particular part of the underlying consumption process, I show that the variance risk premium is equal to the leverage effect times the price of variance risk. The leverage effect is defined as the conditional covariance between market returns and changes in the conditional market variance. It measures the quantity of variance risk. The price of variance risk depends on both the representative agent's willingness to take on risk (risk aversion) and her willingness to substitute consumption over time (the elasticity of intertemporal substitution, or the EIS). The theoretical relation between the variance risk premium and the leverage effect is derived under a general specification of consumption dynamics and is a common feature of many existing models. I argue the relation between the variance risk premium and the leverage effect can be interpreted as the risk-return trade-off for the second moment of index returns as it is similar to the classical risk-return trade-off where the equity risk premium is equal to risk aversion multiplied by the conditional market variance.

To empirically test this theoretical relation, I estimate the intertemporal relation between the variance risk premium and the leverage effect for the S&P 500 from 1996 to 2014. I construct monthly estimates of the variance risk premium and the leverage effect. In the baseline analysis, I measure the variance risk premium by the difference between realized variance computed from intraday data and (the square of) the VIX index. I compute the leverage effect as the realized covariance between S&P 500 return and changes in its conditional variance using daily data within the month. The daily conditional variance of the S&P 500 is estimated from a rolling regression of realized variance on lagged realized variance and the VIX.

Confirming the theoretical relation implied from Epstein-Zin-Weil preferences, I find a statistically significant negative time-series relation between the market variance risk premium and the market leverage effect. A decomposition analysis suggests that changing expectations about future variance play the key role in driving the time-varying leverage effect and variance risk premium.

The negative relation between the variance risk premium and the leverage effect is robust to different implementations. For example, I estimate the variance risk premium using conditional variances generated from leading variance forecasting models, and I find the results based on conditional variances are even stronger than those based on realized variance. The empirical finding is also not sensitive to the proxy for the leverage effect. I find similar results when computing the leverage effect as conditional covariances instead of realized covariances.

The estimated negative relation between the variance risk premium and the leverage effect contains valuable information on preference parameters. If the representative agent's risk aversion is greater than one, then the negative relation in the data implies the elasticity of intertemporal substitution is less than one as well as a preference for early resolution of uncertainty.

Measuring the variance risk premium requires an estimate of expected future return variance under the risk neutral measure. While risk neutral variance can be reliably computed from option prices, investigating the variance risk premium over a long sample period is difficult because of the short time span of option data. Exploiting the relation between the variance risk premium and the leverage effect, I characterize the historical behavior of the variance risk premium by extrapolating the observed empirical relation in the period 1996-2014 to the period 1926-1995. The extrapolated variance risk premium exhibits non-trivial time variations, and the level is intuitively plausible. The two largest spikes in the extrapolated variance risk premium are in October 1929 (Black Tuesday) and October 1987 (Black Monday).

This paper is related to the growing literature on the variance risk premium. Carr and Wu (2009) propose a robust method for quantifying the variance risk premium, and they find the average variance risk premiums are strongly negative for stock indices. Bollerslev, Tauchen and Zhou (2009) and Drechsler and Yaron (2011) document that the variance risk premium predicts future stock market returns. Bollerslev et al. (2014) provide further evidence based on international data. Todorov (2010) investigates the time-series dynamics of the variance risk premium. This paper further deepens our understanding of the variance risk premium by showing the leverage effect is the systematic risk that determines the variance risk premium.

This paper is also related to the long-standing literature on the leverage effect. In the data, the leverage effect at the index level is strongly negative, reflecting the asymmetric responses of the conditional market variance to positive and negative returns. Two prominent explanations have been proposed in the literature. One relates the leverage effect to financial leverage (e.g., Black, 1976; Christie, 1982; Cheung and Ng, 1992). The other one attributes the leverage effect to the time-varying risk premium, also known as the volatility feedback effect (e.g., French, Schwert and Stambaugh, 1987; Campbell and Hentschel, 1992; Bekaert and Wu, 2000). Bandi and Reno (2012) and Yu (2012) report evidence for a time-varying leverage effect. Consistent with existing studies, I also find a negative and time-varying leverage effect in my sample. More importantly, this paper contributes to the literature by highlighting the theoretical link between the variance premium and the leverage effect.

This paper also contributes to the literature by providing new empirical evidence that the EIS is less than one. There is a considerable debate on the magnitude of the EIS. See, among others, Attanasio and Weber (1989), Campbell and Mankiw (1989), Hall (1998), Vissing-Jorgensen (2002), Attanasio and Vissing-Jorgensen (2003), Campbell (2003), Bansal, Khatchatrian and Yaron (2005), Bansal, Kiku and Yaron (2012) and Beeler and Campbell (2012). Unlike existing studies where inferences often rely on consumption and other macroeconomic data, I estimate the EIS based on the relationship between the variance risk premium and the leverage effect, both of which are measured using market data.

Finally, this paper is related to a vast literature that focuses on the risk-return trade-off for the first moment of the stock market return: the behavior of expected excess return on the market (e.g., the equity risk premium) in relation to its conditional variance.[2] In contrast, this paper focuses on the risk-return trade-off for the second moment of the stock market return.

The rest of the paper is organized as follows. Section 2 derives a theoretical relation between the variance risk premium and the leverage effect. Section 3 conducts empirical estimation of this risk-return relation. Section 4 presents robustness results. Section 5 discusses the implications of the empirical findings. Section 6 contains additional discussion of the main results, and Section 7 concludes the paper.

2.2 The Leverage Effect and the Variance Risk Premium: Theory

In this section, I use a consumption-based general equilibrium framework with Epstein-Zin-Weil preferences to derive a risk-return relation between the market variance risk premium and variance risk. Variance risk is determined by the conditional covariance between market returns and changes in the conditional market variance. I refer to this as the leverage effect.[3] The sign of this relation depends on relative values of risk aversion and the elasticity of intertemporal substitution, the key parameters characterizing the preferences of the representative agent.

[2] The empirical evidence regarding the mean-variance relation is inconclusive. Some studies find a positive relation (e.g., Scruggs, 1998; Ghysels, Santa-Clara and Valkanov, 2005; Guo and Whitelaw, 2006; Lundblad, 2007; Ludvigson and Ng, 2007; Pastor, Sinha and Swaminathan, 2008; Rossi and Timmermann, 2015), but others report a negative relation (e.g., Campbell, 1987; Turner, Startz and Nelson, 1989; Glosten, Jagannathan and Runkle, 1993; Harvey, 2001; Brandt and Kang, 2004). Some studies find no statistically significant relation (e.g., French, Schwert and Stambaugh, 1987; Campbell and Hentschel, 1992).

[3] Note that the leverage effect is sometimes defined in terms of a correlation instead of a covariance.

2.2.1 The Model

I consider a discrete time endowment economy with a simple consumption good and a representative agent who has Epstein-Zin-Weil preferences (Epstein and Zin, 1989 and 1991; Weil, 1989) to learn about the equilibrium determinants of variance risk and the variance risk premium.[4] Epstein-Zin-Weil preferences, by disentangling risk aversion from the elasticity of intertemporal substitution, are able to endogenously generate a variance risk premium. The important implications of Epstein-Zin-Weil preferences for asset pricing have been highlighted in recent studies. See, among others, Bansal and Yaron (2004), Drechsler and Yaron (2011), Wachter (2013) and Campbell et al. (2016).

Epstein-Zin-Weil preferences postulate that the representative agent derives utility not only from current consumption but also from the expected discounted future utility. Specifically, the utility function takes the form:

$$U_t = [(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}} \tag{2.1}$$

where δ is the time discount factor, C_t is current consumption in time t, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. When $\gamma = \frac{1}{\psi}$, the utility function in (2.1) can be solved forward to yield the familiar time-separable, constant relative risk aversion (CRRA) power utility.

Epstein and Zin (1991) show that equation (2.1), combined with the intertemporal budget constraint, implies the following Euler equation:

$$E_t[e^{m_{t+1}+r_{i,t+1}}] = 1. \tag{2.2}$$

Note that $r_{i,t+1}$ is the logarithm of the gross return on asset i, and m_{t+1} is the logarithm of the pricing kernel:

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r_{a,t+1} \tag{2.3}$$

---

[4] Epstein-Zin-Weil preferences build on the early work by Kreps and Porteus (1978). The recursive utility specification is developed in continuous time by Duffie and Epstein (1992).

where $\Delta c_{t+1} = \log(C_{t+1}) - \log(C_t)$, and $r_{a,t+1}$ is the logarithm of the gross return on all invested wealth.

I assume consumption dynamics are driven by a vector of state variables $X_{t+1} \in \mathbb{R}^N$ that follows a first-order VAR:

$$\begin{aligned} X_{t+1} &= \mu + FX_t + G_t z_{t+1} \\ G_t G_t' &= h + \sum^n H_n X_{t,n} \end{aligned} \tag{2.4}$$

where $\mu \in \mathbb{R}^N$, $F \in \mathbb{R}^{N\times N}$, $G_t \in \mathbb{R}^{N\times N}$, $h \in \mathbb{R}^{N\times N}$, $H_n \in \mathbb{R}^{N\times N}$, $X_{t,n}$ is the nth element of X_t, and $z_{t+1} \sim N(0, I)$ is a vector of independent Gaussian shocks. While shocks are independent, the state variables can be correlated with each other if off-diagonal elements of G_t are not zero.[5] $G_t G_t'$ is the conditional variance and covariance matrix of z_{t+1}, and it is specified such that (2.4) falls into the affine class of Duffie, Pan and Singleton (2000). Lastly, without loss of generality, consumption growth Δc_{t+1} is the first element of X_{t+1} and dividend growth Δd_{t+1} is the last element of X_{t+1}. The wealth portfolio is a claim to future consumption and a share of stock represents a claim to future dividend.

Equation (2.4) provides a fairly general framework that nests many existing consumption-based models. See, among others, Bansal and Yaron (2004), Bansal et al. (2014), Bollerslev, Tauchen and Zhou (2009), Campbell et al. (2016), and Zhou and Zhu (2015). Eraker and Shaliastovich (2008) and Drechsler and Yaron (2011) provide detailed discussion on solving this type of model.

To endogenously generate a time-varying variance risk premium that is consistent with the data, it is common to adopt a two-factor structure for the underlying consumption volatility dynamics. Bollerslev, Tauchen and Zhou (2009) and Bollerslev, Sizova and Tauchen (2011) make the volatility of consumption volatility stochastic. Zhou and Zhu (2015) consider a specification where consumption volatility is the sum of two volatility components.

[5] To ensure that the variance premium and any return-volatility correlation are endogeneous to the model, I assume consumption growth-related state variables are uncorrelated with consumption volatility-related state variables. This assumption is also used by Bollerslev, Tauchen and Zhou (2009) and Drechsler and Yaron (2011), for example.

The aforementioned studies can be conveniently mapped into (2.4). Below I show, within this framework, the variance risk premium is a simple function of the leverage effect regardless of the specification of the consumption dynamics. Instead of adding additional volatility factors, an alternative way to generate a variance risk premium is to use jumps (e.g., Drechsler and Yaron, 2011).

2.2.2 The Leverage Effect and the Variance Risk Premium

Given the preferences in (2.1) and the exogenous consumption and dividend dynamics in (2.4), both the variance risk premium and the leverage effect are endogenously determined. Appendix A shows that the equilibrium market return process is given by

$$r_{m,t+1} = \text{constant} + B_m X_{t+1} \tag{2.5}$$

where B_m $(1 \times N)$ is the loading of the market return on state vector X_{t+1}. The conditional variance of next period market return ($\sigma^2_{m,t}$) is given by

$$\sigma^2_{m,t} = B_m G_t G_t' B_m' = B_m(h + \sum^{n} H_n X_{t,n}) B_m'. \tag{2.6}$$

Following the literature, the variance risk premium is defined as the difference between physical and risk-neutral expectations of one-period ahead conditional variances:

$$E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}].$$

Throughout the paper, risk neutral quantities are denoted by Q, and all other quantities are taken to be under the physical measure. To compute the variance risk premium, I also need to solve for the model dynamics under the risk neutral measure. Appendix A shows that the dynamics of the state variables under the risk neutral measure are:

$$X_{t+1} = \mu + F X_t - G_t G_t' \Lambda' + G_t \widetilde{z}_{t+1} \tag{2.7}$$

where $\widetilde{z}_{t+1} \sim N(0, I)$, and Λ represents the market price of risk for shocks to the state variables. From (2.4) and (2.7), it follows that

$$E_t(X_{t+1}) - E^Q_t(X_{t+1}) = G_t G_t' \Lambda'. \tag{2.8}$$

Using (2.6) and (2.8), I compute the variance risk premium:

$$\begin{aligned} E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] &= E_t[B_m(h + \textstyle\sum^n H_n X_{t+1,n})B'_m] - E^Q_t[B_m(h + \textstyle\sum^n H_n X_{t+1,n})B'_m] \\ &= \sum^n B_m H_n [G_t G'_t(n)\Lambda']B'_m. \end{aligned} \tag{2.9}$$

where $G_t G'_t(n)$ denotes the nth row of $G_t G'_t$.

