EFFECT OF INTERSTELLAR NEUTRAL HYDROGEN ON THE TERMINATION OF THE SOLAR WIND

A Thesis

Presented to

the Faculty of the Department of Physics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

by

Cary Lloyd Semar

January 1969

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Abstract

The effect of interstellar hydrogen penetrating the supersonic solar wind region from the boundary shell is considered. The density of neutral hydrogen is taken to be one particle per cubic centimeter in interstellar space and the density in the supersonic solar wind region is attenuated by a factor exp(-9a/r) at a distance r from the sun. A set of flow equations is obtained in a manner similar to that of Biermann _et_. _al_. for the solar wind-comet problem. The exchange of mass, momentum, and energy due to photoionization and charge exchange are included with appropriate source terms and the equations are solved numerically to find the flow velocity of the solar wind. The solar wind velocity is seen to drop by an order of magnitude from one to 100 a.u.

Neglecting the magnetic field pressure of the interstellar medium and setting M(mach number) = 1 in the flow equations gives a value for the radius of the supersonic solar wind region of about 50 a.u.

The radius of the solar wind region is also calculated

using the momentum balance method. It is seen that the radius of the supersonic solar wind region is dependent on the thickness of the boundary shell and the pressure profile.

It is shown that if a uniform neutral hydrogen density of one particle per cubic centimeter is assumed, the radius of the supersonic solar wind region increases over the value given above. This peculiar result is attributed to the nonlinearity of the flow equations.

1. INTRODUCTION

The problem of the termination of the solar wind has been explored by several authors, including Axford, Dessler, and Gottlieb (1963) and Patterson, Hanson, and Johnson (1963). A review by Dessler, (1967) gives a comprehensive survey of the present state of our ideas about the solar wind.

The solar wind is a plasma of ionized hydrogen with a frozen-in magnetic field which evolves in time in such a way that we can think of the particles as being "attached" to the particles in the flow, and the magnetic field moves with the fluid velocity of the solar wind. Any particles which become ionized and enter the flow will rotate around the field lines and be carried along by them.

The solar wind must push aside the interstellar magnetic field in the vicinity of the sun. In figure 1, we see that there are three distinct regions corresponding to the supersonic solar wind (region I); the boundary shell or subsonic solar wind (region II); and the undisturbed interstellar medium (region III). The surface between region I and region II corresponds to a standing shock or a transition from supersonic to subsonic flow in the solar wind.

Several mechanisms to maintain the configuration have been considered. Axford, Dessler, and Gottlieb (1963) have performed calculations on a model which assumes an essentially uniform solar wind velocity beyond 1 a.u. and that the principle mechanism for termination of the solar wind is the pressure of the interstellar magnetic field. By balancing the momentum flux of the solar wind, ρV^2, and the energy density of the galactic B-field, $\frac{B^2}{8\pi}$, the distance of the shock from the sun, R_s, can be written

$$R_s = \frac{V_a}{B}\sqrt{8\pi\rho_a} \qquad (1.1)$$

where V_a is the velocity of the solar wind at 1 a.u., ρ_a is the density of the solar wind at 1 a.u. and R_s is given in astronomical units. The results of Axford et. al. give values for R_s of about 60 a.u. for $V_a = 4 \times 10^7$ cm/sec, $B = 10^{-5}$ gauss, and $\rho_a = 5$ cm^{-3}.

Another mechanism is discussed by Dessler (1967). If the boundary shell is thicker than the mean free path of a hydrogen atom for charge exchange, then most of the neutral hydrogen from region III will undergo charge exchange in the boundary shell and impart their momentum to the solar wind, thus exerting an additional force on the boundary shell to augment that due to the galactic B-field. The momentum flux of neutral hydrogen is comparable in magnitude to the pressure of the interstellar magnetic field (Dessler, 1967).

The effect of cosmic ray friction has been considered by Axford and Newman (1965). Their conclusion was that the cosmic ray pressure is probably not very significant if B is much greater than 1 gamma.

Dessler (1967) points out that the velocity of the solar wind will be reduced by the interaction with the neutral hydrogen flux coming through the boundary shell, if the boundary shell is not thick enough to ionize all the incoming neutrals. The incoming neutrals will interact with the solar wind through charge exchange and remove momentum and energy from the flow. The purpose of this paper is to investigate the effect of neutral hydrogen atoms in region I interacting with the solar wind via charge exchange and photoionization processes.

The distribution of neutral hydrogen in the vicinity of the sun has been treated by Patterson, Johnson, and Hanson (1963) assuming a random distribution of velocities in the neutral hydrogen emitted from the inner surface of the boundary shell.

There is the possibility that a thin boundary shell could permit most of the neutral hydrogen to come through into region I (Kern, 1967). In that event the neutral hydrogen will have a bulk velocity relative to the sun due to the motion of the sun through the interstellar medium. The velocity of the sun relative to the interstellar medium is very likely to be that inferred from the proper motion of the stars, 20 km/sec. (Dessler, 1967). In this paper, we shall assume an essentially uniform flow of neutral hydrogen from the apex direction of the sun with a velocity of 20 km/sec.

2. THE FLOW EQUATIONS

The solar wind expands supersonically outward from the sun into the interstellar medium, with which it interacts via charge exchange and photoionization processes. The solar wind and the interstellar medium interchange mass, momentum, and energy. Equations to describe these transport phenomena can be constructed in an *ad hoc* fashion by considering the exchange of particles in a small differential volume element. The problem is strikingly similar to the comet problem as presented by *Biermann, Brosowski, and Schmidt* (1967). The difference is that the particles interchanged are of equal mass and the ions are flowing into an extended region of neutrals rather than the reverse as in the case of the comet.

Photoionization causes the injection of mass into the flow, but due to the equality of the masses of solar wind ions and neutral hydrogen atoms from the interstellar medium, there is no net contribution to the mass flux due to charge exchange. Consequently, we write the continuity equation in the form

$$\nabla \cdot \rho \vec{V} = S_p \tag{2.1}$$

where ρ and $\vec{V}$ are the density and velocity of the solar wind. S_p is the rate of mass injection per unit volume due to photoionization.

The neutral hydrogen atoms possess a velocity $V_\odot$ relative to the sun, consequently momentum is contributed to the flow by both charge exchange and photoionization. The solar wind ions which become neutralized in the charge exchange process, however, represent a considerable momentum loss which must be taken into account in the momentum equation. The divergence of the momentum flux density is given by

$$\nabla \cdot \rho \vec{V}\vec{V} + \nabla p = -S_s \vec{V} - (S_s + S_p)\, \vec{V}_\odot \qquad (2.2).$$

Here, p is the scalar pressure and S_s is the rate at which mass is exchanged between the solar wind and interstellar medium per unit volume due to charge exchange. Using equation (2.1) and expanding the divergence of the dyadic, equation (2.2) may be rewritten

$$(\rho \vec{V} \cdot \nabla)\, \vec{V} + \nabla p = -(S_s + S_p)(\vec{V} - \vec{V}_\odot) \qquad (2.3).$$

In order to construct the energy equation, we let $E = \frac{1}{2}\rho V^2 + \frac{\gamma}{\gamma - 1}\, p$ be the energy density in the solar wind. We have incorporated the magnetic field pressure into the scalar p, thereby requiring that the magnetic field be essentially transverse to the flow and $\gamma = 2$ (Biermann, Brosowski, and Schmidt, 1967). Particles removed from the flow by charge exchange carry off energy per unit mass E/ρ and particles entering the flow have a kinetic energy density $\frac{1}{2}\rho\vec{V}_{\odot}^{2}$. We incorporate these into the energy equation as sources and sinks and obtain

$$\nabla\cdot E\vec{V} = -\frac{E}{\rho}S_s + (S_s + S_p)\frac{\vec{V}_{\odot}^{2}}{2} \qquad (2.4).$$

For a discussion of the source terms, S_s and S_p, see appendix A.

The velocity of the solar wind being of the order of 10^7 cm/sec and the velocity of the sun 10^6 cm/sec (Dessler, 1967), the terms involving $V_{\odot}$ will be neglected in the flow equations. The equations which will be used are

$$\nabla\cdot\rho\vec{V} = S_p \qquad (2.5)$$

$$(\rho\vec{V}\cdot\nabla)\vec{V} + \nabla p = -(S_s + S_p)\vec{V} \qquad (2.6)$$

$$\nabla\cdot E\vec{V} = -\frac{E}{\rho}S_s \qquad (2.7).$$

For the apex direction, the velocities of the solar wind and incoming neutrals are nearly parallel. For this reason, a spherically symmetric model will be adopted. Equations (2.5), (2.6), and (2.7) may be rewritten in the scalar form:

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \rho V) = S_p \tag{2.8}$$

$$V\frac{dV}{dr} + \frac{dp}{dr} = -(S_s + S_p)V \tag{2.9}$$

$$1/r^2 \frac{d}{dr}(r^2 EV) = -\frac{E}{\rho} S_s \tag{2.10}$$

3. DENSITY OF NEUTRAL HYDROGEN NEAR THE SUN

In order to find the source terms S_s and S_p we must develop an expression for the density of neutral hydrogen in the vicinity of the sun, i.e. within 100 a.u. We are adopting a model in which the sun is moving through a uniform gas of neutral hydrogen with a velocity $V_{\odot}$. The density of the gas is ρ_{HO}. Danby and Camm (1956) have calculated the density of matter near a star for such a situation, neglecting any possibility of removal.

Let b be the impact parameter of an atom with the sun and take θ to be the angle between the apex line and the surface of a cone coaxial with the apex line and with the vertex of the cone at the sun. The distance from the sun to a point on the surface of the cone is r. The density of matter at some point on the surface of the cone is (Danby and Camm, 1956)

$$\rho_H(r,\theta) = \frac{\rho_{HO}}{\sin\theta}\frac{\partial b}{\partial r} \quad .$$

According to Patterson, Johnson, and Hanson (1963) particles moving along a trajectory past the sun will be ionized at the rate

$$\frac{dN}{dt} = -\,(1/T_p + 1/T_s)\,\frac{Na^2}{r^2}$$

where $1/T_p$ and $1/T_s$ are the photoioniation and charge exchange parameters respectively. Along a particular trajectory, a particle at time t is at a particular angle θ such that

$$\frac{dN}{dt} = \frac{dN}{d\theta}\frac{d\theta}{dt} \; .$$

Substituting this result into the previous equation yields

$$\frac{dN}{d\theta} = -\,N\,\frac{a^2}{r^2\dot{\theta}}\,\frac{1}{T}$$

where $1/T = 1/T_p + 1/T_s$.

