

ECONOMIC OPTIMAL LEVELS OF CONTROL OF SULFUR DIOXIDE EMISSIONS
FROM THE COMBUSTION OF FOSSIL FUELS

A Dissertation
Presented to
the Faculty of the Department of Economics
University of Houston

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Sigmund Alan Horvitz

August 1971

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ABSTRACT

The 1970 Amendments to the federal Clean Air Act direct the Administrator of the Environmental Protection Agency to promulgate uniform standards governing the ambient concentrations of a number of air pollutants including sulfur dioxide. This statute suggests the question of what constitutes optimal levels of control of air pollutants and provides the direction for the study, the subject of which is economic optimal levels of control of sulfur dioxide emissions.

In Chapter I the sources of sulfur dioxide concentrations, the effects of sulfur dioxide concentrations, and the technology of control of sulfur dioxide emissions are examined. The social control of sulfur dioxide emissions and concentrations is the subject of Chapter II.

The literature on optimal levels of control of sulfur dioxide emissions and concentrations is the subject of Chapter III. Much of this work is directed toward numerical solutions. The result is that there is not in the literature a statement, rigorously derived, of optimal control policy criteria.

Analytic solutions for optimal levels of control of sulfur dioxide emissions are derived in Chapter IV. The analysis proceeds within a static framework. The conditions under which the static solution for a series of instantaneous

optima approaches the dynamic solution for the optimal path of control is the subject of Chapter VII.

In Chapter V the implications of the analysis for a priori decision making are examined. Here rules which are operationally useful in making a priori judgements about optimal levels of control are derived. The use of the rules in a priori decision making is demonstrated.

The numerical solution of the model is the subject of Chapter VI. Here the results of a case study of optimal rates of emission at the sources of sulfur dioxide emissions from the combustion of fossil fuels are reported.

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CHAPTER I
INTRODUCTION

Harmful Effects ¹

External effects are incidental byproducts of the activity of one economic unit which directly affect the activities of other economic units. Where external effects are present incremental social cost and incremental private cost diverge with the result that the market fails to allocate resources efficiently. Too much of some goods and too little of other goods are produced.

One class of external effects are produced by adverse changes in the environment arising from production processes. The costs of these adverse effects are borne by economic units which can be viewed as separate from the producer of the external effects. The market generates the wrong prices with the result that resources are misallocated.

¹This section is based upon R. H. Coase, "The Problem of Social Cost," The Journal of Law and Economics, III (October, 1960) 1-44, Allen V. Kneese, "Air Pollution--General Background and Some Economic Aspects," in The Economics of Air Pollution, ed. by Harold Wolozin (New York: W.W. Norton & Co., 1966), pp. 23-39, E. J. Mishan, "The Postware Literature on Externalities: An Interpretive Essay," Journal of Economic Literature, IX (March 1971), 1-28, and Gordon Tullock, "Social Cost and Government Action," The American Economic Review, Papers and Proceedings of the Eighty-First Annual Meeting of the American Economic Association, Chicago, Ill., Dec. 28-30, 1968, LIX (May, 1969), 189-197

The function of control efforts, collectively taken, in the case of harmful effects, is to bring private and social cost into equality in order to reduce or eliminate the adverse effects. Reallocation of resources through the imposition of control gives rise, however, to losses as well as gains. The policy objective is to maximize the difference between total cost of control and total social cost avoided. This is achieved when control is carried to the point where the marginal cost of control is just equal to the marginal social cost avoided.

Air pollutants constitute a principal source of adverse changes which give rise to external effects. Five types of air pollutants account for more than 90% of the pollutants from all sources in the air.² In 1966 209.6 million tons of the five classes of pollutants--carbon monoxide, hydrocarbons, nitrogen oxides, sulfur oxides, and particulates--were emitted into the air.³ Data on 1966 emissions are given in Table 1.

²Dermot A. O'Sullivan, "Air Pollution," Chemical & Engineering News, XLVIII (June 8, 1970), 41.

³Ibid.

Table 1

THE COMPONENTS OF AIR POLLUTION IN 1966

POLLUTANT	1966 Concentrations In Millions of Tons
Carbon Monoxide	101.6
Hydrocarbons	31.5
Nitrogen Oxides	16.7
Sulfur Oxides	31.2
Particulates	28.6
TOTAL EMISSIONS	209.6

Source: Dermot A. O'Sullivan, "Air Pollution,"
Chemical & Engineering News, XLVIII (June 8, 1970), 41.

Among the activities giving rise to these concentrations are transportation, fuel combustion by stationary sources, industrial processes and garbage incineration. The largest contributor is transportation which in 1966 accounted for approximately one half of the concentration of air pollutants. The principal pollutants to which this activity gave rise in 1966 were carbon monoxide, hydrocarbons and nitrogen oxides.

Sulfur Dioxide Emissions and Concentrations

The sources of sulfur dioxide emissions in 1966 are given in Table 2. Though mobile sources were the largest contributors to the air pollution problem in 1966, stationary

sources were the primary sources of sulfur dioxide concentrations. Among stationary sources

Table 2

SOURCES OF SULFUR DIOXIDE CONCENTRATIONS IN 1966

SOURCES	1966 Emissions of Sulfur Dioxide in Million Tons
Mobile	.4
Stationary	30.8
Fuel Combustion	22.9
Industrial Processes	7.2
Garbage Inceneration	.1
Other	.6
ALL SOURCES	31.2

Source: Dermot A. O'Sullivan, "Air Pollution," Chemical & Engineering News, XLVIII (June 8, 1970), 41

fuel burning installations were the most important sources.