I now compute the leverage effect, defined as the conditional covariance between market return and change in its conditional variance: $\mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$. Using (2.5) and (2.6),

$$\begin{aligned} \mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}) &= \mathrm{cov}_t(B_m X_{t+1}, B_m(h + \sum^n H_n X_{t+1,n})B'_m) \\ &= \sum^n B_m H_n [G_t G'_t(n) B'_m] B'_m. \end{aligned} \tag{2.10}$$

where as before $G_t G'_t(n)$ denotes the nth row of $G_t G'_t$.

From (2.9) and (2.10), Appendix A shows that:

$$E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] = (1-\theta) \times LM \times \mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}) \tag{2.11}$$

where LM is a leverage multiplier. LM arises because the consumption claim and a share of the market have different loadings on the state variables, and it is related to the notion that equity represents a levered exposure to consumption process.

2.2.3 The Risk-Return Trade-Off Between the Variance Risk Premium and the Leverage Effect

For the purpose of this paper, I will set the leverage multiplier equal to one ($LM = 1$). Appendix B shows that this is equivalent to modeling the stock market as a claim to future consumption (Lucas, 1978; Mehra and Prescott, 1985; Bollerslev, Tauchen and Zhou, 2009). In this case, (2.11) becomes:

$$E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] = (1-\theta) \times \mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}). \tag{2.12}$$

Equation (2.12) says the market variance risk premium is equal to the price of variance risk times the quantity of variance risk. The price of variance risk is determined by $(1-\theta)$, which depends on both risk aversion and the EIS of the representative agent. The quantity of variance risk is determined by the leverage effect: $\mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$. Equation (2.12) suggests that it is only the part of variance variations that co-moves with the market that is compensated.

Equation (2.12) is not critically model-dependent in the sense that it is derived under a fairly general framework without assuming a particular specification of the underlying consumption dynamics. The relation between the variance risk premium and the leverage effect holds regardless whether the variance risk premium is caused by stochastic volatility (Bansal and Yaron, 2004), or the stochastic volatility of volatility (Bollerslev, Tauchen and Zhou, 2009), or multiple volatility components (Zhou and Zhu, 2015) in the consumption process. Section 6.2 and 6.3 further illustrate the link between the variance risk premium and the leverage effect using the models proposed by Bansal and Yaron (2004) and Bollerslev, Tauchen and Zhou (2009).

Equation (2.12) is reminiscent of the classical risk-return trade-off for the equity premium (e.g., Merton, 1980), which is based on the following equation:

$$E_t(r_{m,t+1}) - E^Q_t(r_{m,t+1}) = \gamma\sigma^2_{m,t} \tag{2.13}$$

where γ is a measure of aggregate risk aversion, and $\sigma^2_{m,t}$ is the conditional variance of market returns. Equation (2.13) is a statement on the first moment of stock market return, and it describes a risk-return trade-off between the equity risk premium and its systematic risk determined by the conditional market variance. On the other hand, (2.12) is a statement on the second moment of the stock market return, and it characterizes a risk-return relation between the variance risk premium and its systematic risk which is determined by the leverage effect.

2.3 Empirical Analysis

This section begins by describing the data and the construction of the variables used in the empirical analysis. Section 3.2 presents the main empirical results on the intertemporal relation between the variance risk premium and the leverage effect. Section 3.3 performs the decomposition of the leverage effect and relates the variance risk premium to each component of the leverage effect.

2.3.1 Data and Sample

To empirically test (2.12), I construct monthly estimates of the variance premium and the leverage effect from 1996 to 2014. Following Bollerslev, Tauchen and Zhou (2009), at the end of each month I measure the variance risk premium (VRP_t) as the difference between realized variance (RV_t) and risk neutral expected variance (QV_t):

$$VRP_t = RV_t - QV_t.$$

This measure of the variance risk premium is simple and model free. It does not require one to specify a correct model for forecasting variance under the physical measure. In the robustness section, I verify my empirical findings continue to hold when using conditional forecasts of physical variance in the construction of the variance risk premium.

I compute realized variance (RV_t) by summing the squared 5-min log returns on SPY over the month.[6] SPY is an exchange traded fund that targets the performance of the S&P 500. Following the literature, I treat the return from the close of the previous trading day to the open of a new trading day and the return over a weekend as one 5-min interval. Finally, following existing studies I multiply realized variances by 10^4 to convert them in monthly percentage-squared form. As demonstrated in the literature (see, among others, Andersen et al., 2001; Andersen et al., 2003; Barndorff-Nielsen, 2002), realized variance based on high-frequency data provides a better ex-post measure of return variations. I download the intraday data on SPY from the TAQ database. While the TAQ database starts in 1993, data on SPY is limited in the first few years. As a result, my sample begins in 1996 and ends in 2014. Appendix C contains more details on extracting realized variance from intraday data.

[6] Although more sophisticated sampling techniques are available, I follow the recommendation from Liu, Patton and Sheppard (2015) and use the 5-min realized variance.

Table 2.1: Descriptive Statistics

Panel A: Summary Statistics					
	N	Mean	Std.	Skew.	AR-1
RV	228	34.72	52.11	7.24	0.59
QV	228	43.03	37.59	3.22	0.79
VRP	228	-8.31	28.94	6.95	0.15
COV	228	-15.84	76.45	-10.22	0.41
SP	228	0.63	4.45	-0.68	0.08
Panel B: Correlations					
	RV	QV	VRP	COV	SP
RV	1				
QV	0.84	1			
VRP	0.71	0.22	1		
COV	-0.89	-0.64	-0.78	1	
SP	-0.37	-0.43	-0.11	0.38	1

Notes to Table: Panel A of Table 2.1 reports mean, standard deviation (Std.), skewness (Skew.) and first-order autocorrelation coefficient (AR-1) for realized variance (RV), risk neutral variance (QV), the variance risk premium (VRP), the leverage effect (COV) and S&P 500 returns (SP) at monthly frequency. Panel B reports correlations among these variables. RV is calculated as the sum of the squared 5-min log returns on SPY over the month. I multiply realized variances by 10^4 to express them in monthly percentage-squared term. SPY is an ETF that targets the performance of the S&P 500. QV is the squared VIX divided by 12 observed on the last trading day of the month. VRP is the difference between RV and QV. COV is the realized covariance between S&P 500 returns and changes in the conditional variance of the S&P 500 using daily data within the month. The daily conditional market variance is estimated via equation (2.14) based on a rolling window of the past 22 trading days. SP is monthly return of the S&P 500 (in percent). The sample period is from January 1996 to December 2014, a total of 228 months.

I measure the risk neutral expectation of future return variance (QV_t) based on the closing value of the VIX index on the last trading day of each month. Since the VIX is expressed in annualized volatility term (in percent), I divide the squared VIX by 12 to make it comparable to the realized variance. The VIX data is obtained from the Chicago Board Options Exchange (CBOE) website. The CBOE developed the first-ever volatility index in 1993, which then was based on the Black-Scholes-Merton implied volatilities from at-the-money S&P 100 index options. In 2003, the CBOE modified the methodology and started publishing a new VIX index that is based on prices of S&P 500 index options. In this paper, I use the new VIX. The new methodology is motivated by a number of studies that highlight the fact that the market's risk neutral expectation of the total return variation can be recovered from option prices in a model free manner without using any particular option pricing model. For more details on the model free method, see among others Dupire (1994), Neuberger (1994), Carr and Madan (1998), Britten-Jones and Neuberger (2000), and Jiang and Tian (2005).

I calculate the leverage effect (COV_t) as the realized covariance between market return and changes in its conditional variance using daily data over the month:

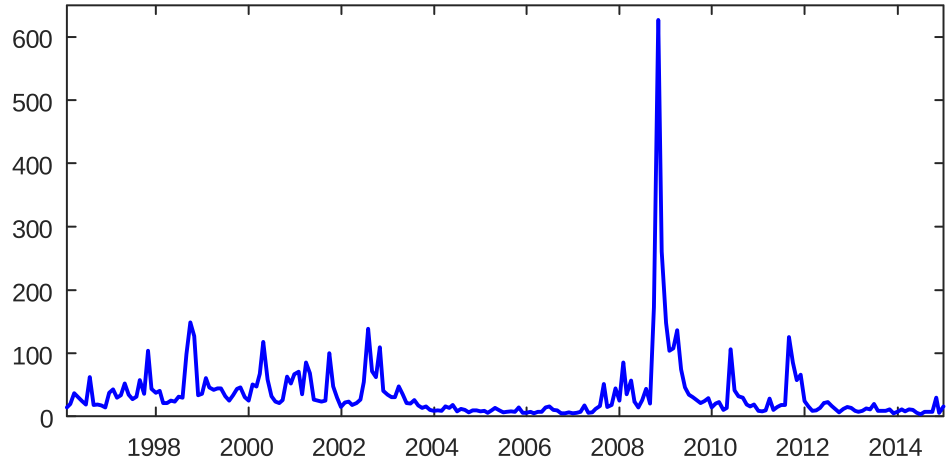
$$COV_t = \text{cov}(r_i, \hat{\sigma}_i^2 - \hat{\sigma}_{i-1}^2)$$

where r_i and $\hat{\sigma}_i^2$ are the log return of the S&P 500 (in percent) and the conditional market variance (in monthly percentage-squared term) in day i of month t, respectively. Daily conditional variances $\hat{\sigma}_i^2$ are obtained as predictive values from the following forecasting model:

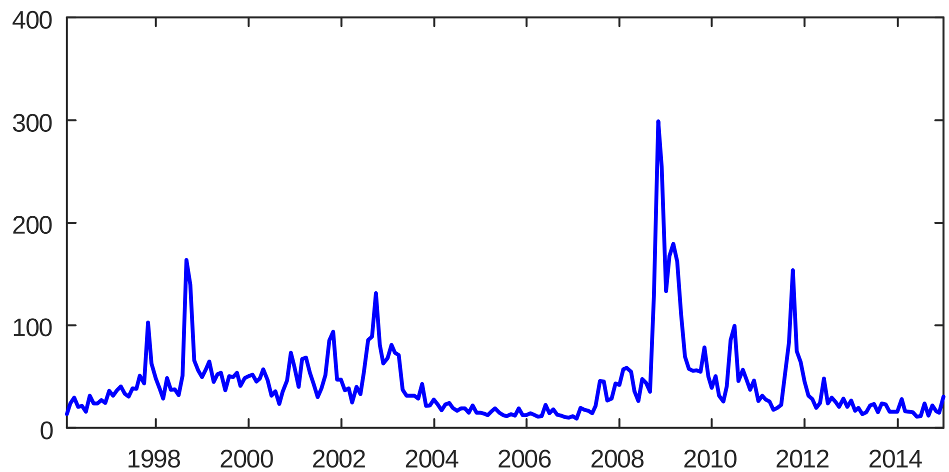
$$RV_i = \delta_0 + \delta_1 RV_{i-1} + \delta_2 QV_{i-1} + \epsilon_i \tag{2.14}$$

where RV_{i-1} and QV_{i-1} are lagged realized variance and squared VIX in day $i-1$. To avoid any looking ahead biases, I compute $\hat{\sigma}_i^2$ at daily frequency using data over the past 22-trading days.