By conservation of angular momentum, $r^2\dot{\theta} = bV_{\odot}$.

$$\frac{dN}{d\theta} = -\,N\,\frac{a^2}{bV_{\odot}T}$$

$$N = N_0 e^{-\frac{a^2\theta}{bV_{\odot}T}} \; .$$

We see that the density of particles along a trajectory must be attenuated by the factor exp $-(\frac{a^2}{V_\odot T}\,\frac{\theta}{b})$. We conclude that the density of neutral hydrogen in the vicinity of the sun will be that given by (3.1) attenuated by the exponential factor above.

$$\rho_H(r,\theta) = \frac{\rho_{HO}}{\sin\theta}\,\frac{\partial b}{\partial r}\,\exp -\left(\frac{a^2}{V_\odot T}\,\frac{\theta}{b}\right) \qquad (3.2).$$

The impact parameter, b, corresponding to a particular trajectory through any point (r,θ) can be found from the orbit equation (Kern, 1967).

$$b = \frac{r}{2}\left(\sin\theta + \sqrt{\sin^2\theta + \frac{4GM}{V_\odot^2 r}(1-\cos\theta)}\right) \quad (3.3).$$

For the apex direction, θ approaches zero. As θ approaches zero the impact parameter approaches $b=\theta r$. Substituting this into (3.2) yields

$$\rho_H(r,0) = \rho_{HO}\exp -\left(\frac{a^2}{V_\odot T}\,\frac{1}{r}\right) \qquad (3.4).$$

This is the expression for the density of neutral hydrogen which will be used in this paper to construct the source terms in the next section.

4. THE MASS FLUX

The source term, S_p, has the form (Appendix A)

$$S_p = \frac{a^2 \rho_H}{r^2 T_p} \tag{4.1}$$

Equation (2.8) may be written

$$\frac{d}{dr}(r^2 \rho V) = a^2 \frac{\rho_H}{T_p} \tag{4.2}$$

and upon integrating from a to r we have

$$\frac{r^2 \rho V}{a^2 \rho_a V_a} = 1 + \frac{1}{\rho_a V_a T_p} \int_a^r \rho_H dr \tag{4.3}.$$

The function ρ_H is given by equation (3.4). The integral can be performed using a change of variables and integral tables (Abramowitz and Stegun). We define the function A(r) to be

$$A(r) = \frac{r^2 \rho V}{a^2 \rho_a V_a} \tag{4.4}$$

The function A(r) is plotted in figure 2. Using (4.4) we can plot the mass flux which is $1/r^2$ A(r). We see that the mass flux (figure 3) is enhanced by photoionization to a value greater than that we would obtain by assuming a simple $1/r^2$ dependence.

5. THE ENERGY FLUX AND ENERGY PER UNIT MASS

The charge exchange source term can be written (Appendix A):

$$S_s = \frac{T_p}{T_s} S_p \quad . \tag{5.1}$$

Substituting this result into equation (2.10) we obtain, using equation (2.8)

$$\frac{d}{dr}(r^2 EV) = -\left(\frac{E}{\rho}\right)\frac{d}{dr}(r^2 \rho V) \tag{5.2}.$$

This equation is readily integrated and using (4.4) we have

$$\frac{r^2 EV}{a^2 E_a V_a} = A^{-T_p/T_s} \tag{5.3}$$

from which we may obtain the energy flux plotted in figure 4.

Dividing both sides of equation (5.3) by A(r) we have

$$\frac{E}{\rho} = \frac{E_a}{\rho_a} A^{-T_p/T} \tag{5.4}.$$

The ratio of energy density to mass density is clearly decreasing. The way the equations have been constructed the solar wind is being cooled by charge exchange and mass is being added due to photoionization.

6. SOLVING THE EQUATION OF MOTION

In order to solve equation (2.9) for the fluid velocity, V, we must eliminate the unknown quantity p. Recalling that $E = \frac{1}{2}\rho V^2 + \frac{\gamma}{\gamma-1}p$ we may express p in terms of the energy density, mass density, and fluid velocity. At this point, we do not know ρ but we do have ρV. Consequently, the pressure may be expressed in terms of known quantities and the velocity which is to be calculated from the solution of (2.9).

$$p = \frac{\gamma-1}{\gamma}\left(E - \tfrac{1}{2}\rho V^2\right). \qquad (6.1).$$

With (6.1), equation (2.9) may be written

$$\left(3/2 - \frac{E}{\rho V^2}\right)\frac{d}{dr}\ln V = \frac{2}{r}\left(\frac{E}{\rho V^2} - \tfrac{1}{2}\right) + \left(\frac{E}{\rho V^2}\frac{T_p}{T_s} + \tfrac{1}{2} - 2\frac{T_p}{T} \right) \frac{d}{dr}\ln(r^2\rho V). \qquad (6.2)$$

For the details of the derivation of equation (6.2) see appendix B.

The quantity $\frac{E}{\rho V^2}$ may be expressed

$$\frac{E}{\rho V^2} = \tfrac{1}{2} + \frac{\gamma}{\gamma-1}\,\frac{p/\rho}{V^2}\ .$$

The velocity of sound is $C^2 = \frac{\gamma p}{\rho}$. Letting $M = \frac{V}{C}$ we may write $\frac{E}{\rho V^2} = \frac{1}{2} + 1/M^2$. Substituting this expression into equation (6.2) we have

$$(1 - 1/M^2)\frac{d}{dr}\ln V = \frac{2}{r}\frac{1}{M^2} + \left[(1/M^2 + \tfrac{1}{2})\frac{T_p}{T_s} + \tfrac{1}{2} - 2T_p/T\right]\frac{d}{dr}\ln(r^2\rho V). \tag{6.3}$$

If we set $M = 1$, the left hand side of the equation vanishes. In order to satisfy the equation, we require

$$\frac{2}{r} + \left(\frac{3T_p}{2T_s} + \tfrac{1}{2} - \frac{2T_p}{T}\right)\frac{d}{dr}\ln\,(r^2\rho V) = 0 \tag{6.4}$$

Equation (6.4) may be solved for r. This determines the distance from the sun at which the solar wind goes from supersonic velocity to subsonic velocity. A graphical solution of equation (6.4) yields

$$r_s = 50 \text{ a.u.}$$

For the details of the solution of (6.4), see appendix C.

Returning to equation (6.2), note that if we divide by $(3/2 - E/\rho V^2)$ we have

$$\frac{d}{dr}\ln V = \frac{\frac{2}{r}\left(\frac{E}{\rho V^2} - \frac{1}{2}\right) + \left(\frac{E}{\rho V^2}\frac{T_p}{T_s} + \frac{1}{2} - \frac{2T_p}{T}\right)\frac{d}{dr}\ln\,(r^2\rho V)}{\frac{3}{2} - \frac{E}{\rho V^2}}. \tag{6.5}$$

Equation (6.5) is a first order differential equation in r and V. All the quantities on the right except V are known as functions of r.

$$\frac{E}{\rho} = \frac{E_a}{\rho_a} \quad A^{-T_p/T} \tag{6.6}$$

and

$$r^2 \rho V = a^2 \rho_a V_a A(r).$$

We can calculate V from this equation (6.5) using an iterative procedure. Let $V(r + h) = V(r) + h\frac{dV}{dr}$ where h is some small interval. The velocity of the solar wind is taken at r = a to be $V_a = 5 \times 10^7$ cm/sec. Beginning with this value, the velocity has been calculated out to 100 a.u., using $h = 5 \times 10^{12}$ cm from r = 1 to r = 10 a.u, h = 1 a.u from r = 10 to r = 20 a.u., h = 5 a.u. from r = 20 to r = 40 a.u., and h = 10 a.u. from r = 40 to r = 100 a.u.

The results of the calculation are given in figure 5. With the velocity in hand, the values for the other fluid quantities can be calculated. The density, ρ, and energy density, E, are plotted in figures 6 and 7.

Note that if we take $r^2\rho V$ to be constant, equation (6.5) reduces to

$$\frac{d}{dr} \ln V = \frac{\frac{2}{r}(E/\rho V^2 \; - \; \frac{1}{2})}{\frac{3}{2} - \frac{E}{\rho V^2}} \tag{6.7}.$$

This is the equation for the velocity when mass loading and charge exchange are neglected. Equation (6.7) may be integrated and put in the form

$$\frac{r}{a} = \left[\frac{\dfrac{E}{\rho V_a^2} - \frac{1}{2}}{\dfrac{E}{\rho V_a^2} - \frac{1}{2} V^2/V_a^2} \right]^2 \frac{V}{V_a}$$

As r approaches infinity

$$\frac{E}{\rho V_a^2} - \frac{1}{2} \frac{V^2}{V_a^2} = 0 \qquad (6.8).$$

Neglecting interaction processes, the ratio E/ρ is a constant. Recalling that $E/\rho V^2 = \frac{1}{2} + 1/M^2$ and taking $M = 5$ at 1 a.u., we may write $E_a/\rho_a V_a^2 = .54$. Substituting this into equation (6.8) and calculating the velocity at maximum we have

$$\frac{V_{MAX}}{V_a} = 1.04.$$

Comparing this figure with the values for V/V_a calculated from (6.5), we see the effect of the interstellar medium on the velocity of the solar wind. The calculation of the shock distance from equation (6.4) indicates that the presence of the neutral hydrogen flux in the solar wind cavity is capable of creating a standing shock and thereby determining the inner limit of the boundary shell.

7. MAGNETIC FIELD-SOLAR WIND MOMENTUM FLUX BALANCE

We will now find r_s by another method. The position of the shock transition, assuming it exists, corresponds to the point at which the solar wind pressure balances the magnetic field pressure of the interstellar medium plus the pressure of the neutral hydrogen (Dessler, 1967). Since the neutral hydrogen does not interact readily with the solar wind ions, we shall neglect the pressure of the interstellar neutral hydrogen and consider only the effect of the galactic magnetic field.