The National Air Pollution Control Administration in 1970 estimated that sulfur dioxide emissions from stationary sources in 1970 will be 36.6 million tons.⁴ The sources

⁴U. S. Congress, Senate, Committee on Public Works, Air Pollution 1970-Part I, Hearings before a subcommittee of the committee on Public Works, Senate, on S.3229, S.3466, and S.3546, 91st Cong., 2nd Sess., 1970, p. 172.

of the estimated emissions are given in Table 3. Fuel combustion is, again, the primary source. Power plants using coal are the most important sources among fuel burning installations.⁵

Table 3

ESTIMATED EMISSIONS OF SULFUR DIOXIDE IN 1970

SOURCES	Estimated 1970 Emission of Sulfur Dioxide in Million Tons
Combustion of Coal and Oil by Power Plants	20.0
Other Combustion of Coal	4.8
Other Combustion of Oil	3.4
Non Combustion Sources	8.4
ALL SOURCES	36.6

Source: U. S. Congress, Senate, Committee on Public Works, Air Pollution 1970-Part I, Hearings before a subcommittee of the Committee on Public Works, Senate, on S.3229, S.3466, and S.3546, 91st Cong., 2nd Sess., 1970, p. 172.

⁵Ibid., p. 174. The combustion of coal accounts for more than 90 percent of the estimated 1970 sulfur dioxide pollution from power generation. Coal combustion accounts for more than 80 percent of the estimated 1970 sulfur dioxide pollution by all stationary fuel combustion. The combustion of coal accounts for more than 60 percent of the estimated sulfur dioxide pollution by all stationary sources.

The National Air Pollution Control Administration has estimated that total sulfur dioxide pollution without abatement will increase to 60.9 million tons in 1980, 86.4 million tons in 1990, and 125.8 million tons in the year 2000.⁶ The contribution by stationary source fuel combustion for each of these years will be 49.9 million tons in 1980, 69.4 million tons in 1990, and 101.2 million tons in the year 2000.⁷

It is important to note that stationary fuel combustion--primarily the combustion of coal--accounted for 41.4 million tons or approximately 20 percent of all air pollution in 1966.⁸ Of this 41.4 million tons, 9.2 million tons were particulates, 6.7 million tons were nitrogen oxides, 1.9 million tons were carbon monoxide, 0.7 million tons were

⁶Ibid., p. 172.

⁷Ibid.

⁸O'Sullivan, "Air Pollution," 41.

hydrocarbons, and 22.9 million tons were sulfur dioxide.⁹

The Effects of Sulfur Dioxide Concentrations

The National Air Pollution Control Administration has reported the evidence of the effect of sulfur dioxide on visibility, health, materials, animals and vegetation.¹⁰ This evidence indicates an association between sulfur dioxide pollution and certain morbidity and mortality rates. It also indicates that sulfur oxides corrode metals, disintegrate paints, weaken fibers, discolor building materials, suppress plant growth and yield, and reduce visibility.

In one study of the effect of sulfur dioxide pollution upon man it was concluded that for an annual mean concentration of 0.037 ppm to 0.092 ppm accompanied by smoke concentrations increased frequency of respiratory symptoms

⁹ Ibid.

¹⁰ See United States Department of Health, Education, and Welfare, Public Health Service, Environmental Health Service, National Air Pollution Control Administration, Air Quality Criteria for Sulfur Oxides (Washington, D. C.: Government Printing Office, 1970) and United States Department of Health, Education, and Welfare, Public Health Service, Environmental Health Service, National Air Pollution Control Administration, National Emissions Standard Study Report, A Report to the Congress of the United States by the Secretary of Health, Education and Welfare, in compliance with Public Law 90-148, Clean Air Act, As Amended (Appendices -3 Vols; Washington, D. C.: U. S. Department of Health, Education, and Welfare, Public Health Service, Environmental Health Service, National Air Pollution Control Administration, 1970), I, pp. B-2-B-25.

and lung disease may occur.¹¹ Another study concluded that an annual mean concentration of 0.040 ppm accompanied by smoke concentrations may increase mortality from bronchitis and lung cancer.¹²

Lave and Seskin have reviewed the studies of the health effects of air pollution.¹³ They conclude that the studies indicate that mortality from bronchitis would be reduced by about 50 percent if concentrations of air pollutants were reduced by approximately 50 percent.¹⁴ They also find that the studies indicate that approximately 25 percent of mortality from lung cancer can be eliminated by a 50 percent reduction in air pollution.¹⁵ Their conclusion is that these studies indicate a strong relationship between all respiratory

¹¹United States Department of Health, Education, and Welfare, Public Health Service, Environmental Health Service, National Air Pollution Control Administration, Air Quality Criteria for Sulfur Oxides, p. 162.

¹²Ibid.

¹³Lester B. Lave and Eugene P. Seskin, "Air Pollution and Human Health," Science, CLXIX (August, 1970), 723-733.