Panel A of Table 2.1 reports summary statistics for RV_t, QV_t, VRP_t, COV_t and monthly returns of the S&P 500 (SP_t). The sample means for the realized and risk neutral variances



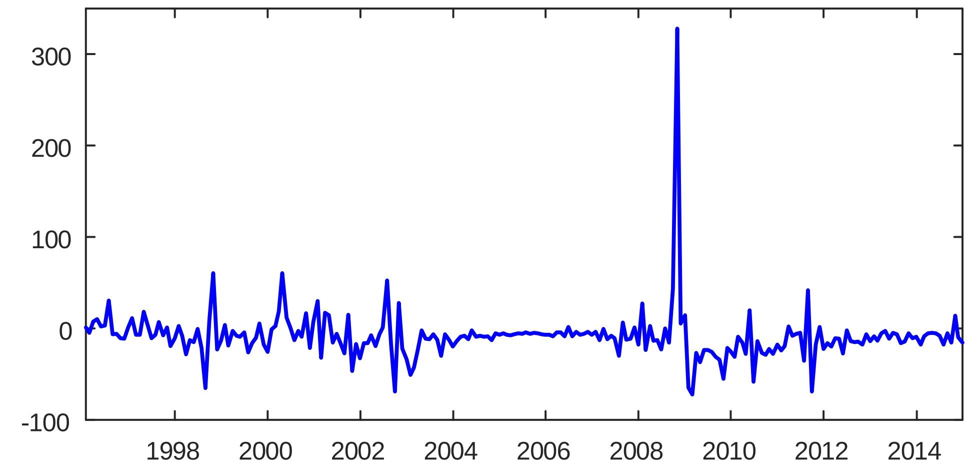
Panel A: Realized Variance



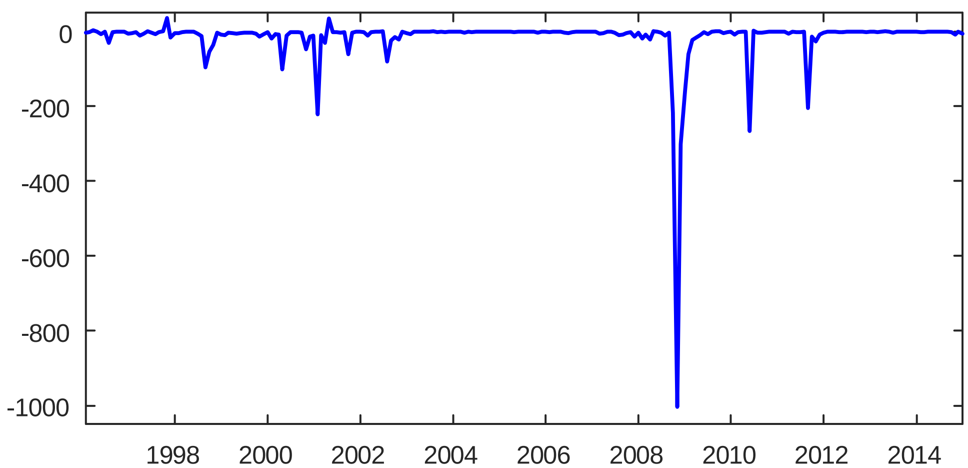
Panel B: Risk Neutral Variance

Notes: This figure plots monthly realized (RV) and risk-neutral variances (QV). RV is calculated as the sum of the squared 5-min log returns on SPY over the month. I multiply realized variances by 10^4 to express them in monthly percentage-squared term. SPY is an ETF that targets the performance of the S&P 500. QV is the squared VIX divided by 12 on the last trading day of the month. The sample period is January 1996 to December 2014.

Figure 2-1: Realized and Risk Neutral Variances



Panel A: Variance Risk Premium



Panel B: Leverage Effect

Notes: This figure plots monthly market variance risk premium (VRP) and leverage effect (COV). VRP is the difference between RV and QV (in monthly percentage-squared term). COV is realized covariance between S&P 500 return and changes in its conditional variance as computed in Table 2.1.

Figure 2-2: The Variance Risk Premium and the Leverage Effect are 34.72 and 43.03, respectively, yielding an unconditional average variance risk premium of -8.31. While stock return variance processes are fairly persistent, the variance risk premium is much less persistent. The first-order autocorrelation coefficient is only 0.15 for VRP_t. These findings are consistent with the existing studies. Panel A also shows that the leverage effect COV_t is negative on average, with an unconditional mean of -15.84 and an AR(1) coefficient of 0.41. Lastly, over the sample period the average monthly return of the S&P 500 is 0.63 percent.

Panel B of Table 2.1 reports correlation coefficients among RV_t, QV_t, VRP_t, COV_t and SP_t. The variance risk premium VRP_t is positively correlated with both physical variance RV_t and risk-neutral variance QV_t. The correlation coefficient is 0.71 and 0.22 respectively. On the other hand, VRP_t is strongly negatively correlated with the leverage effect COV_t, with a coefficient of -0.78. Panel B also indicates that S&P 500 return is negatively correlated with RV_t, QV_t, and VRP_t. In contrast, the correlation is positive between S&P 500 return and the leverage effect.

To better understand the dynamic behavior of these variables, I plot RV_t and QV_t in Figure 2-1, and VRP_t and COV_t in Figure 2-2. Figure 2-1 shows that physical and risk-neutral variances share a lot of commonalities. The correlation between the two time series is 0.84. Todorov (2010) finds that the variance risk premium varies considerably over time. Bandi and Reno (2012) and Yu (2012) document a time-varying leverage effect. Consistent with these studies, Figure 2-2 shows that both VRP_t and COV_t exhibit substantial temporal fluctuations. The striking feature of Figure 2-2 is that the variance risk premium and the leverage effect tend to move in opposite directions, especially during the recent crisis period.

2.3.2 Empirical Results

I test the risk-return relation between the market variance premium and the market leverage effect derived in (2.12) by running the following time-series regression:

$$VRP_t = \alpha + \beta COV_t + \epsilon_t \tag{2.15}$$

where VRP_t and COV_t are computed in Section 3.1.[7] According to theory, VRP_t and COV_t move one-to-one with each other and therefore α should be insignificant. On the other hand, β should be significant although theory provides less guidance on the sign of β. In theory, β can be either positive or negative, depending on the representative agent's willingness to take on risk and her willingness to substitute wealth over time.

Table 2.2 contains results for the estimation of (2.15). Panel A shows that over the full sample period, there is a negative and statistically significant relation between the variance premium and the leverage effect. The estimated coefficient β is -0.29, with a highly significant OLS t-statistic of -18.49. The t-statistic is -12.35 when using Newey and West (1987) standard errors that adjust for autocorrelation and heteroskedasticity. The lag selection follows Newey and West (1994). Panel A also indicates that a substantial fraction of variations in the variance risk premium can be tied to the leverage effect (e.g., the R^2 is 60%), though the estimate on the intercept (α) is negative and statistically significantly different from zero.

Panel B shows that the significance of β is robust to excluding the Great Recession (from December 2007 to June 2009). As is evident in Figure 2-2, the Great Recession features a strong negative correlation between the variance risk premium and the leverage effect, and it is possible that the empirical finding is entirely driven by this particular period. After excluding the Great Recession, the estimated coefficient β is close to the full sample estimate with a magnitude of -0.21, and it remains statistically significant with OLS and Newey-West t-statistics of -5.64 and -3.95, respectively. However, R^2 does drop substantially to only 13%, and the intercept is still significant.

Panel C and D of Table 2.2 report estimation results for two subperiods that are of approximately the same length: 1996 to 2004 and 2005-2014. While the significance of β is robust in both sub-samples, the relation is much stronger in the more recent period of 2005-2014 judging by t-stats and R^2.

In summary, confirming the theoretical relation implied from a consumption-based

[7] I obtain simialr results when suppressing the intercept in the regression.

Table 2.2: Regressing the Variance Risk Premium on the Leverage Effect

	α	β	*Adj.* R^2
Panel A: 1996-2014			
	-12.96	-0.29	60%
OLS	(-10.48)	(-18.49)	
Newey-West	(-7.43)	(-12.35)	
Panel B: Without the Great Recession			
	α	β	*Adj.* R^2
	-11.13	-0.21	13%
OLS	(-9.59)	(-5.64)	
Newey-West	(-8.31)	(-3.95)	
Panel C: 1996-2004			
	α	β	*Adj.* R^2
	-9.40	-0.24	11%
OLS	(-4.91)	(-3.74)	
Newey-West	(-5.05)	(-2.32)	
Panel D: 2005-2014			
	α	β	*Adj.* R^2
	-15.85	-0.30	75%
OLS	(-9.70)	(-19.04)	
Newey-West	(-6.95)	(-11.86)	

Notes to Table: Table 2.2 reports results of the following regression:

$$VRP_t = \alpha + \beta COV_t + \epsilon_t$$

for various sample periods. The market variance risk premium (VRP_t) and the market leverage effect (COV_t) are computed in Table 2.1. Both OLS t-statistics and Newey-West t-statistics that adjust for autocorrelation and heteroskedasticity are reported in brackets. Panel A covers the full sample period from January 1996 to December 2014. Panel B excludes the Great Recession which is from December 2007 to June 2009. Panel C and D represent two sub-periods: January 1996 to December 2004 and January 2005 to December 2014.

model with Epstein-Zin-Weil preferences, I find a statistically significant intertemporal relation between the market variance risk premium and the market leverage effect for the period from 1996 to 2014. I also find the sign of the relation is negative. While a negative risk-return relation may seem counter to common intuition, it does not necessarily contradict the theory. As discussed, the trade-off between the variance premium and variance risk can be either positive or negative, depending on the relative magnitudes of risk aversion and the elasticity of intertemporal substitution. Section 5.1 further discusses how one can utilize the empirical estimate on β to infer the representative agent's preferences.

2.3.3 Decomposing the Leverage Effect

Recall from Section 2, the variance risk premium is related to the leverage effect through the following equation:

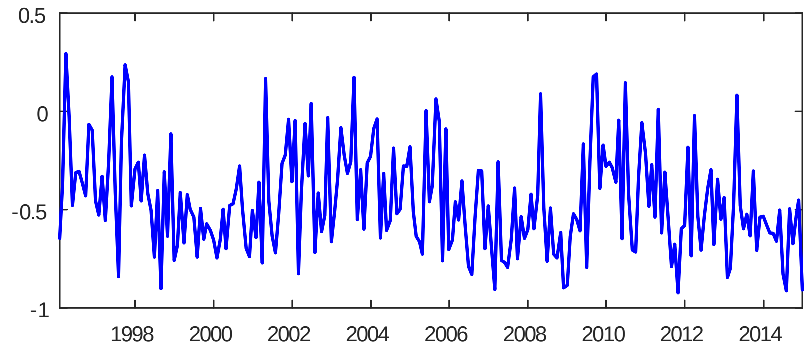
$$E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] = (1-\theta) \times \mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}).$$

Expanding the covariance term, it follows that:

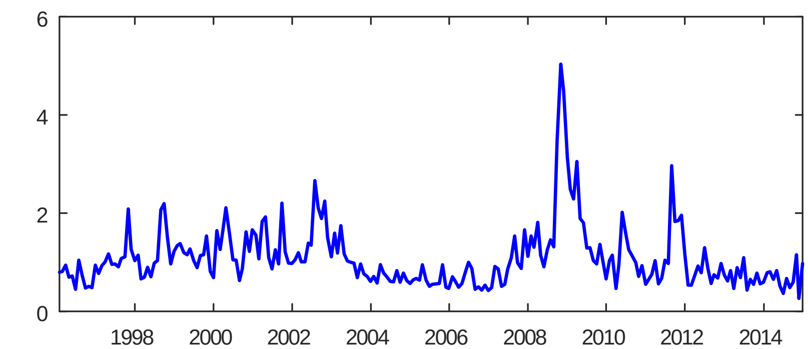
$$E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] = (1-\theta) \times \sigma(r_{m,t+1}) \times \sigma(\sigma^2_{m,t+1} - \sigma^2_{m,t}) \times \rho(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$$

where $\sigma(r_{m,t+1})$ is the standard deviation of market returns, $\sigma(\sigma^2_{m,t+1} - \sigma^2_{m,t})$ is the standard deviation of changes in the conditional variance which is related to kurtosis, and $\rho(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$ is the correlation between changes in the conditional variance and market returns which is related to skewness. This is consistent with the insight from Bakshi and Madan (2006) that the variance risk premium is related to higher-order physical return moments and parameters of the pricing kernel.

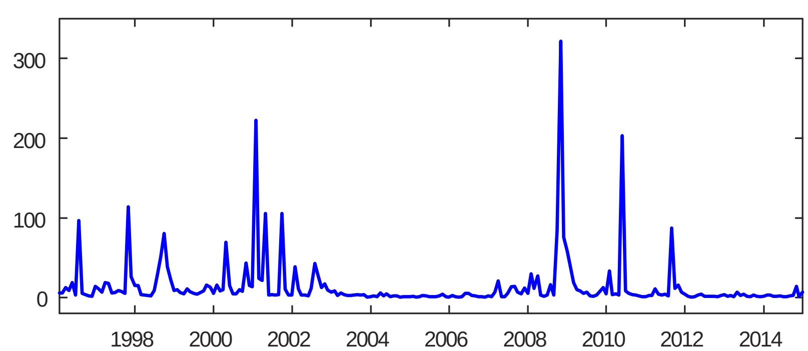
Empirically, I decompose the leverage effect COV_t into the correlation between S&P 500 return and changes in its conditional variance $(\rho_{r,v})$, the standard deviation of S&P 500 returns (σ_r) and the standard deviation of changes in the conditional variance (σ_v). Figure 2-3 plots the three components over time. It seems that σ_v plays the key role in driving the time-varying leverage effect. For example, the huge downward spike of the leverage effect



Panel A: $\rho_{r,v}$



Panel B: σ_r



Panel C: σ_v

Notes: This figure plots the three components of the leverage effect (COV): correlation between S&P 500 return and changes in its conditional variance ($\rho_{r,v}$), the standard deviation of S&P 500 returns (σ_r) and the standard deviation of changes in the conditional variance (σ_v).