The solar wind pressure is

$$p + \rho V^2,$$

which is balanced by the galactic magnetic field pressure,

$$B^2/8\pi .$$

Equating the two, we have

$$p + \rho V^2 = \frac{B^2}{8\pi} \tag{7.1}$$

If this shock transition occurs in a region of high Mach number, i.e., if we have a strong shock, the pressure, p, will be quite small compared to the momentum flux density, ρV^2. In that event, we may establish the shock distance by neglecting p in (7.1) and finding the point at which $\rho V^2 = B^2/8\pi$. Taking the values for ρV and V which have been plotted, ρV^2, the momentum flux density, is shown in figure 8 as a function of r. Taking B = 1 gamma, we find that a balance would occur at about 65 a.u. This result is inconsistent with the results of the previous section, which indicate that the solar wind would not be supersonic beyond 50 a.u., therefore the assumption that p is negligible in equation (7.1) is invalid.

In this model, the most important parameter is the neutral hydrogen flux which determines the behavior of the solar wind in region I. The behavior of the solar wind in this region will be the same whether the boundary shell exists or not, due to the fact that a supersonic flow is unaffected by conditions downstream.

The value of 65 a.u. obtained above was based on the model of a very thin boundary shell. If the boundary shell has a significant thickness, another method must be used to find the balance point. Consider a conical tube of flow with a half angle α. The cone is truncated by the inner and outer surfaces of the boundary shell. The pressure on the inner surface is $p + \rho V^2$ and on the outer surface is $B^2/8\pi$. Assuming the pressure in the boundary shell is p_s, and requiring that the net forces on our truncated cone add up to zero, we may write

$$\int p_s ds_1 + \int (p + \rho V^2) ds_2 + \int \frac{B^2}{8\pi} ds_3 = 0 \qquad (7.2).$$

An equation of this type can be used to determine the shock distance for various values of the thickness of the boundary shell, provided we make some assumption about the pressure profile within the boundary shell. The usual assumption is that the pressure is constant and equal to $B^2/8\pi$ (Dessler, 1967). Assuming we have a strong shock, then the pressure within the boundary shell must rise sharply at the shock point and gradually level off to the value of $B^2/8\pi$ as pictured in figure 9. For such a pressure profile, the assumption of constant $p_s = B^2/8\pi$ is a fair approximation, however, if we substitute for p_s in equation (7.2) and carry out the integration, we regain equation (7.1).

Further study of the behavior of the solar wind in region II is necessary before the effect of the thick boundary shell on the termination distance can be properly assessed, and for that reason it will be deferred for the time being.

A recent paper by G. L. Verschuur (Verschuur, 1968) indicates that the interstellar magnetic field in the local part of the galaxy is about .3 gamma. A pressure balance calculation based on this figure gives a value for the shock distance of about 100 a.u. It appears that if the density of neutral hydrogen in interstellar space is one particle per cubic centimeter or greater, the charge exchange and photoionization processes completely dominate the effect of the interstellar field.

8. CONSTANT NEUTRAL HYDROGEN DENSITY

In this section, we wish to discuss a peculiar effect of the nonlinear nature of the equations. The presence of the neutral hydrogen in the solar system acts to cool and slow down the solar wind. We would expect that increasing the amount of neutral hydrogen present would cause the solar wind to slow down more rapidly and undergo transition from supersonic to subsonic velocities nearer the sun.

On the constrary, if we introduce into our calculations the assumption that the density of neutral hydrogen in the solar system is constant and of order 1 cm^{-3}, the transition point moves farther out from the sun. Taking ρ_H constant, the function A(r) reduces to

$$A(r) = 1 + \frac{\rho_{HO}a}{\rho_a V_a T_p}\left(\frac{r}{a} - 1\right) \qquad (8.1)$$

and equation (6.4) may be written

$$\frac{2}{r} - C\frac{L/a}{1 + L(r/a - 1)} = 0 \qquad (8.2)$$

where

$$C = -\left(\tfrac{3}{2}T_p/T_s + \tfrac{1}{2} - 2T_p/T \right)$$

and

$$L = \frac{\rho_{HO}a}{a V_a T_p} .$$

The vallues of the photoionization and charge exchange constants are given in Appendix C. V_a is taken to be 5×10^7 cm/sec. Using these numbers, and $\rho_{HO}/\rho_a = 5$ we obtain C = 3.17 and L = .027.

Upon solving equation (8.2) for r, we obtain

$$\frac{r}{a} = \frac{1 - L}{L(\frac{C}{2} - 1)} = 62 \qquad (8.3).$$

Comparing (8.3) with the value of r_s found in section 6, it appears that the shock or transition distance is increased for a greater, uniform neutral hydrogen density, or at least in this specific example. This result indicates that one must be careful in forming physical pictures of the attenuation of the solar wind. The result of (8.3) should not be too broadly interpreted, _i.e._, one cannot conclude on the basis of (8.3) that the shock distance will in general increase for higher neutral hydrogen densities.

9. EXPONENTIAL BEHAVIOR OF VELOCITY

The most probable source of error in the preceding calculations is the use of the iterative procedure to determine the velocity of the solar wind. In this section an attempt will be made to apply a correction to the velocity to make it consistent with the determination of the shock distance in section 6.

We find that if we take $V/V_a = D\exp\left(-\frac{r}{m}\right)$ we can choose the constants D and m to give a very good fit to the velocity curve in figure 5 over the range r = 10 a.u. to r = 100 a.u. This indicates that for the purposes of numerical calculations, we may assume an approximately exponential behavior for the velocity over the region indicated.

Recalling that

$$\frac{E}{\rho V^2} = \frac{1}{2} + \frac{1}{M^2} \tag{9.1}$$

and using equation (5.4) we have for M = 1 at r = 50 a.u.

$$\left(\frac{V}{V_a}\right)^2 = \frac{E_a}{\rho_a V_a^2} A^{-T_p/T} \tag{9.2}.$$

Using M = 5 at one a.u., equation (9.1) indicates

$$\frac{E_a}{\rho_a V_a^2} = .54$$

Using the calculated value of A(r) at r = 50 a.u. and the values of T and T_p given in Appendix C we obtain

$$\frac{V}{V_a} = .268 \text{ at } r = 50 \text{ a.u.}$$

This result indicates that at 50 a.u. the cumulative error of our numerical procedure has reached about 40 %. In order to make the velocity consistent with the Mach number calculated at this point, we shall assume exponential behavior of the velocity between 5 a.u. and 100 a.u. At 5 a.u. we take V/V_a = 1.0 and at 50 a.u. V/V_a = .268. The corrected velocity is labeled $\bar{V}$ and plotted in figure 11.

The corrected energy density $\bar{E}$ is shown on figure 4 and the corrected bulk flow kinetic energy density is plotted along with the uncorrected bulk flow kinetic energy density on figure 8. Using these values, we have calculated the Mach number beyond 5 a.u. in figure 12. We see that the Mach number drops sharply beyond 10 a.u. This is the region in which the attenuative effect of the incoming neutral hydrogen flux is most significant.

10. CONCLUSIONS

An interstellar neutral hydrogen density of 1 cm^{-3} is capable of having a strong effect on the termination of the solar wind, and in the light of new figures for the interstellar magnetic field strength, is the dominant mechanism. The solar wind-neutral hydrogen interaction causes a reduction in solar wind velocity and Mach number and is sufficient to establish a transition from supersonic to subsonic velocities at about 50 a.u.

The presence of a strong shock is not likely, rather there is a gradual, continuous diminution of Mach number from supersonic to subsonic velocities, or a weak shock at about fifty a.u.

APPENDIX A

The Source Terms

The source terms S_s and S_p are the rate of mass interchange per unit volume in the solar wind due to photoionization and charge exchange. We write

$$S_p = \frac{a^2}{r^2} \frac{\rho_H}{T_p}$$

and

$$S_s = \frac{a^2}{r^2} \frac{\rho_H}{T_s} \quad .$$

ρ_H is the mass density of neutral hydrogen in region I, $1/T_p$ and $1/T_s$ are the probabilities of photoionization and charge exchange respectively as measured in the vicinity of the earth. The probability of charge exchange is proportional to the particle flux of the solar wind. Since this is one of the parameters we wish to find, we cannot immediately write down the probability of charge exchange. To a first approximation, however, the mass flux goes as $1/r^2$ (see appendix D). It is approximately valid to write the charge exchange probability as proportional to $1/r^2$.

The probability of photoionization depends on the intensity of solar radiation, which goes as $1/r^2$, so a factor a^2/r^2 appears in the source term S_p.

Having made the above approximation, we must bear in mind that we are in effect underestimating the probability of charge exchange. Due to mass loading of the solar wind, particle flux will be somewhat greater than that predicted by a $1/r^2$ proportionality.

We define the constant $1/T$ to be the total probability per second of ionization near the earth.

$$1/T = 1/T_p + 1/T_s.$$

APPENDIX B

Derivation of Equation (6.2)

Using $S_s = \frac{T_p}{T_s} S_p$ (Appendix A) and substituting into (2.9)

$p = \frac{\gamma-1}{\gamma}(E - \frac{1}{2}\rho V^2)$ we have

$$\rho V\frac{dV}{dr} + \frac{\gamma-1}{\gamma}(\frac{dE}{dr} - \frac{1}{2}\frac{d}{dr}(\rho V^2)) = -(\frac{T_p}{T_s} + 1)S_p V \qquad (b.1)$$

which can be rewritten using equation (2.8) in the form

$$\rho V\frac{dV}{dr} + \frac{\gamma-1}{\gamma}(E\frac{d}{dr}\ln E - \frac{1}{2}V^2\frac{d}{dr}\ln \rho V^2) = -(\frac{T_p}{T_s} + 1)\frac{V}{r^2}\frac{d}{dr}(r^2\rho V) \qquad (b.2).$$

Dividing through by ρV^2 we have

$$\frac{1}{V}\frac{dV}{dr} + \frac{\gamma-1}{\gamma}(\frac{E}{\rho V^2}\frac{d}{dr}\ln V^2) = -\frac{T_p}{T}\frac{d}{dr}\ln(r^2\rho V) \qquad (b.3)$$

where we have used $1/T = 1/T_p + 1/T_s$.

$$\frac{1}{V}\frac{dV}{dr} + \frac{\gamma-1}{\gamma}\frac{E}{\rho V^2}\frac{d}{dr}(\ln r^2\rho V) - \frac{2}{r} - \frac{1}{V}\frac{dV}{dr} - \frac{1}{2}\frac{d}{dr}\ln(r^2\rho V)$$
$$-\frac{1}{2}\frac{1}{V}\frac{dV}{dr} + \frac{1}{r} = \frac{d}{dr}\ln(r^2\rho V)^{-T_p/T} \qquad (b.4).$$

We now set $\gamma = 2$ and collect terms involving $\frac{dV}{dr}$ and $\frac{d}{dr}\ln(r^2\rho V)$ and obtain

$$(3/2 - \frac{E}{\rho V^2})\frac{d}{dr}\ln V =$$
$$\frac{2}{r}(\frac{E}{\rho V^2} - \tfrac{1}{2}) - (\frac{E}{\rho V^2}\frac{T_p}{T_s} + \tfrac{1}{2} - \frac{2T_p}{T})\frac{d}{dr}\ln(r^2\rho V) \qquad (b.5)$$

which is equation (6.2).