¹⁴Ibid., 730.

¹⁵Ibid.

disease and air pollution and that it is likely that 25 percent of all morbidity and mortality due to respiratory disease could be eliminated by a 50. percent reduction in air pollution levels.¹⁶

In one study it was concluded that for an annual mean concentration of 0.12 ppm of sulfur dioxide accompanied by high levels of particulate matter, the corrosion rate for steel panels may be increased by 50 percent.¹⁷ In another study it was found that for an annual mean concentration of 0.03 ppm of sulfur dioxide chronic plant injury and excessive leaf drop may occur.¹⁸

The Technology of Control of Sulfur Dioxide Emissions

Among the methods, some of which are now only potentially useful, of reducing sulfur dioxide pollution from stationary combustion sources are the substitution in the combustion process of natural gas and naturally occurring low sulfur coal and oil for high sulfur fuels, the use of fuels which have been desulfurized through refining or cleaning, the

¹⁶Ibid.

¹⁷United States Department of Health, Education and Welfare, Public Health Service, Environmental Health Service, National Air Pollution Control Administration, Air Quality Criteria for Sulfur Oxides, p. 162.

¹⁸Ibid.

desulfurization of flue gas prior to escape into the air, the removal of sulfur and sulfur compounds during combustion, the modification of the combustion process in order to save fuel, the use of blends of low and high sulfur oils, plant relocation, the dispersal of flue gasses by means of tall stacks, and the substitution of atomic energy and hydro-electric energy for fossil fuel energy sources.¹⁹

One technique for the control of sulfur dioxide emissions from the combustion of coal is the substitution in the combustion process for high sulfur coal of naturally occurring low sulfur coal. Although there are relatively large reserves of low sulfur coal in the United States, production has been committed, at least until recently, in large part,

¹⁹The technology of control of sulfur dioxide emissions is discussed in Federal Power Commission, Staff Report, Air Pollution and the Regulated Electric Power and Natural Gas Industries, (Washington, D. C.: Federal Power Commission, 1968), United States Department of Health, Education and Welfare, Public Health Service, Consumer Protection and Environmental Health Services, National Air Pollution Control Administration, Control Techniques for Sulfur Oxide Air Pollutants (Washington, D. C.: U. S. Department of Health, Education, and Welfare, Public Health Service, Consumer Protection and Environmental Health Service, National Air Pollution Control Administration, 1969), and United States Department of Health, Education, and Welfare, Public Health Service, Environmental Health Service, National Air Pollution Control Administration, National Emissions Standard Study Report, I, p. 0-79, III, p. F-502

to the steel industry²⁰ and the export market.²¹ An additional problem is that a three year lag exists to bring new mines into production.²²

One technique for the control of sulfur dioxide emissions from the combustion of residual fuel oil is the substitution of desulfurized residual fuel oil in the combustion process for high sulfur residual fuel oil. Residual fuel oil is desulfurized in relatively small quantities to about 1.0 percent sulfur.²³ The lead time for new production is two to three years.²⁴

²⁰Norman Plaks, "Control of Sulfur Oxides and Nitrogen Oxides" (paper presented at the American Society of Heating, Refrigerating and Air Conditioning Engineers, ASHRAE Annual meeting, San Francisco, California, January 18-22, 1970), p. 3.

²¹Federal Power Commission, Staff Report, Air Pollution and the Regulated Electric Power and Natural Gas Industries, p. 16.

²²Ibid.

²³United States Department of Health, Education, and Welfare, Public Health Service, Consumer Protection and Environmental Health Services, National Air Pollution and Control Administration, Control Techniques for Sulfur Oxide Air Pollutants, pp. 4-78 - 4-82.

²⁴Federal Power Commission, Staff Report, Air Pollution and the Regulated Electric Power and Natural Gas Industries, p. 80.

CHAPTER II

SOCIAL CONTROL OF EMISSIONS AND CONCENTRATIONS OF SULFUR DIOXIDE

Rules governing emissions and concentrations of sulfur dioxide have been promulgated by federal, state and local authorities. The state and local laws were promulgated pursuant to the 1967 Federal Air Quality Act, which required the states to adopt ambient air quality standards and implementation and enforcement plans. The 1967 federal statute is first considered. The state and local rules are then examined. Next, the 1970 amendments to the Federal Clean Air Act are examined. The 1970 amendments passed ambient rule making authority to federal authorities. Finally, the regulations promulgated pursuant to the 1970 amendments are considered.

State and local rules governing emissions and concentrations of sulfur dioxide are effective under the terms of the Air Quality Act of 1967.¹ This statute directed the Secretary of Health, Education and Welfare to develop and issue to the states such criteria of air quality as in his judgement were necessary for the protection of the public

¹U. S. Code, Vol. XXXXII, sec. 1857, amending the Clean Air Act, as amended.

health and welfare, and information on those pollution control techniques which were required to achieve levels of air quality set out in the criteria.² The statute required the state, upon receipt of the air quality criteria and the information on recommended control techniques from the secretary, to file through its governor a letter of intent that the state would, within one hundred and eighty days, adopt ambient air quality standards governing stationary sources of air pollution within the state.³ The statute further required that, thereafter, but within one hundred and eighty days, the state adopt a plan for the implementation, maintenance, and enforcement of the standards adopted.⁴ The statute provided that the standards adopted by the state were the standards applicable to the state upon a determination by the Secretary that the standards were consistent with the data on air quality criteria and control techniques and that the plan for implementation and enforcement assured achieving the standards within a reasonable time.⁵

A number of states promulgated regulations governing ambient sulfur dioxide concentrations. Two sets of these

²Ibid., sec. 1857c.