Figure 2-3: Decomposing the Leverage Effect

Table 2.3: Decomposing the Leverage Effect

Panel A: Summary Statistics				
	N	Mean	Std.	Skew.
$\rho_{r,v}$	228	-0.45	0.26	0.61
σ_r	228	1.09	0.63	2.63
σ_v	228	13.95	33.63	5.80
Panel B: Regressions				
Intercept	-12.16	-28.58	-16.30	-16.24
	(-4.18)	(-2.93)	(-5.94)	(-3.13)
$\rho_{r,v}$	-8.53			-4.55
	(-1.32)			(-1.05)
σ_r		18.67		-2.27
		(1.91)		(-0.38)
σ_v			0.57	0.60
			(2.71)	(2.33)
Adj. R^2	0%	16%	44%	44%

Notes to Table: Panel A of Table 2.3 reports summary statistics for the three components of the leverage effect (COV): correlation ($\rho_{r,v}$) between S&P 500 return and changes in its conditional variance, the standard deviation of S&P 500 returns (σ_r) and the standard deviation of changes in the conditional variance of the S&P 500 (σ_v). Panel B contains results of regressing the variance risk premium on the three components. Newey-West t-statistics that adjust for autocorrelation and heteroskedasticity are reported in brackets. The sample period is January 1996 to December 2014.

during the recent crisis is largely due to an unprecedented increase in σ_v. Panel A of Table 2.3 reports summary statistics for the three components of the leverage effect.

Panel B of Table 2.3 reports results of regressions of the variance risk premium on the three components of the leverage effect. Taken separately, the correlation $\rho_{r,v}$ itself has no explanatory power for the variance risk premium. The standard deviation of returns σ_r shows modest explanatory power, although it becomes insignificant after controlling for the other two components. The standard deviation of changes in conditional variances σ_v is strongly related to the variance risk premium, accounting for 44% variations in the variance risk premium. Note that 44% is lower than the R^2 with the leverage effect as reported in panel A of Table 2.2. Of course, this is not entirely surprising because theory suggests it is the covariance (product of the three components) that should matter.

2.4 Robustness

In this section, I investigate the robustness of the negative relationship between the variance risk premium and the leverage effect documented in Section 3. I show the empirical finding is robust to alternative measures of the variance risk premium and the leverage effect.

2.4.1 Measuring the Variance Premium

In the main analysis, the variance risk premium is computed as the difference between realized variance over the month (RV_t) and the squared VIX (QV_t). Table 2.4 examines whether empirical results are sensitive to the measurement of the variance risk premium.

Column (1) of Table 2.4 considers a measure of ex-post variance risk premium: the difference between future realized variance (RV_{t+1}) and the squared VIX (QV_t). Column (2) uses the variance risk premium measure in Bollerslev, Tauchen and Zhou (2009). The realized variance used in this paper is based on intraday returns on a S&P 500 ETF, whereas in Bollerslev, Tauchen and Zhou the realized variance is based on the S&P 500 index itself. The data is downloaded from Professor Hao Zhou's website.

A more serious concern is the use of realized variance as a proxy for conditional expected variance. The physical variance process might not be a martingale and thus lagged realized variance is a potentially biased estimate of future realized variance. The biases in the conditional variance will of course render the subsequent estimate of the variance premium imprecise.

To address this concern, columns (3) to (5) of Table 2.4 construct the variance risk premium based on the conditional forecasts of future realized variance. Following the literature, I estimate the conditional expectation of future variance under the physical measure by projecting realized variances on a set of predetermined conditioning variables. In particular, I adopt the following three variance forecasting models:

$$RV_t = \delta_0 + \delta_1 RV_{t-1} + \delta_2 QV_{t-1} + \epsilon_t \tag{2.16}$$

$$RV_t = \delta_0 + \delta_1 RV_{t-1} + \delta_2 RV^W_{t-1} + \delta_3 RV^D_{t-1} + \delta_4 QV_{t-1} + \epsilon_t \tag{2.17}$$

$$RV_t = \delta_0 + \delta_1 RV_{t-1} + \delta_2 RV^W_{t-1} + \delta_3 RV^D_{t-1} + \epsilon_t \tag{2.18}$$

where RV_t and QV_{t-1} are defined in Section 3.1. RV^W_{t-1} and RV^D_{t-1} represent average realized variances over the past 5 trading days and 1 trading day respectively, measured on the last trading day of month $t-1$. The three models come from Drechsler and Yaron (2011), Bekaert and Hoerova (2014) and Corsi (2009). To avoid any looking-ahead biases, I estimate (2.16), (2.17) and (2.18) at monthly frequency in a rolling fashion using only past data. More precisely, every month I estimate the models based on past 24 months data and calculate the forecast for 1-month ahead realized variance based on the estimated models. I then subtract the squared VIX from these conditional forecasts to arrive at conditional variance risk premiums. Because I use the first 24 months to initialize the rolling regression, the resulting sample period for columns (3) to (5) is from December 1997 to December 2014.

Table 2.4 shows that the negative relationship between the variance risk premium and the leverage effect is not affected by different computations of the variance risk premium. Across all specifications, the slope estimate is negative and highly significant. Note that results are in fact stronger when I compute the variance risk premium using conditional forecasts of future return variance. Figure 2-4 plots the variance risk premium measures

Table 2.4: Robustness: Measuring the Variance Risk Premium

	(1)	(2)	(3)	(4)	(5)
α	-15.08	-20.68	-16.60	-20.00	-21.38
	(-6.20)	(-10.36)	(-5.79)	(-4.26)	(-4.33)
β	-0.43				
	(-10.06)				
		-0.19			
		(-7.59)			
			-1.02		
			(-5.82)		
				-1.65	
				(-6.06)	
					-1.53
					(-5.73)
Adj. R^2	62%	40%	73%	71%	74%

Notes to Table: Table 2.4 reports results of the following regression:

$$VRP_t = \alpha + \beta COV_t + \epsilon_t$$

with alternative measures of the variance risk premium (VRP_t). Column (1) computes ex-post variance risk premium as the difference between future realized variance (RV_{t+1}) and the squared VIX (QV_t). Column (2) considers the variance risk premium measure used in Bollerslev, Tauchen and Zhou (2009) where realized variance is computed based on intraday data on the index. Columns (3) to (5) compute variance risk premiums based on conditional expected variances instead of realized variance. The forecasting models for the conditional variance are from Drechsler and Yaron (2011), Bekaert and Hoerova (2014) and Corsi (2009), and they are given in (2.16), (2.17) and (2.18) respectively. The conditional variances are estimated using a rolling window of past 24 months data. Newey-West t-statistics that adjust for autocrrelation and heteroskedasticity are reported in brackets. The sample period is January 1996 to December 2014 for column (1) and (2), and is December 1997 to December 2014 for columns (3) to (5).

used in columns (1) to (3) in Table 2.4.

2.4.2 Measuring the Leverage Effect

In the main analysis, the leverage effect is computed as realized covariance between S&P 500 return and change in its conditional variance using daily data within the month:

$$COV_t = \text{cov}(r_i, \hat{\sigma}_i^2 - \hat{\sigma}_{i-1}^2)$$

where $\hat{\sigma}_i^2$ is estimated at daily frequency using the variance forecasting model in (2.14) based on a rolling window of past 22 trading days. Table 2.5 reports the results of regressing the variance risk premium on eight alternative measures of the leverage effect.

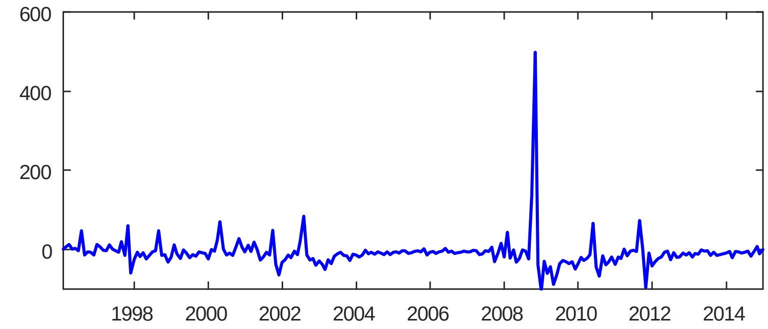
Column (1) of Table 2.5 uses the squared VIX as a proxy for $\hat{\sigma}_i^2$ and compute the leverage effect as realized covariance between S&P 500 returns and changes in the VIX. The VIX provides genuinely forward-looking information and is a natural candidate for conditional variance (see, for example, Ang, Hodrick, Xing and Zhang, 2006). Of course, the VIX contains a potentially time-varying variance risk premium component, and it is likely to be a biased estimate of conditional expected variance under the physical measure.

Instead of using a rolling window, Column (2) of Table 2.5 estimates the variance forecasting model in (2.14) month by month using daily data within each month and takes the fitted values as estimates of $\hat{\sigma}_i^2$.

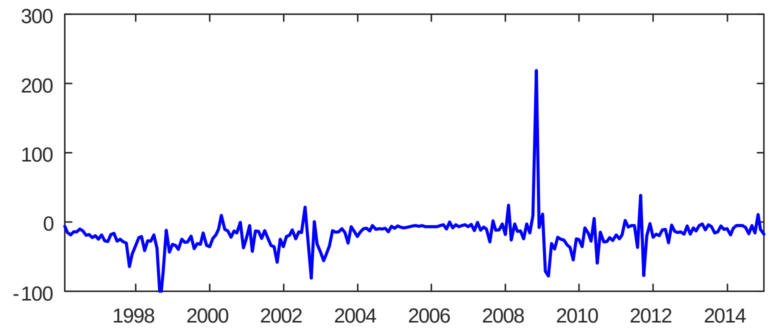
Column (3) considers a new variance forecasting model for estimating $\hat{\sigma}_i^2$. Besides daily realized variance and the VIX, I add the average realized variance over past month and week as additional predictors to the main forecasting model in (2.14). Figure 2-5 plots realized covariances used in columns (1) to (3).

Column (4) considers daily realized variance as a proxy for $\hat{\sigma}_i^2$ and calculate the leverage effect as realized covariance between S&P 500 returns and changes in realized variances.

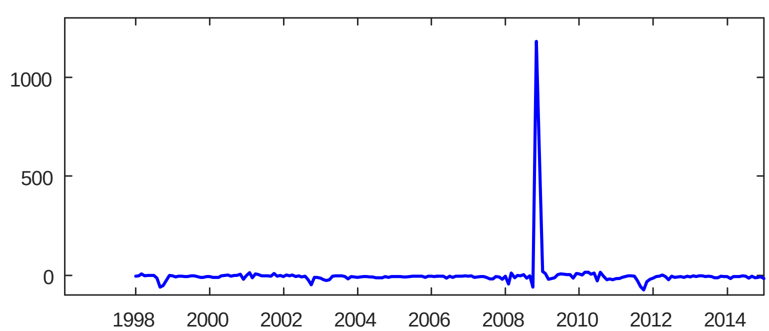
So far I have only focused on realized covariances. Some measures of realized covariance tend to be fairly persistent, which justifies the use of lagged realized covariance as a proxy



Panel A



Panel B



Panel C

Notes: This figure plots alternative measures of the variance risk premium. Panel A plots the difference between ex-post realized variance and the VIX. Panel B plots the variance risk premium used in Bollerslev, Tauchen and Zhou (2009). Panel C plots the difference between expected physical variance and the VIX.