APPENDIX C

Solution of Equation (6.4)

The equation to be solved is:

$$\frac{2}{r} + \left(\frac{3}{2} \frac{T_p}{T_s} + \tfrac{1}{2} - 2\frac{T_p}{T} \right) \frac{d}{dr} \ln(r^2 \rho V) = 0 \qquad \text{(c.1).}$$

For notational convenience, we define the constant

$$C = \frac{3T_p}{2T_s} + \tfrac{1}{2} - 2\,\frac{T_p}{T}\ .$$

Using

$$1/T_p = .45 \times 10^{-6}\ \text{sec}^{-1}$$

$$1/T_s = .75 \times 10^{-6}\ \text{sec}^{-1}$$

$$1/T = 1/T_p + 1/T_s$$

(Kern, 1967) we find $C = 3.17$.

From equation (2.8) we have, using the explicit form of the source terms in Appendix B:

$$\frac{d}{dr}(r^2 \rho V) = \frac{a^2 \rho_H}{T_p} \qquad \text{(c.2)}$$

which becomes, upon integration

$$r^2 \rho V = a^2 \rho_a V_a + \frac{a^2}{T_p} \int_a^r \rho_H\, dr \qquad \text{(c.3).}$$

Taking the natural logarithm of equation (d.3) and differentiating with respect to r we have

$$\frac{d}{dr}\ln(r^2\rho V) = \frac{(a^2/T_p)\rho_H}{a^2\rho_a V_a + \frac{a^2}{T_p}\int_a^r \rho_H dr} .$$

Upon dividing numerator and denominator by $a^2\rho_a V_a$ this equation becomes

$$\frac{d}{dr}\ln(r^2\rho V) = \frac{\frac{1}{\rho_a V_a T_p}\rho_H}{A(r)} \qquad (d.4)$$

where A(r) is the function discussed in section 4 and ρ_H is that given in equation (3.4).

$$\rho_H = \rho_{HO}\exp -\left(\frac{9a}{r}\right)$$

taking $a = 1.5 \times 10^{-13}$cm, $V_O = 2 \times 10^6$ cm/sec and $1/T_p = .45 \times 10^{-6}\,\text{sec}^{-1}$ in equation (3.4).

Equation (d.4) becomes, upon substitution

$$\frac{d}{dr}\ln(r^2\rho V) = \frac{\frac{\rho_{HO}}{\rho_a V_a T_p}\exp -\left(\frac{9a}{r}\right)}{A(r)} \qquad (d.5).$$

We now introduce the constant $L = \dfrac{\rho_{HO}a}{\rho_a V_a T_p}$. The density of the interstellar medium is taken to be 1 particle per cubic centimeter and the density of the solar wind at 1 a.u. is taken to be 5 particles per cubic centimeter. Taking the velocity of the solar wind, V_a, at one a.u. to be 5×10^7 cm/sec we find L = .027. Using the definitions of C, L, and equation (ɑ.5), equation (ɑ.1) is rewritten

$$\frac{2}{N} - \frac{CL}{A(r)} \exp -(9/N) = 0$$

where N = r/a. This equation is equivalent to

$$N = \frac{2A(r)}{CL} \exp (9/N) \qquad (ɑ.6)$$

which is solved by graphical procedures. For the result, see figure (10).

APPENDIX D

Calculation of Mass Flux

From equation (2.8) we found the mass flux of the solar wind to be given by

$$\frac{\rho V}{\rho_a V_a} = \frac{a^2}{r^2} A(r)$$

where the function A(r) is given by

$$A(r) = 1 + \frac{1}{\rho_a V_a T_p} \int_a^r \rho_H dr \qquad (d.1)$$

In order to find A(r) we must evaluate the integral

$$\int_a^r \rho_H dr \ .$$

Using (3.4) we have

$$\int_a^r \rho_H dr = \int_a^r \rho_{HO} \exp -(9a/r) \ dr \qquad (d.2).$$

We now introduce the new variable u = 9a/r and obtain

$$\int_a^r \rho_H dr = -9a \rho_{HO} \int_9^{9a/r} \frac{\exp -u}{u^2} du \qquad (d.3).$$

Upon integrating by parts we have

$$\int_a^r \rho_H dr = 9a\,\rho_{HO}\left[\frac{\exp -(9a/r)}{9a/r} - \frac{\exp -9}{9} + \int_9^{9a/r} \frac{\exp -u}{u}\,du\right] \qquad (d.4).$$

The last term in (d.4) can be expressed in terms of the difference of two exponential integrals (Abramowitz and Stegun, 5.1.1).

$$\int_a^r \rho_H dr = 9a\,\rho_{HO}\left[\frac{r}{9a}\exp -(9a/r) - \frac{1}{9}\exp -9 + E_1(9) - E_1(9a/r)\right].$$

With this result, we can now calculate the function A(r) which is plotted in figure (2) and the mass flux.

