Two Approaches for Optimal Synthesis of a Thin Wire Antenna

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INTRODUCTION

- In this study, we explore a strategy for determining the current distribution of a thin-wire antenna based on a given radiation pattern. By this we mean seeking for a current distribution on the antenna so that the generated radiation pattern closely approximates a prescribed far field pattern.
- The integral equation that models the relationship between the current distribution and the generated radiation pattern will be analyzed using the method of moments. The unknown current distribution was approximated with a truncated series leading to a system of linear equations.
- This linear system is then solved using Tikhonov regularization. This study directly compares the results obtained from two series representations, namely the Taylor and Fourier series. Our results show that the accuracy of this strategy is primarily dependent upon the approximating series used.
- Further research on this problem may lead to the development of more effective techniques in terms of accuracy and stability. Additionally, focus on inverse problems where radiation patterns are measured with noise will allow for characterization of the source, such as its location, length, and current.

THEORY

- From Papas, given a thin-wire antenna with current (z) on $|z| \leq$, the radiation pattern is given by

$$F(\theta) = \sin\theta \int_{-} e^{-z\cos\theta} \; z \;, ( ) \qquad \theta \in [0,\pi]$$

  - This relation show that the radiation pattern is uniquely determined for every real angle on the interval.
  - With the boundary conditions $(z) = 0$ when $|z| = $.
- To find an $\boldsymbol{f}$ such that the above conditions are satisfied, we can approximate $\boldsymbol{f}$ as a truncated series of the form

$$(z) \approx \sum_{=0} \; z( )$$

  - where is some known function
  - and represents the number of modes.
- Thus, we have

$$F(\theta) = \sin\theta \int_{-} e^{-z\cos\theta} \sum_{=0} (z)$$

$$= \sin\theta \sum_{=0} \int_{-} e^{-z\cos\theta} \; z \; z. \qquad ( )$$

- We then cast this equation into the form $\boldsymbol{Ax} = \boldsymbol{b}$

$$A = \begin{pmatrix} 11 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & \end{pmatrix} \; x = \begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix} \; = \begin{pmatrix} F(\theta_1) \\ F(\theta_2) \\ \vdots \\ F(\theta) \end{pmatrix}$$

  - where is the length of the $\boldsymbol{\theta}$ mesh, and

$$= \sin\theta \sum_{=0} \int_{-} e^{-z\cos\theta} \qquad (z)$$

METHOD

- First, we prescribe a radiation pattern $\boldsymbol{F}$.
  - Since $\sin\theta$ is a factor, we must use functions which are 0 at 0 and π.
  - The two functions we used are

$$F_1(\theta) = \sum_{\kappa=1}^{10} \sin\kappa\theta \qquad\qquad F_2(\theta) = \frac{\sin 2\theta - \pi}{2\theta - \pi}.$$

Taylor Series

- Taylor Series approximation where $z( ) = z$

$$= \sin\theta \sum_{=0} \int_{-} e^{-z\cos\theta} z$$

  - Through repeated integration, we get

$$= \sin\theta \sum_{=0} \left[e^{-\cos\theta} \left(\frac{z}{-\cos\theta} - \frac{-1}{(-\cos\theta)^2} + \cdots + \frac{!}{(-\cos\theta)^{+1}} \right) \right]_{-}$$

- The issues with this series becomes immediately apparent when we take into account the boundary conditions of $\boldsymbol{f}$. We must force the restraints into the system by adding two rows of zeros on $\boldsymbol{A}$ and $\boldsymbol{b}$.

Fourier Series

- Fourier Series approximation where $(z) = \sin\left(n\pi\left(\frac{-z}{}\right)\right)$

$$= \sin\theta \sum_{=0} \int_{-} e^{-z\cos\theta} \sin\left(n\pi \left(\frac{-z}{}\right)\right)$$

  - Through repeated integration, we get

$$= \sin\theta \sum_{=0} \frac{\pi}{(-\cos\theta)^2 + \pi^2 ( )} \left(e^{-\cos\theta} - \cos n\pi \; e^{\cos\theta} \right)$$