Figure 2-4: Alternative Measures of the Variance Risk Premium

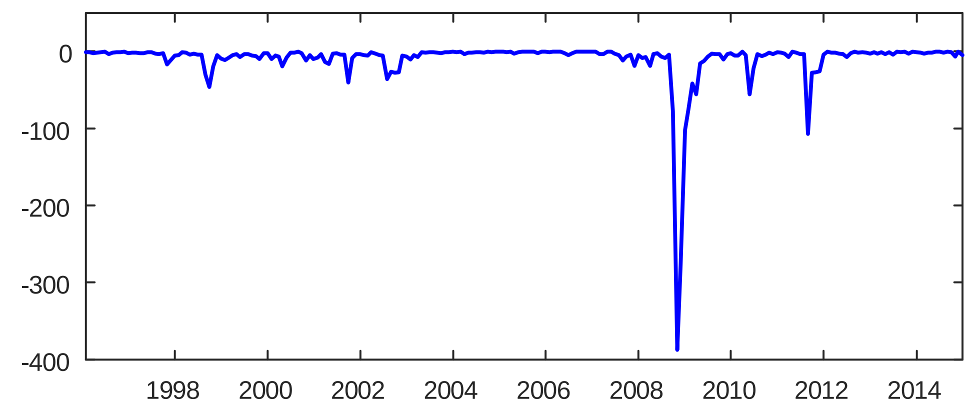
Table 2.5: Robustness: Measuring the Leverage Effect

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	-13.70	-11.61	-12.27	-12.23	-13.16	-13.14	-12.04	-12.70
β	(-7.10)	(-7.76)	(-7.53)	(-5.85)	(-7.90)	(-8.20)	(-7.39)	(-7.47)
	-0.54				-0.28			
	(-4.44)				(-18.21)			
		-0.40				-0.38		
		(-3.07)				(-15.88)		
			-0.16				-0.05	
			(-36.03)				(-23.69)	
				-0.48				-0.26
				(-2.90)				(-10.57)
Adj. R^2	39%	24%	64%	40%	57%	58%	57%	52%

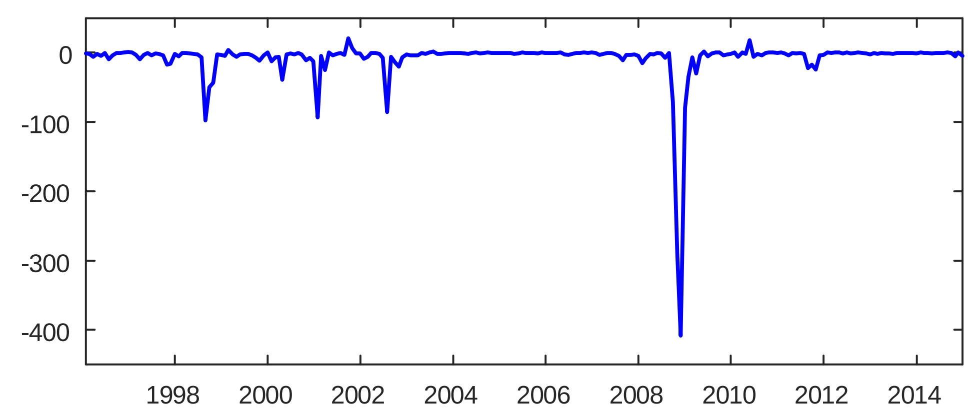
Notes to Table: Table 2.5 reports results of the following regression:

$$VRP_t = \alpha + \beta COV_t + \epsilon_t$$

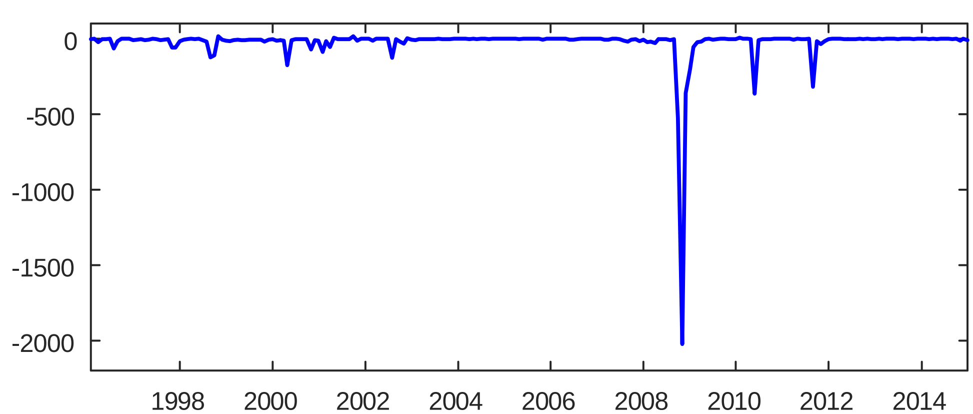
with different measures of the leverage effect (COV_t). Columns (1) to (4) compute the leverage effect as realized covariance between S&P 500 return and changes in its conditional variance using daily data within the month, but with different proxies for daily conditional market variance. Column (1) uses daily squared VIX. Column (2) obtains daily conditional variances as fitted values from estimating the variance forecasting model in (2.16) month by month. Column (3) constructs conditional variances at daily frequency using the forecasting model in (2.17) based on a rolling window of past 22 trading days. Column (4) uses daily realized variance. Columns (5) to (8) report results using expected covariances corresponding to realized counterparts in columns (1) to (4). Expected 1-month ahead covariances are predictive values from the forecasting model in (2.19) based on a rolling window of past 24 months data. Newey-West t-statistics that adjust for autocrrelation and heteroskedasticity are reported in brackets. The sample period is January 1996 to December 2014 for columns (1) to (4) and December 1997 to December 2014 for columns (5)-(8).



Panel A



Panel B



Panel C

Notes: Panels A to C of this figure plot alternative measures of the leverage effect used in columns (1) to (3) in Table 2.5. The sample period is January 1996 to December 2014

Figure 2-5: Alternative Measures of the Leverage Effect for conditional expected covariance.[8] To further ensure empirical findings are not dependent on the use of realized covariances, I also compute conditional covariances. For each realized covariance, I estimate the conditional covariance by projecting realized covariances on lagged realized variances and market returns: :

$$COV_t = \delta_0 + \delta_1 RV_{t-1} + \delta_2 R_{t-1} + \epsilon_t \tag{2.19}$$

where COV_t is realized covariance in month t, and RV_{t-1} and R_{t-1} are realized variance and realized market return in month $t-1$, respectively. I estimate (2.19) based on a rolling window of past 24 months data and use the model estimates to obtain the forecast of next period leverage effect. The choice of the conditioning variables is motivated by Bandi and Reno (2012) who find the time-varying leverage effect is strongly related to the level of market variance, and Yu (2012) who finds that the size and the sign of lagged returns are driving the time-varying leverage effect. The regression results with conditional covariances are reported in columns (5) to (8), corresponding to their realized counterparts in columns (1) to (4). Since I use the first 24 months to initiate the rolling estimation, the resulting sample period for columns (5) to (8) is from December 1997 to December 2014, a total of 205 months.

Table 2.5 shows that regardless of the leverage effect measure, the estimated coefficient β is always negative and statistically significant. In unreported results, I also regress other measures of the variance risk premium against the eight leverage effect measures and obtain similar conclusions.

2.5 Implications

Section 5.1 discusses the implications of a negative relation between the variance risk premium and the leverage effect on investor preferences. Section 5.2 conducts an extrapolation to characterize the historical behavior of the variance risk premium dating back to 1926.

[8]The AR (1) coefficient is 0.66, 0.60, 0.37 and 0.20, respectively for realized covariances in columns (1) to (4).

2.5.1 Preferences

Recall from (2.12), theory states a relationship between the market variance risk premium and the market leverage effect:

$$E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] = (1-\theta) \times \mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, γ and ψ are the representative agent's relative risk aversion and the elasticity of intertemporal substitution, respectively. The sign of the relation $(1-\theta)$ can be either positive or negative depending on relative magnitudes of risk aversion and the EIS.

Table 2.6 shows how the sign of $1-\theta$ is related to γ and ψ, as well as the subsequent implications on investors' attitudes towards the way in which uncertainty about consumption is resolved over time. As is shown in Panel A, if $\gamma = \frac{1}{\psi}$, then $1-\theta = 0$. In this case, Epstein-Zin-Weil preferences collapse to the standard power utility, and the representative agent is indifferent to the timing of the resolution of uncertainty of consumption process.

Panel B considers scenarios where risk aversion is less than one: $\gamma < 1$. In this case, if ψ is between one and $\frac{1}{\gamma}$, then $1-\theta$ is negative. Since $\psi < \frac{1}{\gamma}$, this implies that the representative agent prefers late resolution of consumption uncertainty. On the other hand, when ψ is greater than $\frac{1}{\gamma}$ or less than 1, then $1-\theta$ is positive.

Panel C considers scenarios where risk aversion is greater than one: $\gamma > 1$. In this case, if ψ is between $\frac{1}{\gamma}$ and one, then $1-\theta$ is negative. This implies that the representative agent prefers early resolution of consumption uncertainty as $\psi > \frac{1}{\gamma}$. This also implies that a rise in the market leverage effect lowers the variance risk premium as $1-\theta < 0$. On the other hand, when ψ is greater than 1 or less than $\frac{1}{\gamma}$, $1-\theta$ is greater than zero and a rise in the leverage effect leads to an increase in the variance risk premium.

The above analysis suggests that the empirical behavior of the variance premium in relation to the leverage effect can be used to infer the representative agent's preferences. In the data, $1-\theta$ is estimated to be negative and statistically significant. First, this can be viewed as evidence to reject the null hypothesis that $\psi = \frac{1}{\gamma}$. In other words, Epstein-Zin-Weil preferences are supported by the data.

Table 2.6: Implications for Preferences

	$1-\theta$	Prefer early resolution	Prefer late resolution	Indifferent
Panel A: $\gamma = \frac{1}{\psi}$				
	$1-\theta$	Prefer early resolution	Prefer late resolution	Indifferent
$\gamma = \frac{1}{\psi}$	$= 0$			✓
Panel B: $\gamma < 1$				
	$1-\theta$	Prefer early resolution	Prefer late resolution	Indifferent
$\psi > \frac{1}{\gamma}$	> 0	✓		
$\psi < 1$	> 0		✓	
$1 < \psi < \frac{1}{\gamma}$	< 0		✓	
Panel C: $\gamma > 1$				
	$1-\theta$	Prefer early resolution	Prefer late resolution	Indifferent
$\psi > 1$	> 0	✓		
$\psi < \frac{1}{\gamma}$	> 0		✓	
$\frac{1}{\gamma} < \psi < 1$	< 0	✓		

Notes to Table: Table 2.6 reports implications of different combinations of preference parameters in the Epstein-Zin-Weil utility function on the sign of $1 - \theta$, as well as on investor preference regarding the timing of resolution of consumption uncertainty. In the Epstein-Zin-Weil utility function, ψ is the elasticity of intertemporal substitution, γ is relative risk aversion and $1 - \theta = 1 - \frac{1-\gamma}{1-\frac{1}{\psi}}$.

Moreover, although relative risk aversion is not without controversial, most studies would conclude that a reasonable value of γ should be between 1 and 10 (Mehra and Prescott, 1985). If one believes that the representative agent's risk aversion is greater than one, then a negative estimate on $1-\theta$ implies an EIS that is between $\frac{1}{\gamma}$ and 1. That is, the representative agent is less willing to substitute consumption over time ($\psi < 1$), and she prefers early resolution of the uncertainty of consumption process ($\frac{1}{\gamma} < \psi$).

Note that the above estimate of the EIS is obtained based on market data only. This feature differentiates my analysis from existing studies where consumption and other macroeconomic data are often utilized to make inference on the magnitude of the EIS. The potential issues with consumption data have been long recognized in the literature (e.g., Working, 1960; Breeden, Gibbons and Litzenberger, 1989; Savov, 2011). In fact, the mixed evidence might be in part due to the use of consumption data in empirical tests. For example, Campbell (2003) and Beeler and Campbell (2012) infer the magnitude of the EIS by regressing consumption growth rate on the risk-free rate, and they conclude that the EIS is less than one. On the other hand, Bansal, Khatchatrian and Yaron (2005) and Bansal, Kiku and Yaron (2012) emphasize the negative relation between price dividend ratio and consumption volatility and they conclude that the EIS is greater than 1.

Theory states a conditional relationship between the variance risk premium and the leverage effect

$$E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] = (1-\theta) \times \mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}).$$

One can take unconditional expectations for both sides to derive an unconditional relation

$$E[\sigma^2_{m,t+1}] - E^Q[\sigma^2_{m,t+1}] = (1-\theta) \times \mathrm{cov}(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}).$$

As shown in Table 2.1, on average both the variance risk premium and the leverage effect are negative. This implies that $1-\theta$ is positive, which contradicts the conditional results. Under the assumption that $\gamma > 1$, $1-\theta > 0$ is consistent with two types of preferences. It implies either that $\psi > 1$ and a preference for early resolution of uncertainty, or that $\psi < \frac{1}{\gamma}$ and a preference for later resolution of uncertainty.