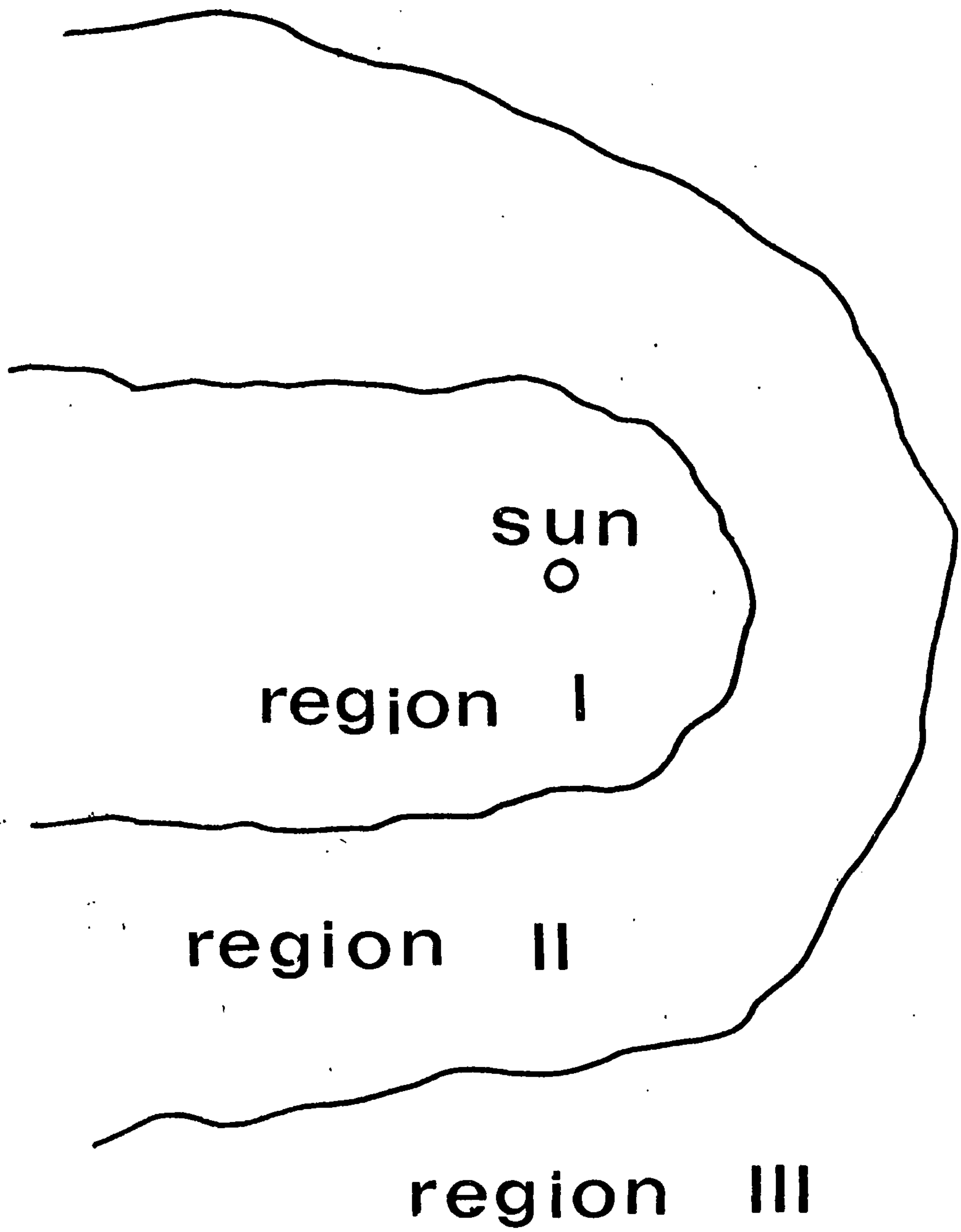


FIG 1

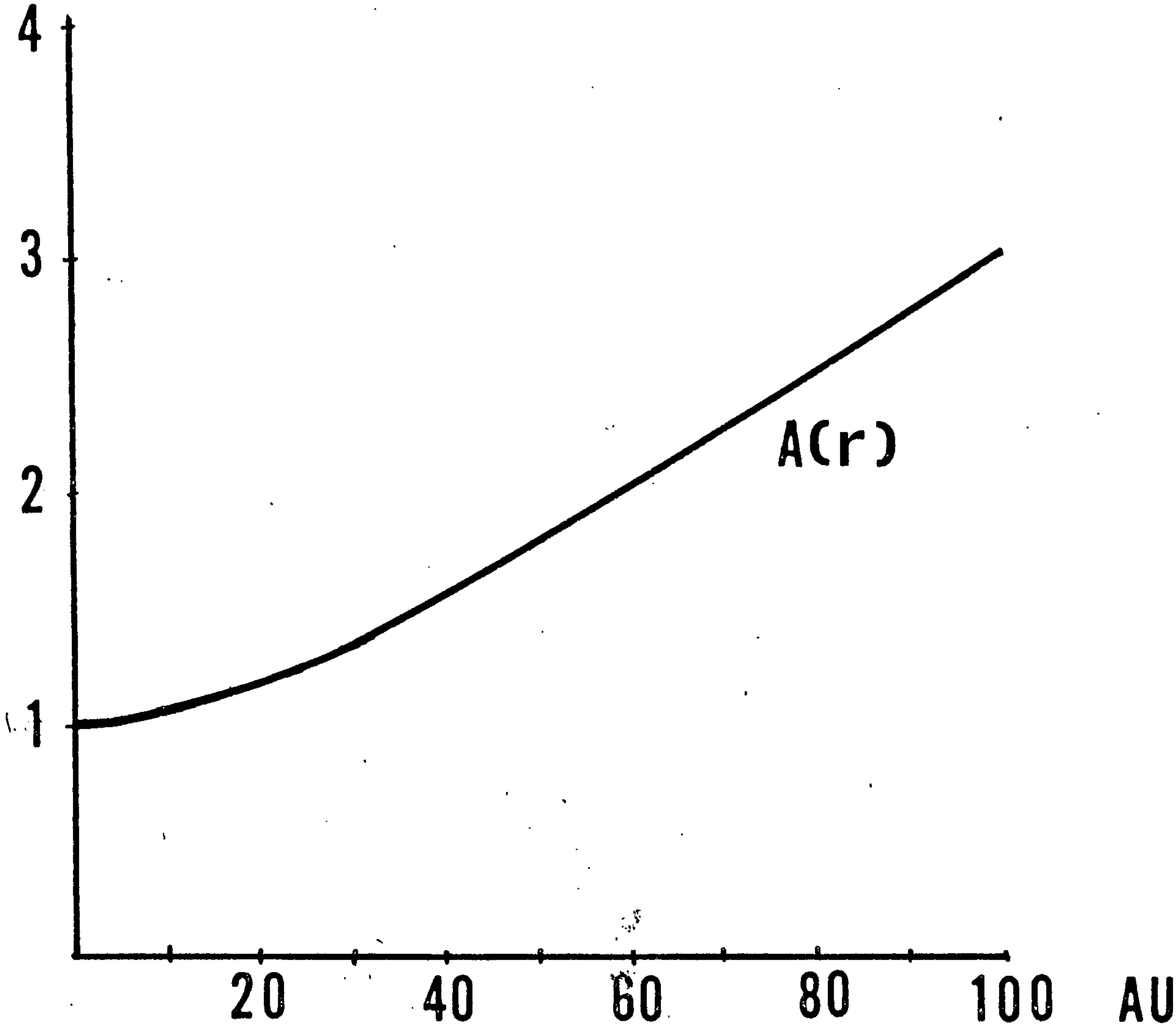


FIG 2

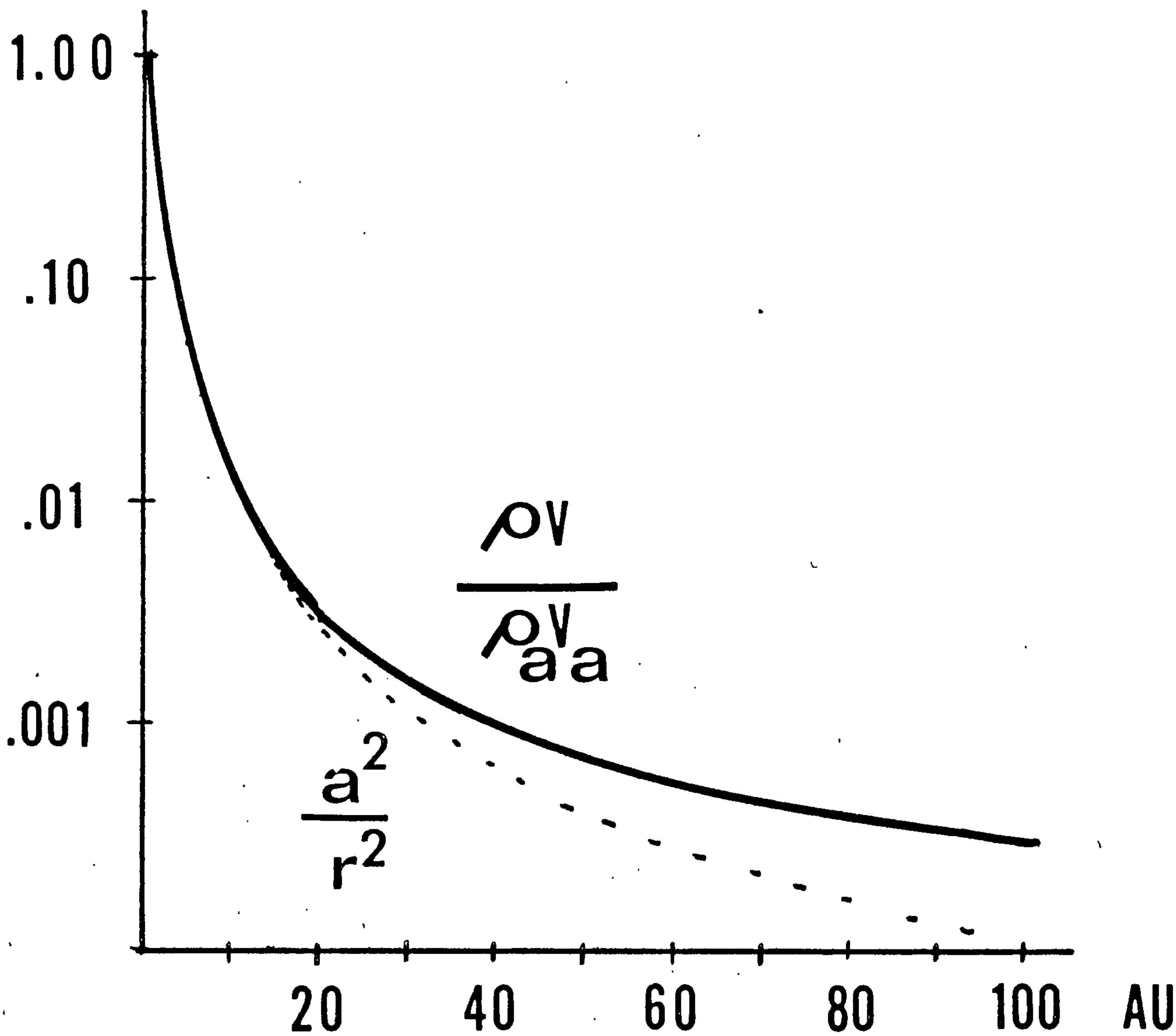


FIG 3

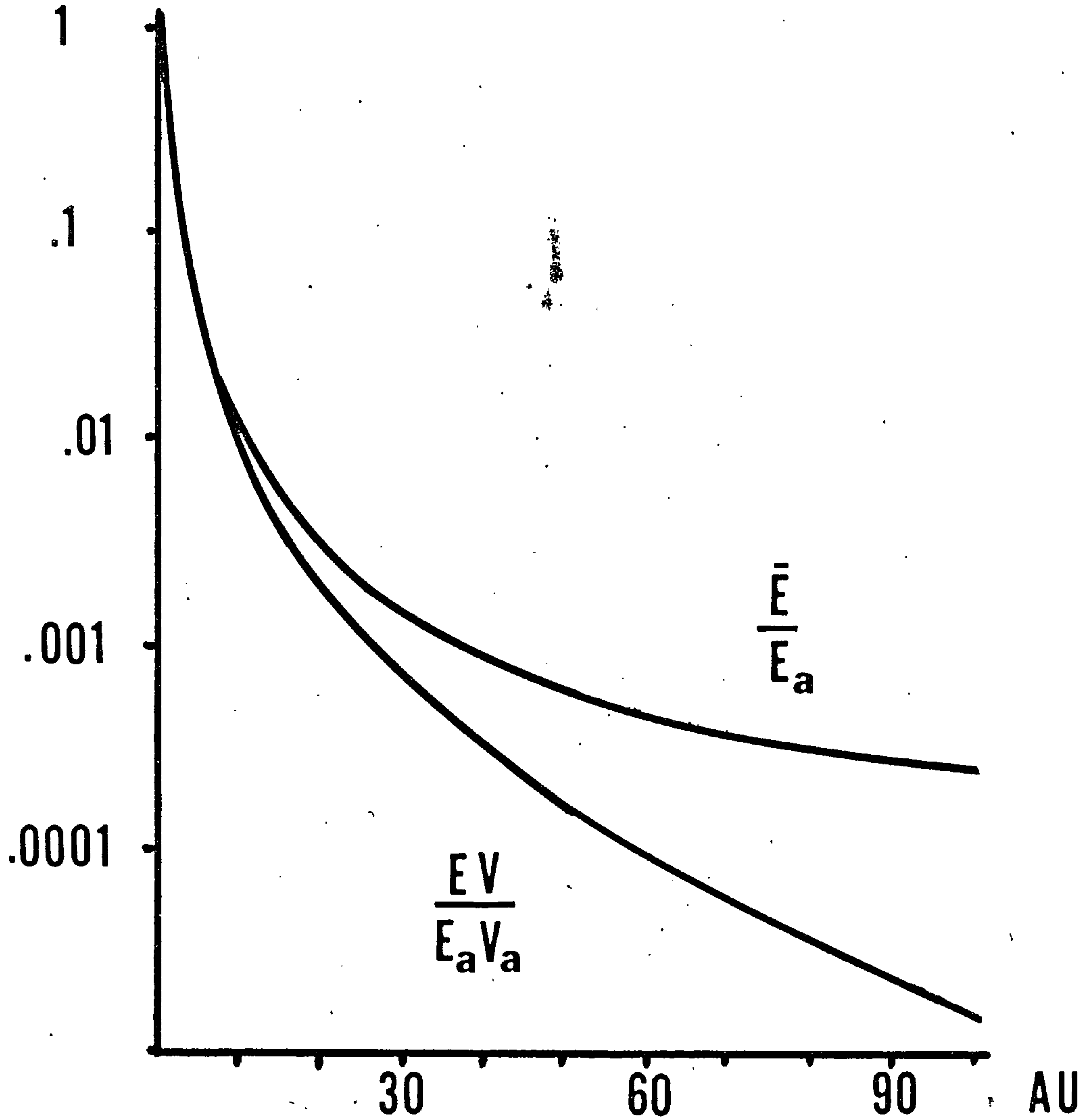