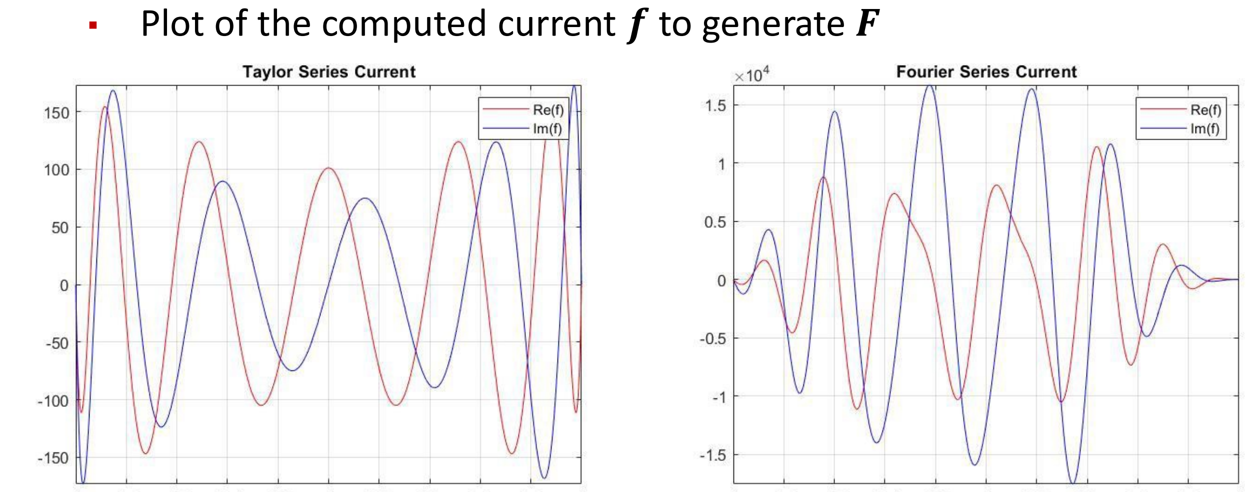
- The boundary conditions are imposed within the function itself.

RESULTS & PLOTS

- We use MATLAB in order to find the least squares solution of our systems.
- Given the near-singular nature of $\boldsymbol{A}$, it becomes necessary to employ Tikhonov Regularization using the L-curve. Doing so has the added benefit of making our solution less sensitive to noise.

Current Plots

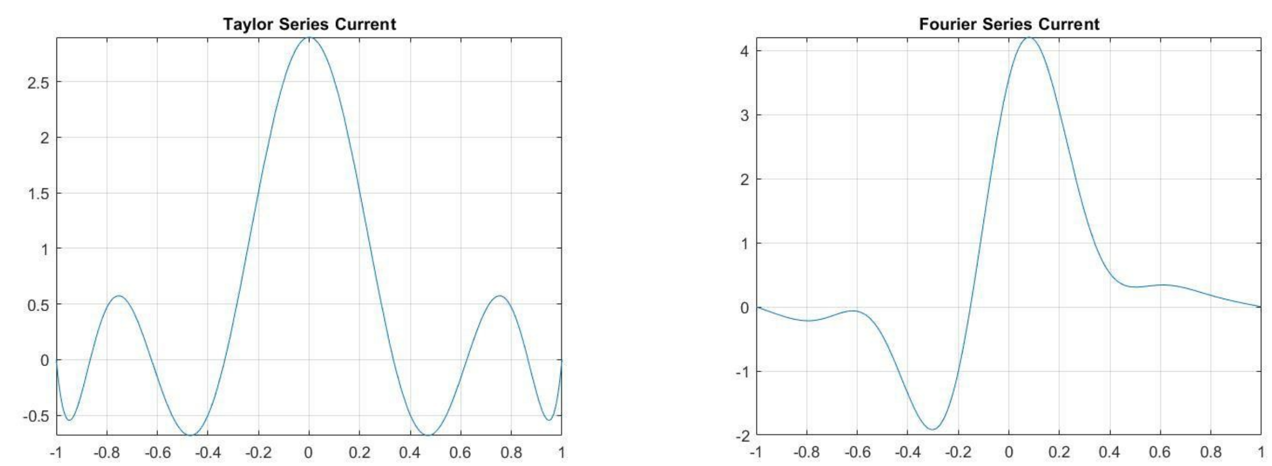
- Upon obtaining the coefficient vector $\boldsymbol{x}$, we are able to construct the required complex current $\boldsymbol{f}$. This also provides an opportunity to visually inspect that the physical constraints are met and that the current represents something the can be physically reproduced.
  - $= 10; \; = 1; \; = 40; \; = 20$
  - Plot of the computed current $\boldsymbol{f}$ to generate $\boldsymbol{F}$



- As expected, the forcing of the boundary conditions on the matrix generated by the Taylor Series yields a current which has issues meeting those conditions. Clearly, the Fourier Series produces a more stable current and the values at -$\boldsymbol{l}$ and $\boldsymbol{l}$ are closer to their expected value at 0.

RESULTS & PLOTS (cont.)