Finally, when deriving the above theoretical equation, I assume the leverage multiplier is equal to one. This will not affect inference as long as the leverage multiplier is positive. While a positive leverage multiplier is intuitively sensible and certainly true in many existing models, it is possible under some specification and parameterization the leverage multiplier might be negative. If the leverage multiplier is negative, a negative relation between the variance risk premium and the leverage effect would suggest the opposite results. Note that the difference between conditional and unconditional results still exist.

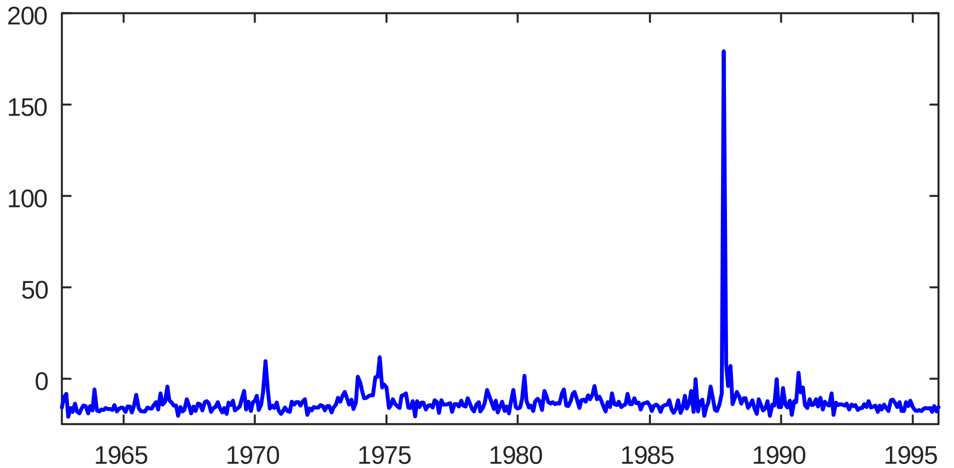
2.5.2 Extrapolating the Variance Risk Premium

This section presents another application of (2.12). To empirically estimate the variance risk premium, one needs to compute an expectation of future return variance under the risk neutral measure. Thanks to the development of options market, this quantity can be estimated reliably with option data. Option data, however, only becomes available very recently and therefore existing studies on the variance risk premium are forced to focus on a short sample period.[9]

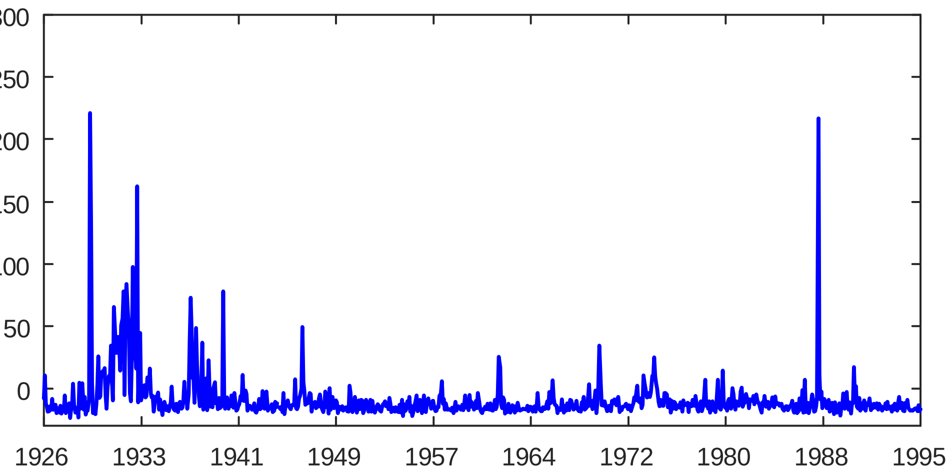
Exploiting the theoretical equivalence between the variance risk premium and the leverage effect, one can estimate the variance risk premium from the leverage effect. The computation of the leverage effect relies only on return data which is readily available for a much longer sample period. I compute the leverage effect for the S&P 500 index going back to 1962 and the CRSP value-weighted market index going back to 1926 when daily return data first becomes available. I extrapolate the variance risk premium to the pre-1996 period based on the observed empirical relationship between the variance premium and the leverage effect in the period of 1996 to 2014.

To construct a long monthly time series of the leverage effect, daily market variances cannot be computed from high-frequency returns because of the short time span of intraday data. Instead, I proxy daily market variance using the French, Schwert and Stambaugh

[9] The VIX starts in 1990. The old VIX (VXO) starts in 1986.



Panel A: S&P 500 Variance Risk Premium 1962-1995



Panel B: CRSP Return Index Variance Risk Premium 1926-1995

Notes: The top panel plots the extrapolated variance risk premium for the S&P 500 index from August 1962 to December 1995. The bottom panel plots the extrapolated variance risk premium for the CRSP Total Return Index from February 1926 to December 1995.

Figure 2-6: Extrapolating the Variance Risk Premium

(1987) measure which is the sum of squared daily log returns, adjusted for autocorrelations:

$$\hat{\sigma}_i^2 = \sum_{j=0}^{21} r_{i-j}^2 + 2\sum_{j=0}^{21} r_{i-j} r_{i-j-1}. \tag{2.20}$$

Note that I compute (2.20) at daily frequency by using the past 22 trading days to generate a time-series of $\hat{\sigma}_i^2$.

Top panel of Figure 2-6 plots the implied variance risk premium for the S&P 500 index from August 1962 to December 1995. Bottom panel plots the implied variance risk premium for the CRSP market index from February 1926 to December 1995. Figure 2-6 shows that the variance risk premium is also time-varying in the pre-option period with occasional large spikes such as October 1987 (Black Monday) and October 1929 (Black Tuesday).

2.6 Discussion

This section includes further analysis on the relationship between the variance risk premium and the leverage effect derived in Section 2. Section 6.1 discusses the variance risk premium in a standard consumption model with power utility. Section 6.2 considers the relationship between the market variance risk premium and the market leverage effect in the long-run risks model of Bansal and Yaron (2004) (BY hereafter). Section 6.3 considers the relationship between the market variance risk premium and the market leverage effect in the stochastic volatility of volatility model of Bollerslev, Tauchen and Zhou (2009) (BTZ hereafter). The two models are chosen because of their importance in the literature as well as the analytical tractability. I follow the notations used in the two papers, respectively.

2.6.1 The Variance Risk Premium with Power Utility

In a standard consumption model (e.g., Rubinstein, 1976; Lucas, 1978; Breeden, 1979), the representative agent is endowed with a power utility function:

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma}$$

where γ is the coefficient of relative risk aversion. In this case, the log pricing kernel takes the form (Campbell, 2003):

$$m_{t+1} = \log\delta - \gamma\Delta c_{t+1} \tag{2.21}$$

where as before $\Delta c_{t+1} = \log(C_{t+1}) - \log(C_t)$. Equation (2.21) suggests that only the transient shock to consumption growth is priced. Equation (2.21) is a special case of the Epstein-Zin-Weil pricing kernel in (2.3) when $\gamma = \frac{1}{\psi}$. As Appendix A shows, the variance risk premium is determined by the conditional covariance between physical variance process and the pricing kernel process:

$$\begin{aligned} E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] &= -\mathrm{cov}_t(\sigma^2_{m,t+1} - \sigma^2_{m,t}, m_{t+1}) \\ &= \gamma\mathrm{cov}_t(\sigma^2_{m,t+1} - \sigma^2_{m,t}, \Delta c_{t+1}) \\ &= 0. \end{aligned}$$

Note that $\mathrm{cov}_t(\sigma^2_{m,t+1} - \sigma^2_{m,t}, \Delta c_{t+1})$ will be equal to zero unless one imposes an exogenous correlation between the two. The fact that the power utility is unable to endogenously generate a variance risk premium has been pointed out by many prior studies. See, among others, Bollerslev, Tauchen and Zhou (2009) and Drechsler and Yaron (2011).

In contrast, substituting for m_{t+1} from the Epstein-Zin-Weil pricing kernel in (2.3), it follows that

$$\begin{aligned} E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] &= -\mathrm{cov}_t(\sigma^2_{m,t+1} - \sigma^2_{m,t}, \theta\log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{a,t+1}) \\ &= \frac{\theta}{\psi}\mathrm{cov}_t(\sigma^2_{m,t+1} - \sigma^2_{m,t}, \Delta c_{t+1}) + (1-\theta)\mathrm{cov}_t(\sigma^2_{t+1} - \sigma^2_{t}, r_{a,t+1}) \\ &= (1-\theta)\mathrm{cov}_t(\sigma^2_{m,t+1} - \sigma^2_{m,t}, r_{a,t+1}). \end{aligned}$$

In summary, the standard power utility framework predicts a zero variance risk premium. In contrast, Epstein-Zin-Weil preferences are able to endogenously generate a variance risk premium because of the extra component in the pricing kernel. The extra component is the direct result of separating the risk aversion from the elasticity of intertemporal substitution.

2.6.2 The Bansal and Yaron (2004) Model

Bansal and Yaron (2004) emphasize the importance of risks that are related to time-varying long-run growth prospects and fluctuating economic uncertainty. Their model is able to quantitatively explain a wide-range of stylized facts in the data and has inspired many subsequent studies. In particular, the BY model specifies the following consumption and dividend dynamics:

$$\begin{aligned} g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1} \\ x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\ g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1} \\ \sigma_{t+1}^2 &= \sigma^2 + v_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1} \end{aligned}$$

where g_{t+1} and $g_{d,t+1}$ are consumption and dividend growth rate, x_{t+1} is the small persistent component in consumption and dividend growth rates, ϕ is the leverage ratio on expected consumption growth and σ_t is the conditional volatility of consumption. e_{t+1}, η_{t+1}, u_{t+1} and w_{t+1} are shocks to the system and are independent of each other. Bansal and Yaron (2004) distinguish the consumption claim from a share of the market, and therefore the computation of the variance risk premium and the leverage effect follows the analysis in Appendix A.

Appendix D casts the BY model into the general framework in Section 2, and computes the variance risk premium according to equation (2.9):

$$E_t[\sigma_{m,t+1}^2] - E_t^Q[\sigma_{m,t+1}^2] = (\beta_{m,e}^2 + \varphi_d^2) A_2 \kappa_1 (1-\theta) \sigma_w^2 \tag{2.22}$$

where $\beta_{m,e}$, A_2 and κ_1 are defined as in Bansal and Yaron (2004). Note that the variance risk premium in (2.22) is constant.

Appendix D also computes the leverage effect according to equation (2.10):

$$\text{cov}_t(r_{m,t+1}, \sigma_{m,t+1}^2 - \sigma_{m,t}^2) = (\beta_{m,e}^2 + \varphi_d^2) A_{2,m} \kappa_{1,m} \sigma_w^2 \tag{2.23}$$

which yields the same result as equation (12) of Bansal and Yaron (2004). Note that the leverage effect is also constant in their model.

Comparing (2.22) and (2.23), one can see that the market variance premium is a linear function of the market leverage effect:

$$E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] = (1-\theta)\frac{A_2\kappa_1}{A_{2,m}\kappa_{1,m}}\mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$$

which not surprisingly is a special case of the general expression derived in (2.11). Note that $\frac{A_2\kappa_1}{A_{2,m}\kappa_{1,m}}$ is the leverage multiplier (LM) and is positive as shown by Bansal and Yaron (2004).

2.6.3 The Bollerslev, Tauchen and Zhou (2009) Model

Bollerslev, Tauchen and Zhou (2009) consider an extension of the long-run risks model of Bansal and Yaron (2004) to account for a time varying variance risk premium that predicts subsequent stock market returns. In particular, they assume the following consumption dynamics:

$$\begin{aligned} g_{t+1} &= \mu_g + \sigma_{g,t} z_{g,t+1} \\ \sigma^2_{g,t+1} &= a_\sigma + \rho_\sigma \sigma^2_{g,t} + \sqrt{q_t} z_{\sigma,t+1} \\ q_{t+1} &= a_q + \rho_q q_t + \varphi_q \sqrt{q_t} z_{q,t+1} \end{aligned}$$

where g_{t+1} and $\sigma_{g,t}$ are consumption growth rate and the conditional volatility of consumption, q_{t+1} is the volatility of the consumption volatility which is the key to generate a time-varying variance risk premium. $z_{g,t+1}$, $z_{\sigma,t+1}$ and $z_{q,t+1}$ are shocks to the system and they are independent of each other. Bollerslev, Tauchen and Zhou (2009) treat the consumption claim the same as the a share of the stock market and therefore the computation of the variance risk premium and the leverage effect follows the analysis in Appendix B.