FIG 4

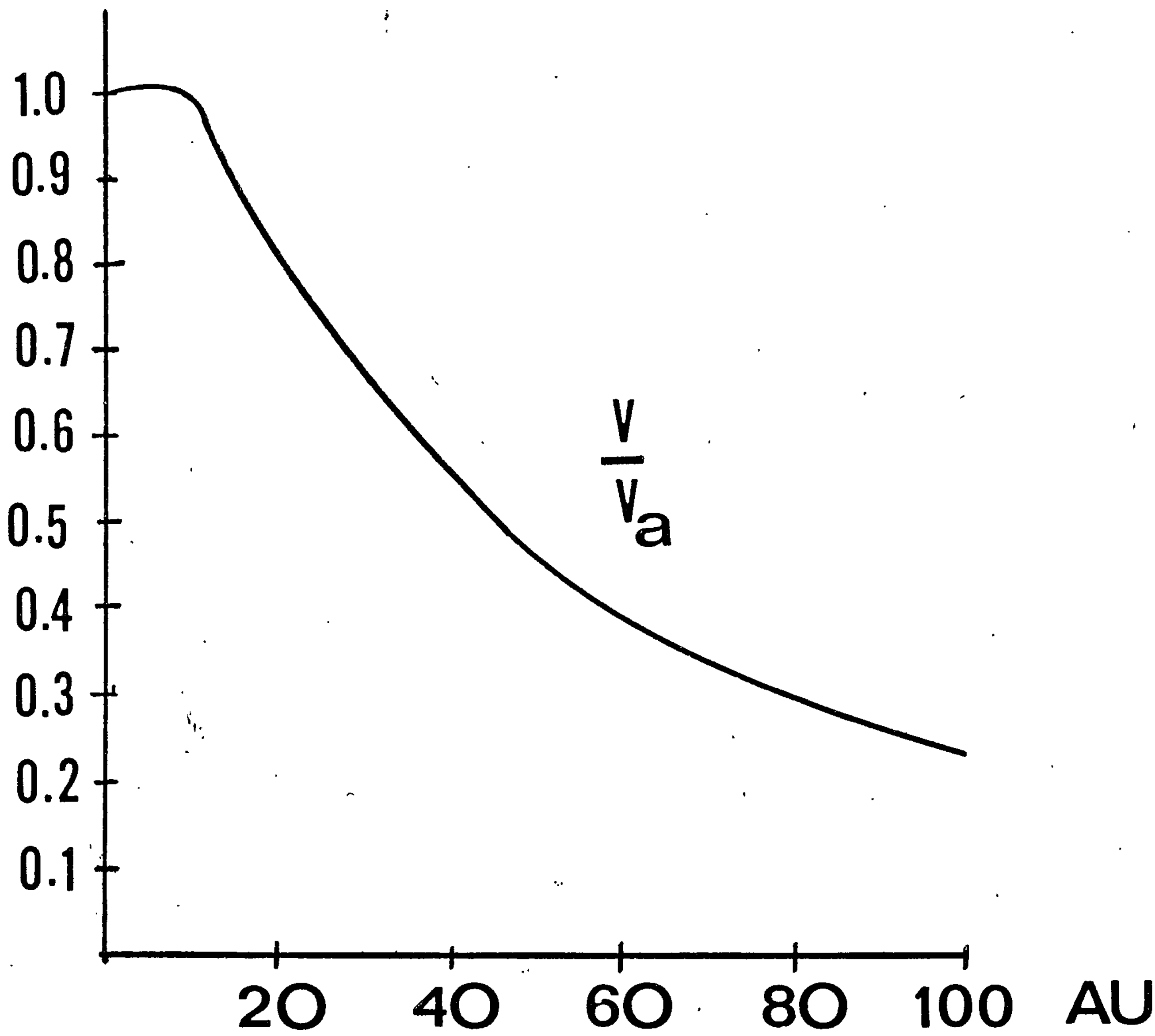


FIG 5

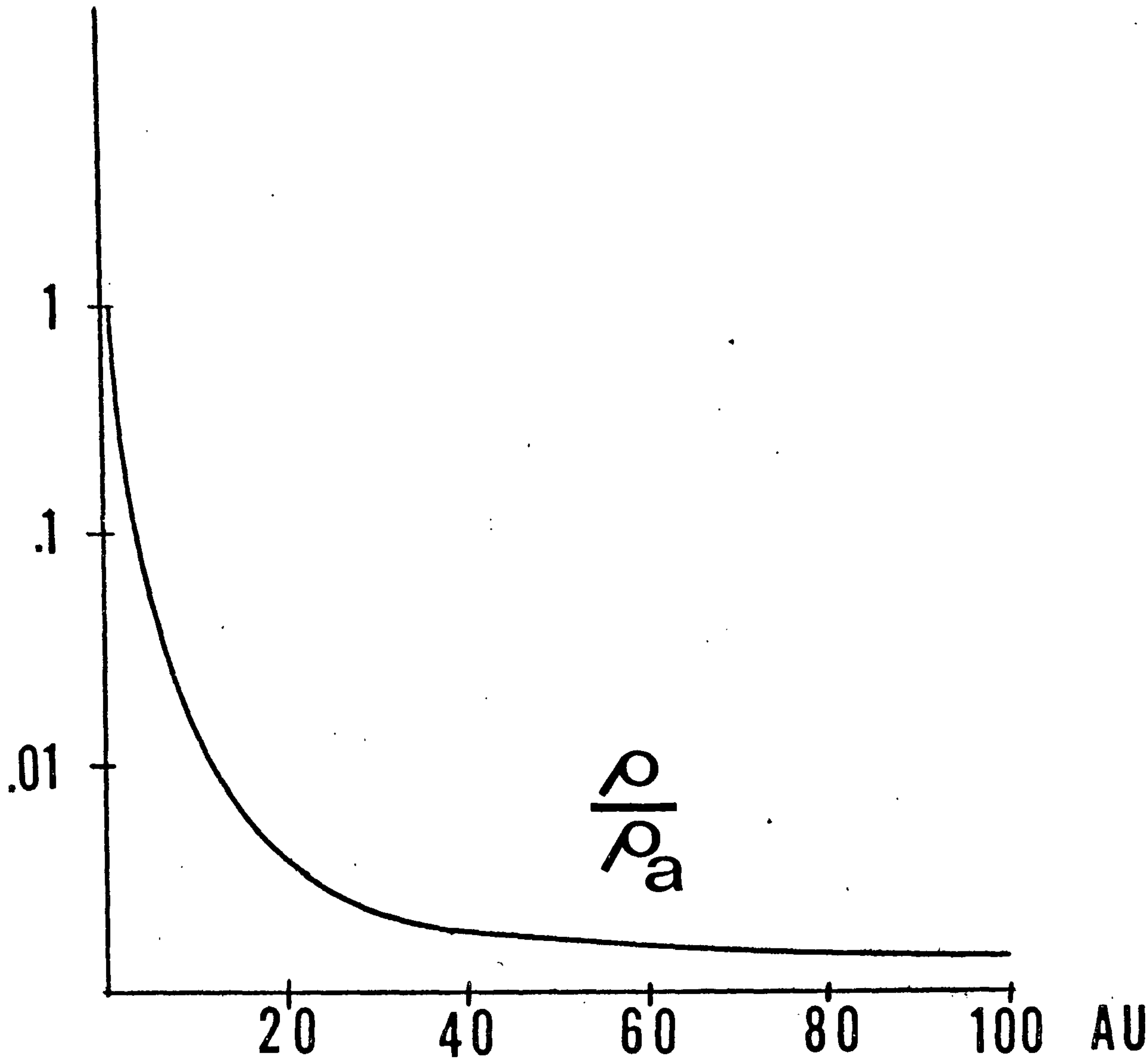


FIG 6

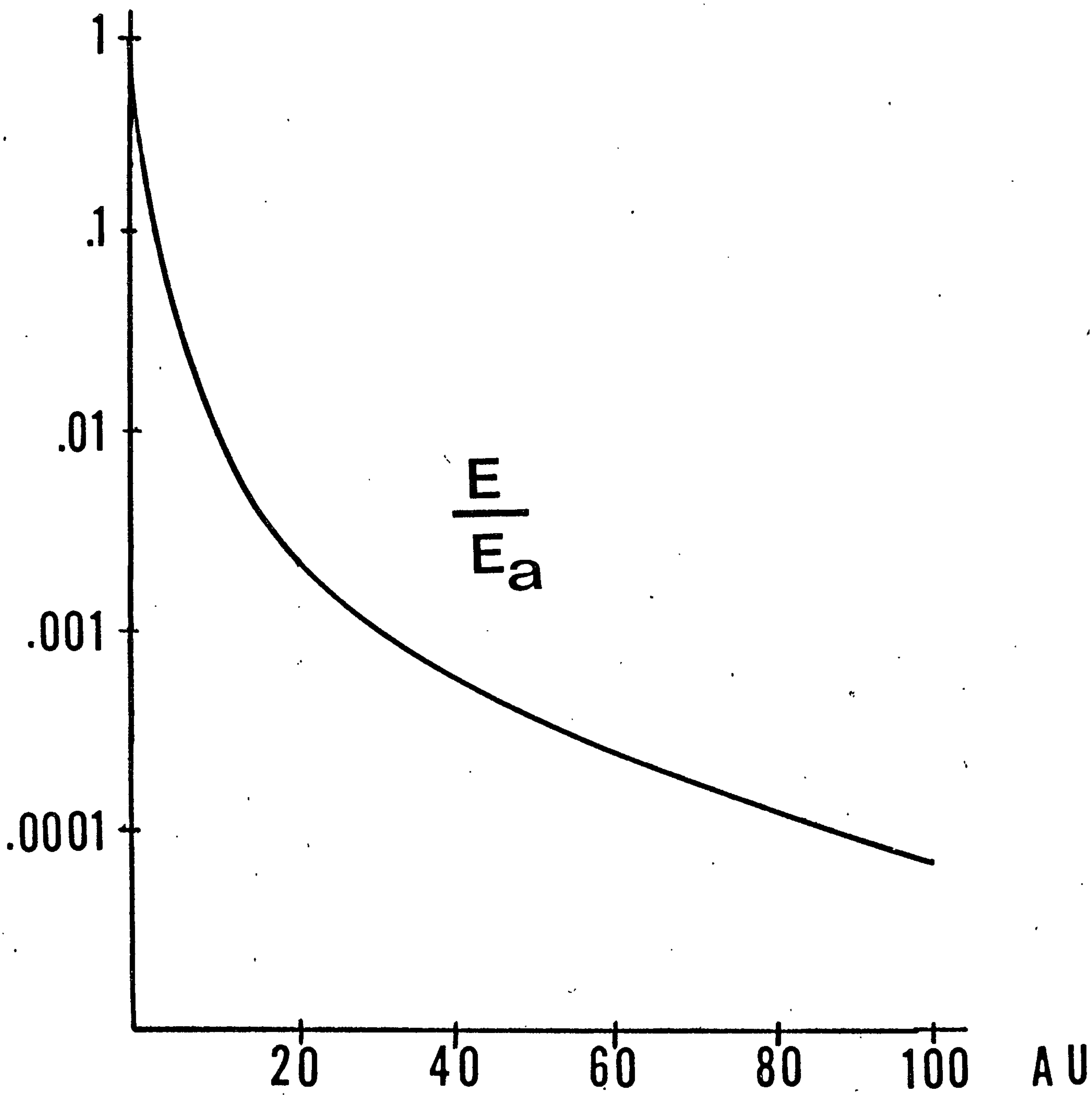