³Ibid., sec. 1857d

⁴Ibid.

⁵Ibid.

regulations are considered.

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The Texas regulation⁶ governing ambient concentrations of sulfur dioxide provides, in part, that in all areas other than industrial areas the ambient air shall not exceed 0.2 ppm of sulfur dioxide in any 24 hour period and that in industrial areas the ambient air shall not exceed 0.3 ppm average during any 24-hour period.

The New Jersey regulation⁷ governing ambient concentrations of sulfur dioxide applies uniformly throughout the state and provides, in part, that

during any one hour period the average concentration of sulfur dioxide shall not exceed 0.25 ppm

during any twelve consecutive months one hour average concentrations may attain 0.25 ppm no more than once.

during any twelve consecutive months one hour average concentrations may attain or exceed 0.20 ppm no more than nine times.

during twelve consecutive months the geometric mean value of all one hour averages shall not exceed 0.017 ppm.

The State and local authorities promulgated regulations governing emissions at the source in order to achieve the ambient air quality standards adopted.

⁶Texas Air Control Board, Regulation III, pp. III-1-III-2. (Mimeographed.)

⁷The Bureau of National Affairs, Environment Reporter, State Air Laws, sec. 451, para. 0761.

In Table 4 the form of the rules governing sources in several jurisdictions and the sources to which the rules apply are given. Under the regulations in effect in Dade County, Florida, Los Angeles County, California, the San Francisco Bay Area, and the City of New York uniform emission standards and fuel regulations are applied to all sources. Under the regulations in effect in New Jersey, and St. Louis varying emission standards and fuel regulations are applied to the sources.

The New Jersey regulation governing sulfur dioxide emissions from coal combustion distinguishes between sources on the bases of type of coal used and the purpose for which the coal is used. It is prohibited under this regulation for any person to use bituminous coal containing sulfur in excess of 1 percent by weight or anthracite coal containing sulfur in excess of 0.7 percent by weight. An exemption from this rule applies to specific power generating facilities that demonstrate that a facility cannot accept coal which complies with the percent by weight sulfur regulations.

The Air Pollution Control Regulation for the St. Louis Metropolitan Area distinguishes between sources on the basis of the size of the fuel burning installation. This regulation prohibits fuel burning installations having a capacity of 2,000 million or more Btu's per hour to emit sulfur dioxide in an amount greater than 2.3 pounds of sulfur dioxide per

Table 4

SELECTED SULFUR DIOXIDE REGULATIONS

Jurisdiction	Form of Regulation	Sources to Which the Regulation Applies
New Jersey	Fuel Regulation governing the sulfur content of coal	The regulations distinguish between bituminous coal users and anthracite coal users and between use by power plants and by others
St. Louis Metropolitan Area, Missouri	Fuel Regulations governing the sulfur content of fuel	The regulations distinguish between fuel burning installations, on the basis of size.
Dade County Florida	Emission Standards	The standards apply to all sources equally
Los Angeles County, California	Emission Standards	The standards apply to all sources equally
City of New York	Fuel Regulations governing the sulfur content of fuel	The regulations apply equally to coal users and to users of residual fuel oil
San Francisco Bay Area, California	Emission Standards	The standards apply to all sources equally

Source: The New Jersey regulation is reported in The Bureau of National Affairs, Environment Reporter, State Air Laws, sec. 451, para. 0701. The other regulations are in U.S. Department of Health, Education & Welfare, Abatement and Control Development Programs, A Compilation of Selected Air Pollution Emission Control Regulations and Ordinances (rev. ed.; Washington, D.C.: Government Printing Office, 1968).

million Btu's of heat input to the installation, and prohibits fuel burning installations having a capacity of less than 2,000 million Btu's per hour to use coal and residual fuel oil containing more than 2 percent sulfur by weight during a two month period in 1968-1969, during a four month period in 1969-70, during a six month period in 1970-71, and thereafter.

Under the Clean Air Amendments of 1970⁸ Congress placed ambient rule making authority in the office of the Administrator of the Environmental Protection Agency subject to the right of the states to adopt more strict standards than those promulgated by the administrator.⁹ The administrator is directed to promulgate within 120 days after the enactment of the Clean Air Amendment regulations prescribing a national primary ambient air quality standard and a national secondary ambient air quality standard for each pollutant for which air quality criteria has been issued.¹⁰ National primary ambient air quality standards are those which in the judgement of the administrator are necessary to protect the public health.¹¹ National secondary ambient air quality

⁸Pub. L. 91-604, Statutes at Large, LXXXIV, sec. 1676, enacted December 1, 1970 amending the Clean Air Act, as amended.

⁹Ibid., sec. 116.

¹⁰Ibid., sec. 109.