Appendix E casts the BTZ model into the general framework in Section 2, and computes the market variance risk premium according to (A.15):

$$E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] = (1-\theta)\kappa_1[A_\sigma + A_q\kappa_1^2(A_\sigma^2 + A_q^2\varphi_q^2)\varphi_q^2]q_t \tag{2.24}$$

where κ_1, A_σ, and A_q are defined as in their paper. Equation (2.24) is the same as equation (16) of Bollerslev, Tauchen and Zhou (2009). Note that in the model the variance risk premium is time-varying because the volatility of consumption volatility q_t is time-varying

Appendix E also computes the leverage effect $\text{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$ within their model based on (A.16):

$$\text{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}) = \kappa_1[A_\sigma + A_q\kappa_1^2(A_\sigma^2 + A_q^2\varphi_q^2)\varphi_q^2]q_t. \tag{2.25}$$

As with the variance risk premium, the leverage effect in their model is also time-varying because of q_t.

Comparing (2.24) and (2.25), again one can see:

$$E_t[\sigma^2_{m,t+1}] - E_t^Q[\sigma^2_{m,t+1}] = (1-\theta)\text{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$$

which is exactly the same as (2.12).

2.7 Conclusion

This paper studies the risk-return relation between the variance risk premium and variance risk. In an endowment economy where the representative agent has Epstein-Zin-Weil preferences, variance risk is determined by the leverage effect, defined as the conditional covariance between market returns and changes in the conditional market variance. The sign of the relation between the variance risk premium and the leverage effect depends on both risk aversion and the elasticity of intertemporal substitution of the agent.

Empirically, I document a negative and statistically significant relationship between the variance risk premium and the leverage effect for the S&P 500 from 1996 to 2014. This suggests that the elasticity of intertemporal substitution is less than one and that investors have a preference for early resolution of uncertainty. Exploiting the relation between the variance risk premium and the leverage effect, I also characterize the variance risk premium dynamics going back to 1926.

The analysis in this paper can be extended in a number of ways. While Epstein-Zin-Weil preferences are a natural starting point for understanding the risk-return trade-off between the variance risk premium and variance risk, it would be interesting to incorporate other preferences such as ambiguity aversion or disappointment aversion (e.g., Miao et al., 2012,;

Bonomo et al., 2015). Second, this paper has only focused on the variance risk premium and time-series implications, and extensions to the skew risk premium (Kozhan et al., 2013) or a cross-sectional analysis would be useful. I plan to address these topics in future research.

Appendix A

This appendix computes the market variance risk premium and the market leverage effect. Following Bansal and Yaron (2004), Drechsler and Yaron (2011) and Bollerslev, Tauchen and Zhou (2009), the solution to finding an equilibrium in this type of model is as follows. First, I solve for the return on the consumption claim $r_{a,t+1}$ as it shows up in the pricing kernel.

I conjecture that the solution for the logarithm of wealth-consumption ratio (the price-dividend ratio for the consumption claim) v_t, is a linear function of the state variables :

$$v_t = A_0 + A^{'} X_t \tag{A.1}$$

where $A^{'}$ is a $1 \times N$ row vector of loadings on the state variables that need to be solved endogenously. Next, I can write the return on the consumption claim as a linear function of price-dividend ratios using the Campbell-Shiller (1988) log-linear approximation:

$$r_{a,t+1} = \kappa_0 + \kappa_1 v_{t+1} - v_t + \Delta c_{t+1} \tag{A.2}$$

where κ_0 and κ_1 are parameters of linearization. Dividend growth is replaced with consumption growth since the consumption claim pays aggregate consumption as its dividend. Substituting for v_t and v_{t+1} from equation (A.1) into equation (A.2), I have

$$\begin{aligned} r_{a,t+1} &= \kappa_0 + \kappa_1 (A_0 + A^{'} X_{t+1}) - A_0 - A^{'} X_t + \Delta c_{t+1} \\ &= \kappa_0 + \kappa_1 A_0 - A_0 - A^{'} X_t + B X_{t+1} \end{aligned} \tag{A.3}$$

where $B = \kappa_1 A^{'} + e_c$, $e_c = [1, 0....0]$ is a $1 \times N$ row vector that selects Δc_{t+1} from X_{t+1}, as Δc_{t+1} is the first element in X_{t+1}. The Euler equation in (2.2) must hold for all returns including the return on the consumption claim. One can solve for A_0 and $A^{'}$ by substituting (A.3) into (2.2) and evaluating the resulting expectation. Depending on specifications, a closed-form solution is not guaranteed. For more details on this, see Drechsler and Yaron (2011).

Once A_0 and $A^{'}$ are known, one can substitute (A.3) into (2.3) to obtain the endogenous pricing kernel that prices all assets:

$$\begin{aligned} m_{t+1} &= \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{a,t+1} \\ &= \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)[\kappa_0 + \kappa_1 (A_0 + A^{'} X_{t+1}) - A_0 - A^{'} X_t + \Delta c_{t+1}] \\ &= \theta \log \delta + (\theta - 1)\kappa_0 + (\theta - 1)(\kappa_1 - 1) A_0 - (\theta - 1) A^{'} X_t - \Lambda X_{t+1} \end{aligned}$$

where I use the fact that $\frac{\theta}{\psi} + 1 - \theta = \gamma$, and let $\Lambda = (1 - \theta)\kappa_1 A^{'} + \gamma e_c$. As before, $e_c = [1, 0....0]$ is the selector vector for Δc_{t+1}. Λ represents the market prices of risks of fundamental shocks to the state variables.

To compute the market variance risk premium and the market leverage effect, one needs to solve for the equilibrium price-dividend ratio and return for the stock market, which represents a claim to future dividends. As before, I apply the Campbell-Shiller approximation to the return on the stock market:

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} v_{m,t+1} - v_{m,t} + \Delta d_{t+1} \tag{A.4}$$

where $v_{m,t}$ is price-dividend ratio of the market. Note that Δd_{t+1} has its own dynamics, and cannot be replaced with Δc_{t+1} as in the case of the consumption claim. I guess the solution for $v_{m,t}$ takes the form:

$$v_{m,t} = A_{0,m} + A_m^{'} X_t \tag{A.5}$$

where $A_m^{'}$ is a $1 \times N$ row vector of loadings on the state variables. Substituting for $v_{m,t}$ and $v_{m,t+1}$ from equation (A.5) into equation (A.4), I have:

$$\begin{aligned} r_{m,t+1} &= \kappa_{0,m} + \kappa_{1,m}(A_{0,m} + A_m^{'} X_{t+1}) - A_{0,m} - A_m^{'} X_t + \Delta d_{t+1} \\ &= r_{0,m} - A_m^{'} X_t + B_m X_{t+1} \qquad \text{(A.6)} \end{aligned}$$

where $r_{0,m}$ includes all constants, $B_m = \kappa_{1,m} A_m^{'} + e_d$, and $e_d = [0, 0....1]$ is a $1 \times N$ row vector that selects Δd_{t+1} from X_{t+1}. As before, one can solve for $A_{0,m}$ and $A_m^{'}$ by substituting (A.6) into the Euler equation (2.2).

Substituting (2.4) into (A.6), I have:

$$r_{m,t+1} = r_{0,m} + (B_m F - A_m^{'})X_t + B_m G_t z_{t+1} \tag{A.7}$$

which implies the conditional variance of next period market return at time t is

$$\sigma^2_{m,t} = B_m G_t G_t' B_m'. \tag{A.8}$$

Recall from equation (2.4), state variables have the following dynamics under the physical measure:

$$X_{t+1} = \mu + F X_t + G_t z_{t+1}. \tag{A.9}$$

Risk neutral probabilities are just physical probabilities re-weighted by investors' risk preferences as characterized by the pricing kernel (e.g., the Radon-Nikodym derivative $\frac{dQ}{dP} = \frac{M_{t+1}}{E_t(M_{t+1})}$). Drechsler and Yaron (2011) show that the dynamics of the state variables under the risk neutral measure are:

$$X_{t+1} = \mu + F X_t - G_t G_t' \Lambda' + G_t \widetilde{z}_{t+1} \tag{A.10}$$

where $\widetilde{z}_{t+1} \sim N(0, I)$ and the shift in mean $-G_t G_t' \Lambda'$ reflects risk adjustment. From (A.9) and (A.10),

$$E_t(X_{t+1}) - E_t^Q(X_{t+1}) = G_t G_t' \Lambda'.$$

The market variance risk premium can be computed as follows:

$$\begin{aligned}
E_t[\sigma^2_{m,t+1}] - E_t^Q[\sigma^2_{m,t+1}] &= E_t[B_m(h + \textstyle\sum^n H_n X_{t+1,n})B_m'] - E_t^Q[B_m(h + \textstyle\sum^n H_n X_{t+1,n})B_m'] \\
&= \sum^n B_m H_n [E_t(X_{t+1,n}) - E_t^Q(X_{t+1,n})] B_m' \\
&= \sum^n B_m H_n [G_t G_t'(n) \Lambda'] B_m'. \qquad \text{(A.11)}
\end{aligned}$$

where in the last line I use $E_t(X_{t+1}) - E_t^Q(X_{t+1}) = G_t G_t' \Lambda'$, and $G_t G_t'(n)$ denotes the nth row of $G_t G_t'$. Note that the market variance risk premium can be conveniently computed as the conditional covariance between the pricing kernel process and physical variance process:

$E_t[\sigma^2_{m,t+1}] - E_t^Q[\sigma^2_{m,t+1}] = -\text{cov}_t(m_{t+1}, \sigma^2_{m,t+1})$. To see this,

$$\begin{aligned} -\text{cov}_t(m_{t+1}, \sigma^2_{m,t+1}) &= -\text{cov}_t(-\Lambda X_{t+1}, B_m(h + \sum^n H_n X_{t+1,n})B'_m) \\ &= \sum^n \text{cov}_t(\Lambda X_{t+1}, B_m H_n X_{t+1,n} B'_m) \\ &= \sum^n B_m H_n [G_t G'_t(n)\Lambda'] B'_m \end{aligned}$$

The market leverage effect $\text{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$ can be computed as follows. Using (A.7) and (A.8),

$$\begin{aligned} \text{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}) &= \text{cov}_t(B_m X_{t+1}, B_m(h + \sum^n H_n X_{t+1,n})B'_m) \\ &= \sum^n B_m H_n [G_t G'_t(n) B'_m] B'_m. \end{aligned} \tag{A.12}$$

From (A.11) and (A.12), one can characterize the relationship between the market variance risk premium and the market leverage effect. Under the assumption that $G_t G'_t$ does not depend on Δc_{t+1} and Δd_{t+1}[10],

$$E_t[\sigma^2_{m,t+1}] - E_t^Q[\sigma^2_{m,t+1}] = (1-\theta)\sum^n \frac{\kappa_1 A'(n)}{\kappa_{1,m} A'_m(n)} B_m H_n G_t G'_t(n) B'_m(n) B'_m \tag{A.13}$$

Equation (A.13) says the variance risk premium is equal to the weighted sum of each component of the leverage effect. The weights are given by $\frac{\kappa_1 A'(n)}{\kappa_{1,m} A'_m(n)}$. $\frac{\kappa_1 A'(n)}{\kappa_{1,m} A'_m(n)}$ arises because the consumption claim and a share of the market have different loadings on the state variables, and it is related to the notion that equity represents a levered exposure to consumption process. With a slight abuse of notation, I write (A.13) as

$$E_t[\sigma^2_{m,t+1}] - E_t^Q[\sigma^2_{m,t+1}] = (1-\theta) \times LM \times \text{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t})$$

Appendix B

This appendix assumes that a share of the market represents a claim to future consumption. In other words, the market portfolio is the same as the wealth portfolio. Now I compute

[10] The assumption that $G_t G'_t$ does not depend on Δc_{t+1} and Δd_{t+1} holds in many existing models including Bansal and Yaron (2004), Bollerslev, Tauchen and Zhou (2009), Drechsler and Yaron (2011), and Zhou and Zhu (2015), to just name a few.

the variance risk premium and the leverage effect for the wealth portfolio. From (A.3), the conditional variance of next period market return at time t is given by:

$$\sigma^2_{m,t} = BG_tG'_tB' \tag{A.14}$$

which of course is known at time t.