- $= 10; \; = 1; \; = 20; \; = 10$
- Plot of the real part of the computed current $\boldsymbol{f}$ to generate $\boldsymbol{F}$

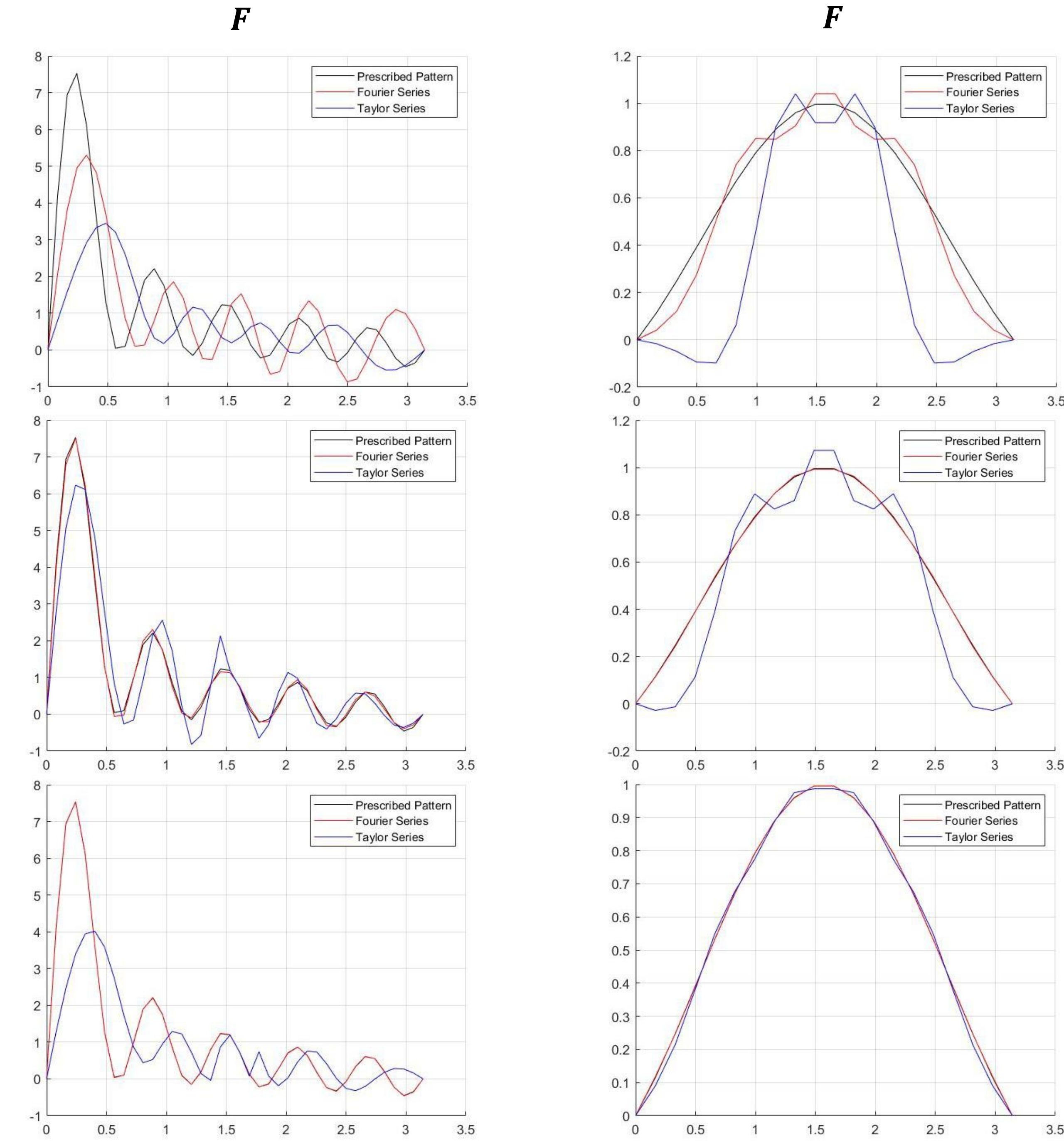


- The imaginary parts of the current obtained using the Taylor Series and Fourier Series approximations were extremely small ($\sim 10^{-8}$ and $\sim 10^{-14}$ respectively).

Generated Pattern Plots

- For $\boldsymbol{F}$, $= 10; \; = 1; \; = 40; \; = 10, 15, 20$
- For $\boldsymbol{F}$, $= 10; \; = 1; \; = 20; \; = 6, 8, 10$

$\boldsymbol{F}$ $\qquad$ $\boldsymbol{F}$



- From $\boldsymbol{F}$ we can see that the Fourier Series approximates better and faster than the Taylor Series. This is also the case for $\boldsymbol{F}$.
- It is of note that the stability of the Taylor Series approximation, in the case of $\boldsymbol{F}$ drops significantly as continues to increase.

REFERENCES

- Charles Herach Papas. Theory of Electromagnetic Wave Propagation. 2nd ed. New York: Dover; 1988. 48-50 p.