¹¹Ibid.

standards are those which in the judgement of the administrator are necessary to protect the public welfare from any known or anticipated adverse effects associated with the presence of the pollutants in the ambient air.¹²

The statute requires each state to adopt and submit to the administrator within nine months after the promulgation of national primary or secondary ambient air quality standards a plan which provides for implementation, maintenance, and enforcement of the standards within the state.¹³ Within four months after the date required for submission of a plan, the administrator must approve or disapprove the plan, approving it if he finds that it provides for the attainment of the primary standard as expeditiously as practicable and no later than 3 years from the date of approval of the plan and the attainment of the secondary standard within a reasonable time.¹⁴

¹² Ibid.

¹³ Ibid., sec. 110.

¹⁴ Ibid. Section 110 provides that, upon application of the governor of a state at the time of submission of a plan implementing a national primary ambient air quality standard, the administrator may extend the 3 year period but for not more than 2 years if he determines, among other things, that one or more emission sources are unable to comply with the requirements of the plan because the necessary technology is not available or will not be available soon enough to permit compliance within the three year period and the state has considered reasonably available alternative means of attaining the primary standard and has concluded that attainment of the primary standard within the three years cannot be achieved.

The administrator of the Environmental Protection Agency promulgated in April, 1971 standards governing a number of pollutants, including sulfur dioxide.¹⁵ The national primary ambient air quality standards for sulfur dioxide are 0.03 ppm annual arithmetic mean and 0.14 ppm maximum 24 hour concentration not to be exceeded more than once per year.¹⁶ The National secondary ambient air quality standards for sulfur dioxide are 0.02 ppm annual arithmetic mean and 0.10 ppm maximum 24 hour concentration not to be exceeded more than once per year, and 0.50 maximum 3 hour concentration not to be exceeded more than once per year.¹⁷

Against the background developed in Chapters I and II there are developed in the chapters that follow criteria for optimality in the control of sulfur dioxide emissions and concentrations from the combustion of fossil fuels. The analysis proceeds within a static framework. Some aspects of the problem of dynamic optimality are discussed in the concluding chapter.

¹⁵Environmental Protection Agency, "National Primary And Secondary Ambient Air Quality Standards," Federal Register, XXXVI. No. 84, April 30, 1971, 8186 - 8201.

¹⁶Ibid., 8187.

¹⁷Ibid.

CHAPTER III

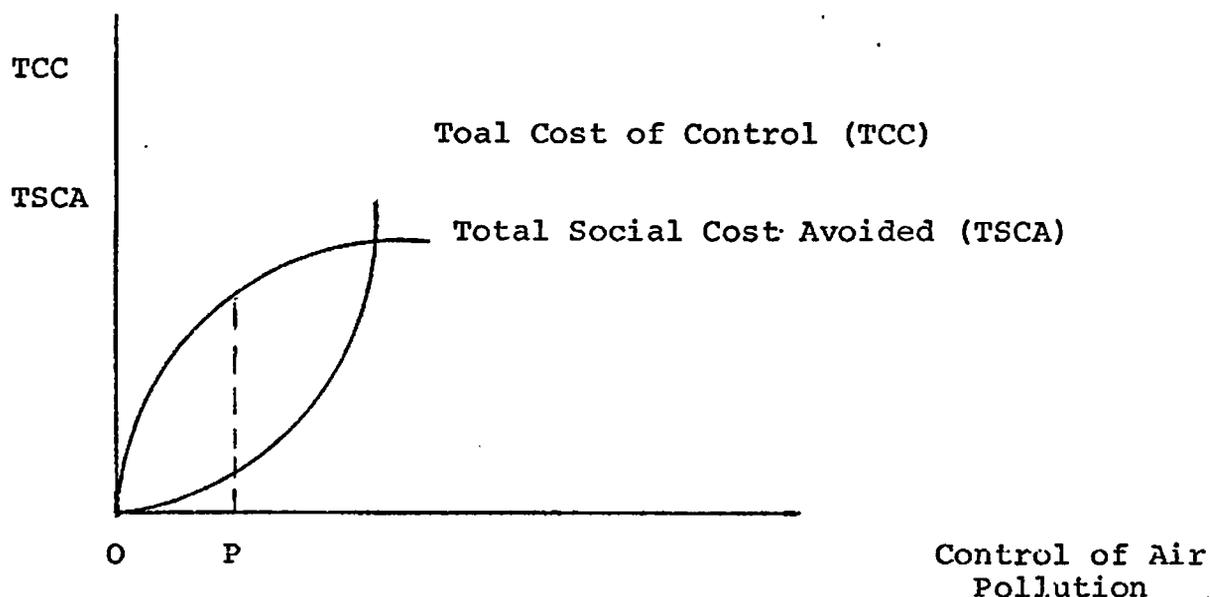
ECONOMIC MODELS OF OPTIMALITY

Optimal levels of control of pollution have been the subject of much discussion in the economics literature. Allen Kneese discusses¹ the conditions for the optimal level of control of all air pollution from all sources but notes that the approach he outlines is most readily applied to an individual pollutant in a specific area. Kneese concludes that the optimal level of control of air pollution is that level which maximizes the difference between total social cost avoided (TSCA) through control and total cost of control (TCC). The optimal level of control of air pollution is given graphically by point P in Figure 1. Total cost of control associated with each level of control is measured in dollars along the vertical axis. Total social cost avoided associated with each level of control is also measured in dollars along the vertical axis.