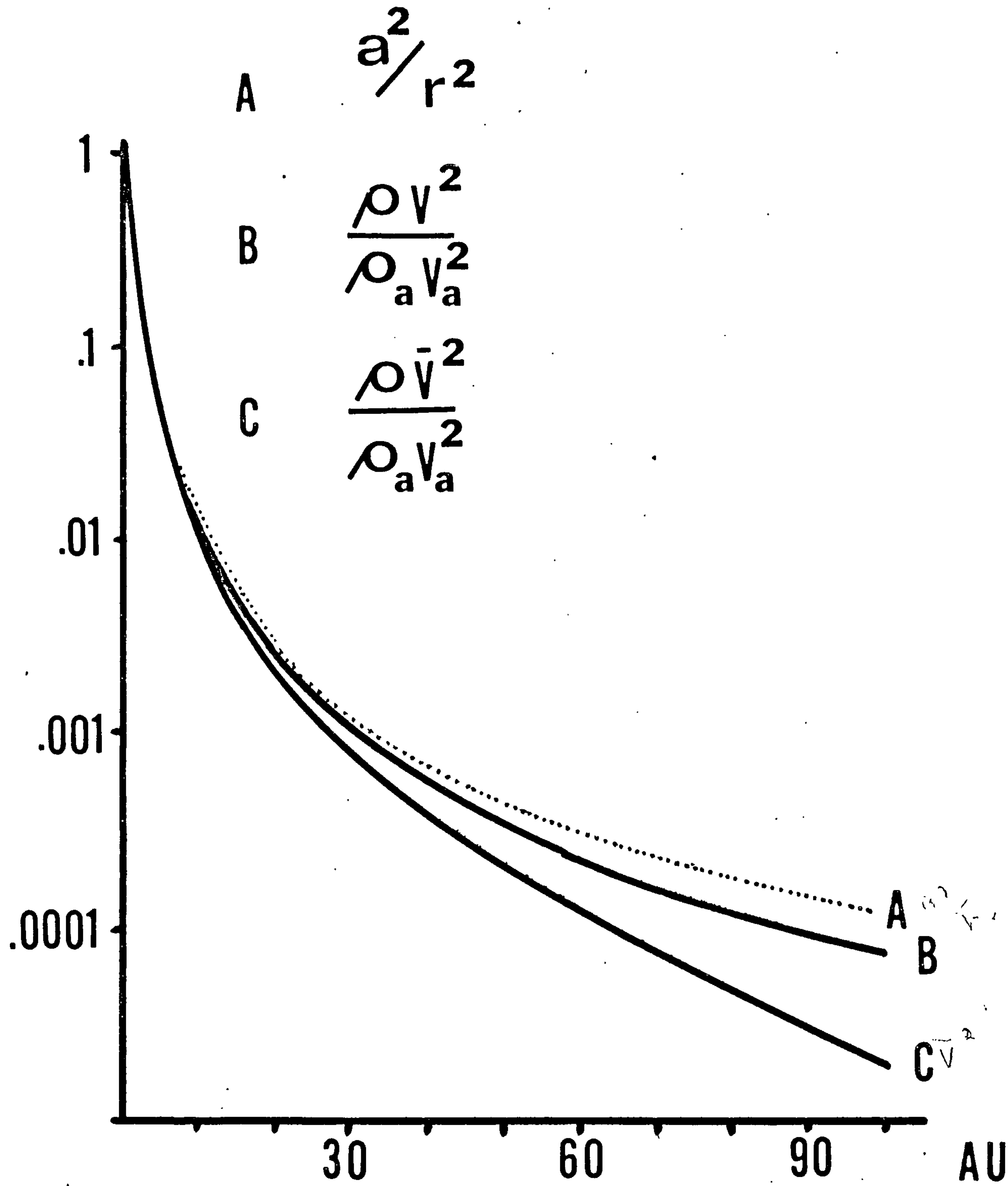


FIG 7



A a^2/r^2
B $\frac{\rho V^2}{\rho_a V_a^2}$
C $\frac{\rho \bar{V}^2}{\rho_a V_a^2}$
1
.1
.01
.001
.0001
30
60
90
AU
A
B
C