It follows that

$$\begin{aligned} E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] &= E_t[B(h + \textstyle\sum^n H_n X_{t+1,n})B'] - E^Q_t[B(h + \textstyle\sum^n H_n X_{t+1,n})B'] \\ &= \sum^n BH_n[E_t(X_{t+1,n}) - E^Q_t(X_{t+1,n})]B' \\ &= \sum^n BH_n[G_tG'_t(n)\Lambda']B' \end{aligned} \tag{A.15}$$

where $G_tG'_t(n)$ denotes the nth row of $G_tG'_t$. As before, the variance risk premium can also be conveniently computed as the conditional covariance between the price kernel process and physical variance process: $E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] = -\mathrm{cov}_t(m_{t+1}, \sigma^2_{m,t+1})$. To see this,

$$\begin{aligned} -\mathrm{cov}_t(m_{t+1}, \sigma^2_{m,t+1}) &= -\mathrm{cov}_t(-\Lambda X_{t+1}, B(h + \sum^n H_n X_{t+1,n})B^{'}) \\ &= \sum^n \mathrm{cov}_t(\Lambda X_{t+1}, BH_n X_{t+1,n} B^{'}) \\ &= \sum^n BH_n[G_tG'_t(n)\Lambda^{'}]B^{'}. \end{aligned}$$

Using (A.3) and (A.14), I compute the leverage effect as follows,

$$\begin{aligned} \mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}) &= \mathrm{cov}_t(BX_{t+1}, B(h + \sum^n H_n X_{t+1,n})B') \\ &= BH_n[\sum^n \mathrm{cov}_t(X_{t+1,n}, X_{t+1})B']B' \\ &= \sum^n BH_n[G_tG'_t(n)B']B' \end{aligned} \tag{A.16}$$

where $G_tG'_t(n)$ denotes the nth row of $G_tG'_t$.

Equations (A.15) and (A.16) implies,

$$\begin{aligned} E_t[\sigma^2_{m,t+1}] - E^Q_t[\sigma^2_{m,t+1}] &= \sum^n BH_n[G_tG'_t(n)\Lambda']B' \\ &= (1-\theta)\sum^n BH_n[G_tG'_t(n)B']B' \\ &= (1-\theta)\mathrm{cov}_t(r_{m,t+1}, \sigma^2_{m,t+1} - \sigma^2_{m,t}) \end{aligned}$$

where as defined before $B = \kappa_1 A' + e_c$, and $\Lambda = [(1-\theta)\kappa_1 A' + \gamma e_c]$. The assumption that $G_t G_t'$ does not depend on Δc_{t+1} ensures that I can move $1-\theta$ out of the summation.

Appendix C

In this paper, realized variance is computed using 5-minute intraday returns. To calculate intraday returns, I construct a total of 78 5-minute intervals during the regular trading hours from 9:30 EST to 16:00 EST. I use the trade price lastly recorded in the interval to compute returns. If there is no trade observed in an interval, the return on that interval is set to zero. Careful cleaning is necessary for high frequency data. Following Barndorff-Nielsen et al. (2011) and Christensen et al. (2010), I apply the following filters before sampling intraday stock prices:

1) delete entries with a correction indicator not equal to zero;

2) delete entries with abnormal sale condition. (Trades where COND has a non-missing code except for "@", "E", "F", "@E", "@F");

3) delete entries with price less than or equal to zero and entries with trade size less than or equal to zero.

Appendix D

This appendix computes the market variance risk premium and the market leverage effect in the Bansal and Yaron (2004) model. First, I cast their model into the general framework laid out in Section 2.1:

$$
\underbrace{\begin{bmatrix} g_{t+1} \\ x_{t+1} \\ g_{d,t+1} \\ \sigma^2_{t+1} \end{bmatrix}}_{X_{t+1}} = \underbrace{\begin{bmatrix} \mu \\ 0 \\ \mu_d \\ (1-v_1)\sigma^2 \end{bmatrix}}_{\mu} + \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & \phi & 0 & 0 \\ 0 & 0 & 0 & v_1 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} g_t \\ x_t \\ g_{d,t} \\ \sigma^2_t \end{bmatrix}}_{X_t} + \underbrace{\begin{bmatrix} \sigma_t & 0 & 0 & 0 \\ 0 & \varphi_e\sigma_t & 0 & 0 \\ 0 & 0 & \varphi_d\sigma_t & 0 \\ 0 & 0 & 0 & \sigma_w \end{bmatrix}}_{G_t} \underbrace{\begin{bmatrix} \eta_{t+1} \\ e_{t+1} \\ u_{t+1} \\ w_{t+1} \end{bmatrix}}_{z_{t+1}}
$$

$$
G_t G_t' = \begin{bmatrix} \sigma^2_t & 0 & 0 & 0 \\ 0 & \varphi_e^2\sigma_t^2 & 0 & 0 \\ 0 & 0 & \varphi_d^2\sigma_t^2 & 0 \\ 0 & 0 & 0 & \sigma_w^2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_w^2 \end{bmatrix}}_{h} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \varphi_e^2 & 0 & 0 \\ 0 & 0 & \varphi_d^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{H_\sigma} \sigma_t^2.
$$

In the BY model, there are three types of shocks that are priced: shocks to consumption growth η_{t+1}, to the long run component of consumption growth e_{t+1} and to the time-varying consumption volatility w_{t+1}. The corresponding market prices of risks are:

$$
\Lambda = [\ \gamma \quad (1-\theta)\kappa_1 A_1 \quad 0 \quad (1-\theta)\kappa_1 A_2\ ].
$$

Moreover, loadings of the market return on fundamental shocks are given by:

$$
B_m = [\ 0 \quad \kappa_{1,m}A_{1,m} \quad 1 \quad \kappa_{1,m}A_{2,m}\ ].
$$

Now I compute the variance risk premium in the BY model based on the general expression in (2.9):

$$
\begin{aligned}
VRP_t &= [\ 0 \quad \kappa_{1,m}A_{1,m} \quad 1 \quad \kappa_{1,m}A_{2,m}\ ] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \varphi_e^2 & 0 & 0 \\ 0 & 0 & \varphi_d^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \left([\ 0 \quad 0 \quad 0 \quad \sigma_w^2\ ] \begin{bmatrix} \gamma \\ (1-\theta)\kappa_1 A_1 \\ 0 \\ (1-\theta)\kappa_1 A_2 \end{bmatrix} \right) \begin{bmatrix} 0 \\ \kappa_{1,m}A_{1,m} \\ 1 \\ \kappa_{1,m}A_{2,m} \end{bmatrix} \\
&= [\kappa_{1,m}^2 A_{1,m}^2 \varphi_e^2 + \varphi_d^2](1-\theta)\kappa_1 A_2 \sigma_w^2 \\
&= (\beta_{m,e}^2 + \varphi_d^2) A_2 \kappa_1 (1-\theta)\sigma_w^2
\end{aligned}
$$

where $\beta_{m,e}^2 = \kappa_{1,m}^2 A_{1,m}^2 \varphi_e^2$, following the notation in the BY model.

To compute the leverage effect in the model, I apply the general expression derived in (2.10):

$$
\begin{aligned}
COV_t &= \begin{bmatrix} 0 & \kappa_{1,m}A_{1,m} & 1 & \kappa_{1,m}A_{2,m} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \varphi_e^2 & 0 & 0 \\ 0 & 0 & \varphi_d^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 & 0 & 0 & \sigma_w^2 \end{bmatrix} \begin{bmatrix} 0 \\ \kappa_{1,m}A_{1,m} \\ 1 \\ \kappa_{1,m}A_{2,m} \end{bmatrix} \right) \begin{bmatrix} 0 \\ \kappa_{1,m}A_{1,m} \\ 1 \\ \kappa_{1,m}A_{2,m} \end{bmatrix} \\
&= [\kappa_{1,m}^2 A_{1,m}^2 \varphi_e^2 + \varphi_d^2]\kappa_{1,m}A_{2,m}\sigma_w^2 \\
&= (\beta_{m,e}^2 + \varphi_d^2)A_{2,m}\kappa_{1,m}\sigma_w^2.
\end{aligned}
$$

Appendix E

This appendix computes the variance risk premium and the leverage effect in the Bollerslev, Tauchen and Zhou (2009) model. As before, I first cast the BTZ model into the general framework laid out in Section 2.1:

$$
\underbrace{\begin{bmatrix} g_{t+1} \\ \sigma_{g,t+1}^2 \\ q_{t+1} \end{bmatrix}}_{X_{t+1}} = \underbrace{\begin{bmatrix} \mu_g \\ a_\sigma \\ a_q \end{bmatrix}}_{\mu} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_\sigma & 0 \\ 0 & 0 & \rho_q \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} g_t \\ \sigma_{g,t}^2 \\ q_t \end{bmatrix}}_{X_t} + \underbrace{\begin{bmatrix} \sigma_{g,t} & 0 & 0 \\ 0 & \sqrt{q_t} & 0 \\ 0 & 0 & \varphi_q\sqrt{q_t} \end{bmatrix}}_{G_t} \underbrace{\begin{bmatrix} z_{g,t+1} \\ z_{\sigma,t+1} \\ z_{q,t+1} \end{bmatrix}}_{z_{t+1}}
$$

$$
\begin{aligned}
G_tG_t' &= \begin{bmatrix} \sigma_{g,t}^2 & 0 & 0 \\ 0 & q_t & 0 \\ 0 & 0 & \varphi_q^2 q_t \end{bmatrix} \\
&= \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{h} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H_\sigma} \sigma_{g,t}^2 + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varphi_q^2 \end{bmatrix}}_{H_q} q_t.
\end{aligned}
$$

In the BTZ model, shocks to consumption growth $z_{g,t+1}$, to consumption volatility $z_{\sigma,t+1}$ as well as to the volatility of consumption volatility $z_{q,t+1}$ are priced. The market prices of those risks are given by:

$$
\Lambda = \begin{bmatrix} \gamma & (1-\theta)\kappa_1 A_\sigma & (1-\theta)\kappa_1 A_q \end{bmatrix}.
$$

Loadings of the market return on fundamental shocks are given by:

$$
B = [\ 1 \quad \kappa_1 A_\sigma \quad \kappa_1 A_q\ ].
$$

In the BTZ model, the variance risk premium arises because of both $\sigma^2_{g,t}$ and q_t, and it can be computed using (A.15) :

$$\begin{aligned}
VRP_t &= \begin{bmatrix} 1 & \kappa_1 A_\sigma & \kappa_1 A_q \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 & q_t & 0 \end{bmatrix} \begin{bmatrix} \gamma \\ (1-\theta)\kappa_1 A_\sigma \\ (1-\theta)\kappa_1 A_q \end{bmatrix} \right) \begin{bmatrix} 1 \\ \kappa_1 A_\sigma \\ \kappa_1 A_q \end{bmatrix} \\
&\quad + \begin{bmatrix} 1 & \kappa_1 A_\sigma & \kappa_1 A_q \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varphi_q^2 \end{bmatrix} \left(\begin{bmatrix} 0 & 0 & \varphi_q^2 q_t \end{bmatrix} \begin{bmatrix} \gamma \\ (1-\theta)\kappa_1 A_\sigma \\ (1-\theta)\kappa_1 A_q \end{bmatrix} \right) \begin{bmatrix} 1 \\ \kappa_1 A_\sigma \\ \kappa_1 A_q \end{bmatrix} \\
&= (1-\theta)\kappa_1 A_\sigma q_t + (1-\theta)\kappa_1 q_t [A_q \kappa_1^2 (A_\sigma^2 + \varphi_q^2 A_q^2)\varphi_q^2] \\
&= (1-\theta)\kappa_1 [A_\sigma + A_q \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2)\varphi_q^2] q_t.
\end{aligned}$$

The leverage effect can be computed using (A.16):

$$\begin{aligned}
COV_t &= \begin{bmatrix} 1 & \kappa_1 A_\sigma & \kappa_1 A_q \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 & q_t & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \kappa_1 A_\sigma \\ \kappa_1 A_q \end{bmatrix} \right) \begin{bmatrix} 1 \\ \kappa_1 A_\sigma \\ \kappa_1 A_q \end{bmatrix} \\
&\quad + \begin{bmatrix} 1 & \kappa_1 A_\sigma & \kappa_1 A_q \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varphi_q^2 \end{bmatrix} \left(\begin{bmatrix} 0 & 0 & \varphi_q^2 q_t \end{bmatrix} \begin{bmatrix} 1 \\ \kappa_1 A_\sigma \\ \kappa_1 A_q \end{bmatrix} \right) \begin{bmatrix} 1 \\ \kappa_1 A_\sigma \\ \kappa_1 A_q \end{bmatrix} \\
&= \kappa_1 A_\sigma q_t + \kappa_1 q_t [A_q \kappa_1^2 (A_\sigma^2 + \varphi_q^2 A_q^2)\varphi_q^2] \\
&= \kappa_1 [A_\sigma + A_q \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2)\varphi_q^2] q_t.
\end{aligned}$$

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