¹See Allen V. Kneese, "How Much is Air Pollution Costing Us in the United States?" in Proceedings: The Third National Conference on Air Pollution, Washington, D. C., December 12-14, 1966 (Washington, D. C.: Government Printing Office, 1967), pp. 530-531, and Robert U. Ayres and Allen V. Kneese, "Environmental Pollution," in U. S. Congress Joint Economic Committee, Federal Programs for the Development of Human Resources, Vol. II (Washington, D. C.: Government Printing Office, 1968), pp. 633-635.

Figure 1

THE OPTIMAL LEVEL OF CONTROL OF AIR POLLUTION--THE TOTAL SOCIAL COST AVOIDED FUNCTION AND THE TOTAL COST OF CONTROL FUNCTION



Source: Robert U. Ayres and Allen V. Kneese, "Environmental Pollution," in U. S. Congress Joint Economic Committee, Federal Programs for the Development of Human Resources, Vol. II (Washington, D. C.: Government Printing Office, 1968), p. 635.

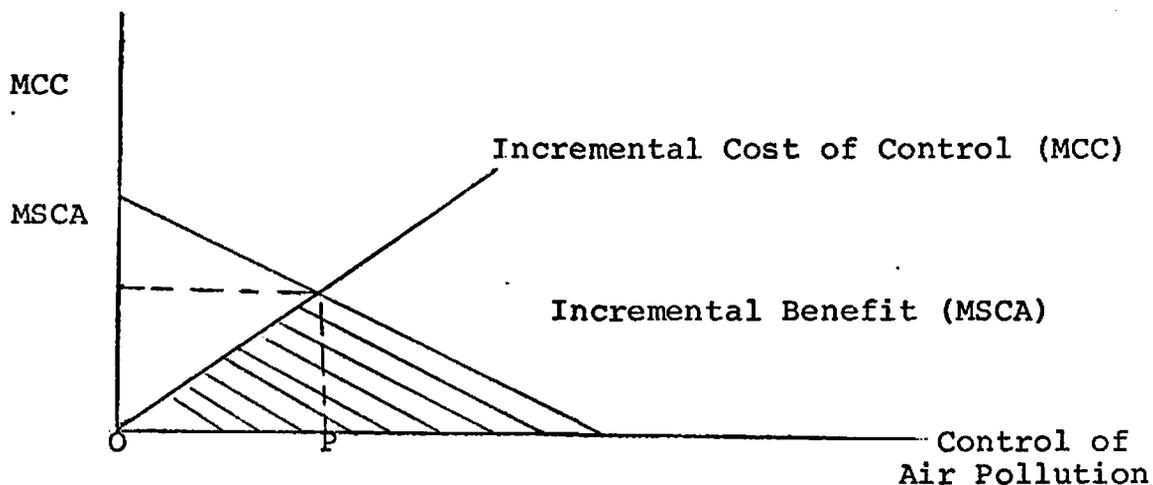
Along the horizontal axis control of air pollution is measured in terms of an index of air pollution. A movement from the southwest corner of the diagram east along the horizontal axis entails a reduction in air pollution--an increase in the control of air pollution. At point P the slopes of the two functions are equal. Here net benefit--the difference between total benefit and total control cost--is a maximum. Kneese notes that for point P to constitute the optimal level of control the total control cost associated with a given level of

control must be the minimum total control cost for that level of control.

The first derivatives of the two functions are graphed in Figure 2. Marginal cost of control (MCC) and marginal social cost avoided (MSCA) associated with every level of control are measured in dollars along the vertical axis. Control is measured along the horizontal axis.

Figure 2

THE OPTIMAL LEVEL OF CONTROL OF AIR POLLUTION--THE
INCREMENTAL SOCIAL COST AVOIDED FUNCTION
AND THE INCREMENTAL COST OF CONTROL
FUNCTION



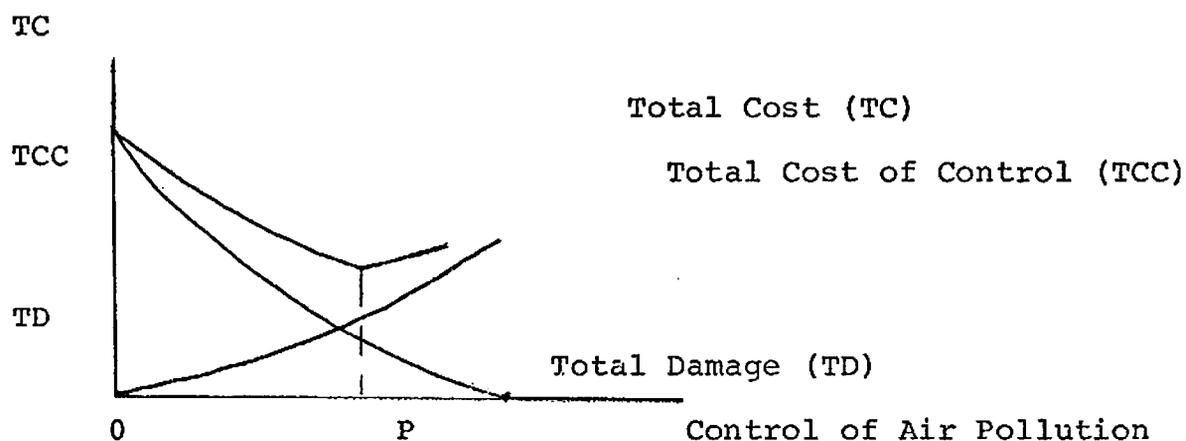
Source: Allen V. Kneese, "How Much is Air Pollution Costing Us in the United States?" in Proceedings: The Third National Conference on Air Pollution, Washington, D. C., December 12-14, 1966 (Washington, D. C.: Government Printing Office, 1967), p. 531.