FIG 8

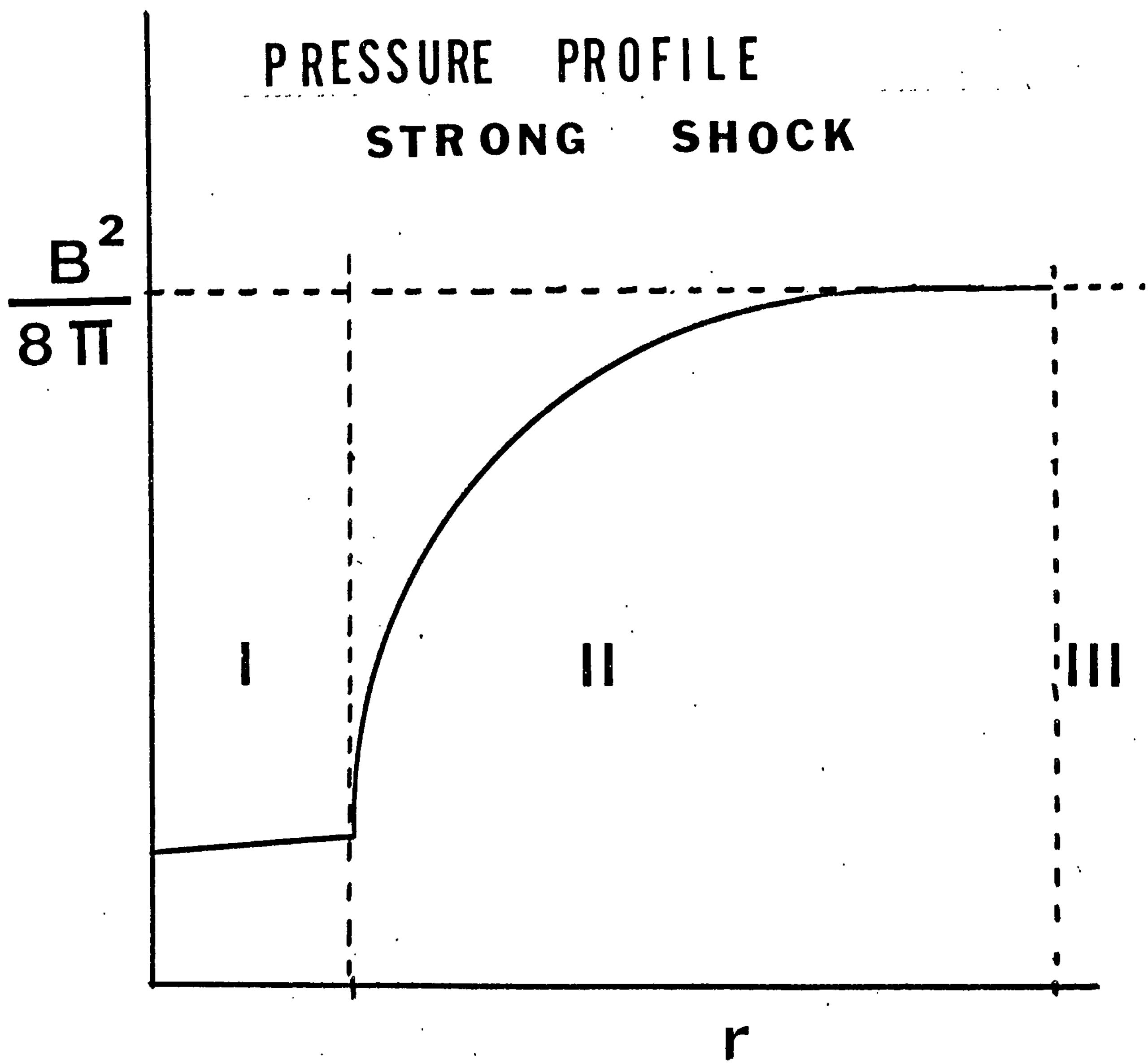


FIG 9

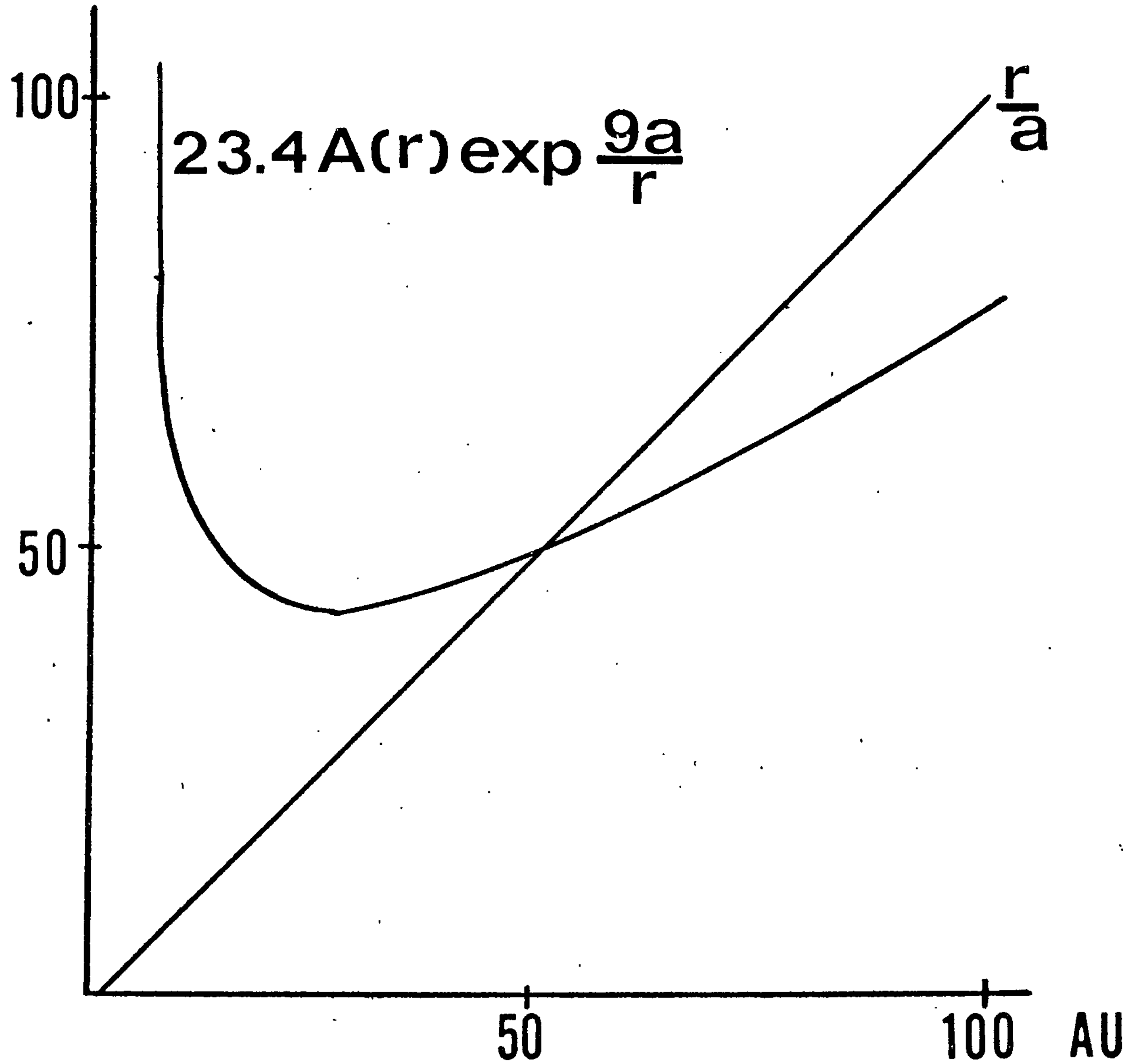


FIG 10

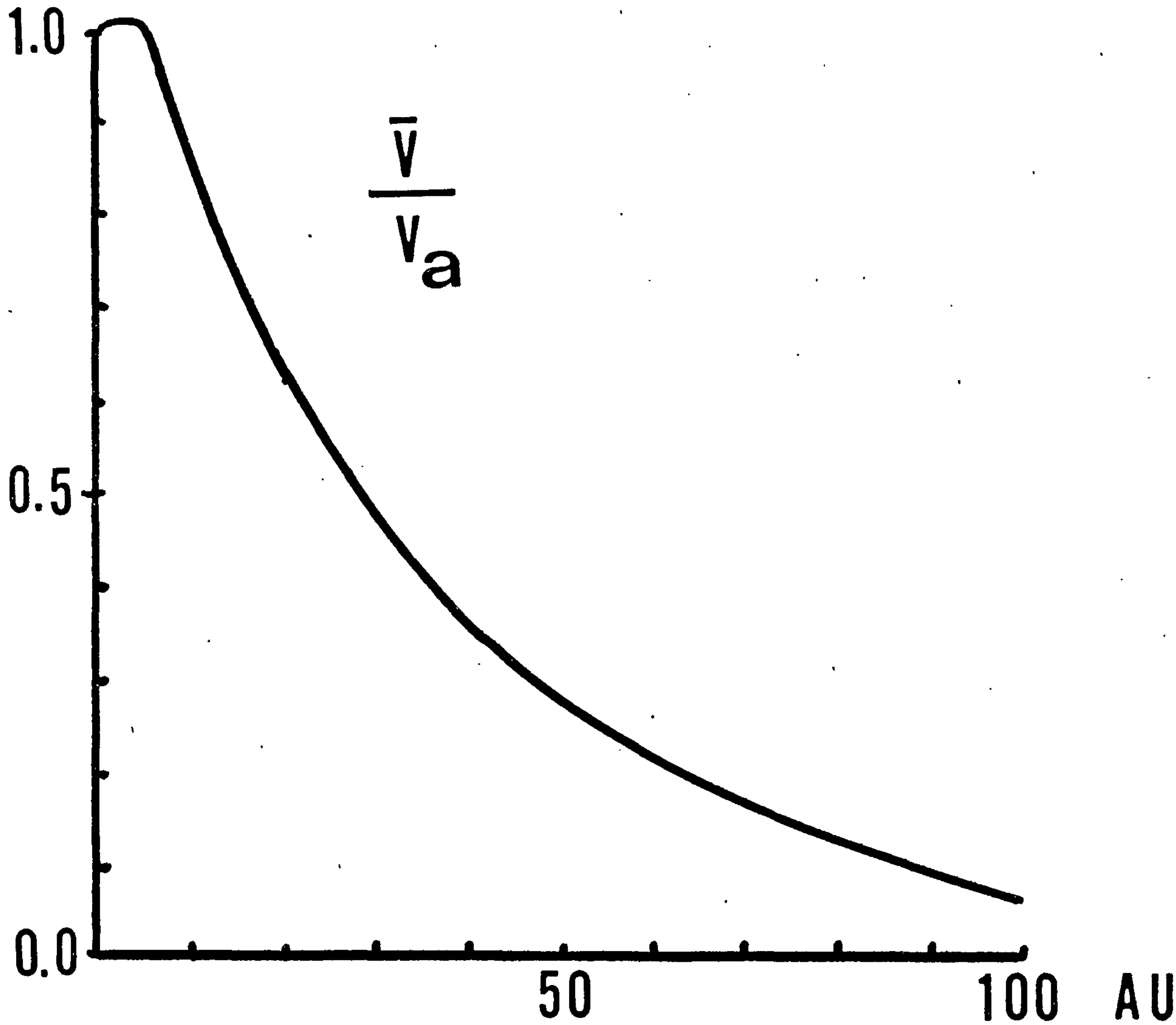


FIG 11

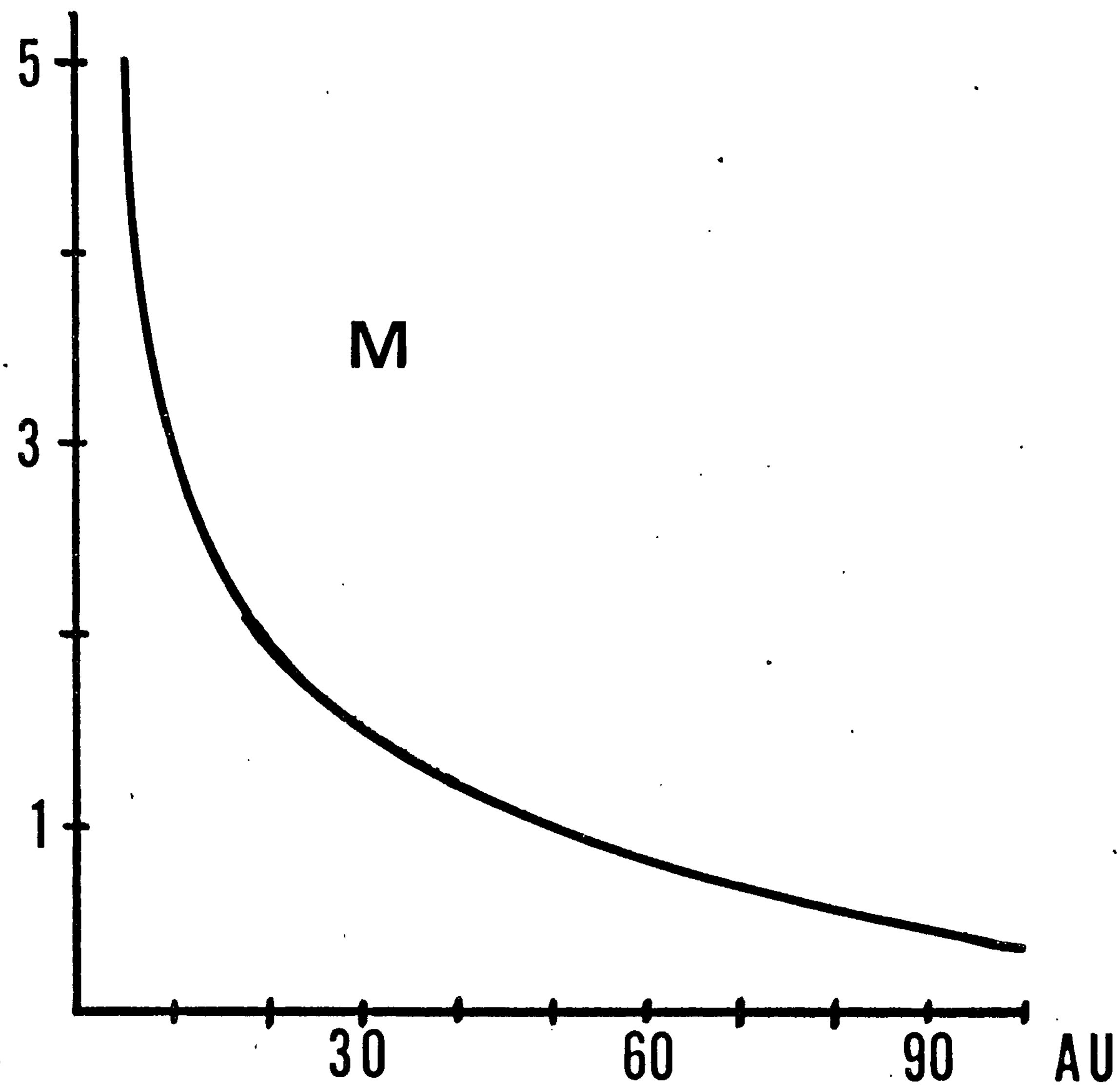


FIG 12

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