The rising MCC function and the falling MSCA function intersect at point P--the optimal level of control--where at the margin of control the incremental cost of control and the in-

cremental benefit from control are equal.²

Figure 3

THE OPTIMAL LEVEL OF CONTROL OF AIR POLLUTION--THE TOTAL COST OF CONTROL FUNCTION AND THE TOTAL DAMAGE FUNCTION



Source: Azriel Teller, "Air Pollution Abatement: Economic Rationality and Reality," Daedalus, The Journal of the American Academy of Arts & Sciences, CXCIV (Fall, 1967), 1086.

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Underlying the shapes of the TCC function and the TSCA function are the assumptions that total control cost increases at an increasing rate as control is increased, TSCA increases at a decreasing rate as control is increased, and these rates of change are constant. Harold Wolozin has hypothesized that as control efforts move the community from a point of relative saturation of air pollutants to a point where air pollution is just detectable, outlays for abatement increase more and more rapidly as successively more difficult to obtain levels of clean air are reached, and they soar as the point where air pollution is just detectable is approached. Hence a nonlinear MCC function. Wolozin suggests that the MSCA function is also nonlinear. It is conceivable, he argues, that there is a range of rapid rise in damage cost relative to pollution levels followed by a leveling off as higher levels of pollution are encountered. Harold Wolozin, "The Economics of Air Pollution: Central Problems," Law and Contemporary Problems, XXXIII (Spring, 1968); 229-231. Statistical investigations of the shape of the total social cost avoided function are cited in Chapter V.

Point P in Figure 2 entails minimum combined cost of control efforts and cost of uncontrolled emissions. Total cost at point P is given graphically in Figure 2 by the crosshatched area under the MCC and the MSCA functions and by point P' in Figure 3.³

Colin Wright⁴ defines optimality in a static system by deriving the first order condition that minimizes the quadratic form of the expression

$$T (P) = D (P) + A (P)$$

where T is total cost of pollution for a given emission rate

³Robert Ayres examines the components of cost of air pollution. He concludes that the total cost of air pollution is the sum of three components--losses in dollars of non-market benefits, net direct dollar expenditures, and net indirect dollar costs. Net indirect dollar costs include the costs attributable to the pattern of expenditure associated with air pollution. This component of costs may be positive or negative. Ayres notes, for example, that if unskilled labor is in short supply the use of it in a laundry entails the waste of resources, but if it is in oversupply the alternative to employment in the laundry might be unemployment. Robert U. Ayres "Air Pollution in Cities," Natural Resources Journal, IX (January, 1969), 12-16

⁴Colin Wright, "Economic and Political Aspects of Dynamic Pollution Control," in Study of the Social and Economic Effects of Changes in Air Quality, 1969-70 Annual Report (Corvallis, Oregon: Oregon State University, 1970), 37-39.

(P), D is damage at this rate and A is abatement cost incurred at this rate. Wright concludes that optimality requires that control be carried to the point where the marginal damage from uncontrolled emissions is just equal to the marginal cost saving for uncontrolled emissions.

The conditions for minimizing the total cost over time of air pollution have been investigated. Wright derives the optimal path of control using the classical calculus of variations.⁵

Azriel Teller investigated the relative costs associated with two approaches to sulfur dioxide abatement-- equiproportional abatement and selective abatement.⁶ Equiproportional abatement requires that each source reduce emissions in the same proportion as the desired reduction in the ambient sulfur dioxide concentration. Implicit in this approach are the assumptions that a source which emits z percent of the emissions is responsible for z percent of the concentration and that each source faces the same incremental cost of control function. Under selective abatement it is

⁵Wright, "Economic and Political Aspects of Dynamic Pollution Control," 39-55.

⁶Azriel Teller, "The Use of Linear Programming to Estimate the Cost of Some Alternative Air Pollution Abatement Policies," in Proceedings of the IBM Scientific Computing Symposium on Water and Air Resource Management, 1967 (White Plains, N.Y.: IBM Data Processing Division, 1968), pp. 345-353.

assumed, on the other hand, that the cost of a unit reduction in emissions is not necessarily the same for all sources, and that the effect upon a particular receptor of a unit reduction in emissions is not necessarily the same for all sources.

Teller employed a variation of the linear programming model outlined below to determine for the Nashville, Tennessee Metropolitan area the relative costs of selective abatement and equiproportional abatement. In his analysis he assumes that there are two types of fuel--fuel a, a high sulfur, high polluting fuel, and fuel b, a low sulfur, low polluting fuel. Control of sulfur dioxide emissions is restricted to the substitution of the low sulfur fuel for the high sulfur fuel. In the study of the Nashville area he assumes that low sulfur coal is substituted for high sulfur coal.

An outline of the model and a glossary of terms follow:

$$1. \text{ Minimize } \sum_{j=1}^n [C_{jb}T_{jb}^V + C_{ja}T_{ja}^V]$$

Subject to

$$2. T_{ja}^V K_{ja} + T_{jb}^V K_{jb} \geq H_j \quad (j = 1, 2, \dots, n)$$

$$3. \sum_{j=1}^n M_{ij} [T_{ja}^V E_{ja} + T_{jb}^V E_{jb}] \leq S_i \quad (i = 1, 2, \dots, m)$$

where a is the fuel of high sulfur content originally used, b is the less polluting substitute fuel, C_{jb} is the cost per ton of fuel b , C_{ja} is the cost per ton of fuel a , T_{ja}^V is the variable number of tons of fuel a , T_{jb}^V is the variable number of tons of fuel b , K_{ja} is the Btu content per ton of fuel a , K_{jb} is the Btu content per ton of fuel b , H_j is the Btu requirement of source j , E_{ja} is the emission per ton of fuel a from source j , E_{jb} is the emission per ton of fuel b from source j , S_i is the air quality standard at receptor i , and M_{ij} is the meteorological parameter relating emissions at source j to air quality at receptor i .

The objective is to minimize the total cost of fuel for all sources (Equation 1) subject to the constraints that for each receptor the concentration received of sulfur dioxide from all sources taken together is not greater than the sulfur dioxide standard (Equation 3), and that the combustion of fuels a and b entail the same Btu value as fuel a alone prior to substitution (Equation 2).

Teller concludes that in the Nashville area selective abatement is less costly than equiproportional abatement. This is attributable to the fact that under selective abatement fuel is substituted at those sources where it has the greatest per dollar effect. It is substituted at those sources where incremental costs of control are small and con-

trol has a large impact upon concentrations received.

A number of investigators have taken the optimal level of control of sulfur dioxide emissions as given and employed linear programming models to determine the least cost means of achieving the exogenously determined targets. Robert Kohn generates a function giving the relationship between the projected incremental cost of control of sulfur dioxide emissions in 1975 from all sources and the level of control of sulfur dioxide emissions from all sources for the St. Louis, Missouri area.⁷ A point on this function is, for a given level of control, the first derivative of the function which gives for every level of control of sulfur dioxide emissions the minimum total cost of control of sulfur dioxide emissions.

Jackson and Wohlers⁸ employ a linear programming model to find the least cost solutions for achieving instantaneously over time exogenously determined levels of reduction in a number of pollutants, including sulfur dioxide, in the Dela-

⁷Robert E. Kohn, "Achieving Air Quality Goals at Minimum Cost," Washington University Law Quarterly, MCMLXVIII (Spring, 1968), 325-360.

⁸Walter E. Jackson and Henry C. Wohlers, Determination of Regional Air Pollution Control Costs and the Cost of Air Pollution in the Delaware Valley (Philadelphia, Pa.: Drexel University, 1970).

ware Valley. As in the Kohn and the other⁹ linear programming models, supplies of fuels over time and fuel prices over time are exogenously determined and not related to the configuration of control and the cost of compliance. Estimates of maximum incremental fuel supplies and fuel prices are built into the model and are constraints in the least cost analysis.

The Kohn one-pollutant sulfur dioxide model constitutes a part only of a multi-pollutant linear programming model.¹⁰ The objective of the larger model is to minimize the cost of controlling several classes of pollutants--sulfur

⁹See, for example, William Sajor and Ellison S. Burton, "Evaluating Air Quality Implementation Plans by Computer Simulation" (paper presented at the 97th Annual Meeting of the American Public Health Association (Statistics Section), Philadelphia, Pa., November 1, 1969), Ernst and Ernst, A Cost-Effectiveness Study of Air Pollution Abatement in the National Capital Area (Washington, D.C.: Ernst and Ernst, 1969), Ernst and Ernst, Costs and Economic Impact of Air Pollution Control, Fiscal Years 1970-1974 (Washington, D.C.: Ernst and Ernst, 1969), and Ernst and Ernst, A Cost-Effectiveness Study of Air Pollution Abatement in the Greater Kansas City Area (Washington, D.C.: Ernst and Ernst, 1969).

¹⁰Robert E. Kohn, "A Mathematical Programming Model for Air Pollution Control," School Science and Mathematics, LXIX (June, 1969), 487-494.

dioxide, carbon monoxide, hydrocarbons, nitrogen oxides, and particulate matter--subject to the constraints, among others, that sulfur dioxide is reduced by 485 million pounds, carbon monoxide is reduced by 1.3 billion pounds, hydrocarbons are reduced by 280 million pounds, nitrogen oxides are reduced by 75 million pounds, and particulate matter is reduced by 180 million pounds.

The fact of the interdependencies¹¹ among pollutants provides strong justification for an explicit multi-pollutant approach to the matter of establishing optimal levels of control of individual air pollutants. The appropriateness of the approach is grounded in the need to determine as precisely as is possible the cost of control and the social cost avoided associated with every level of control of the individual pollutant.¹²

¹¹For example, when low sulfur coal is substituted for high sulfur coal in the effort to control sulfur dioxide emissions from the combustion of coal, emissions of particulate matter may be reduced because given the relatively high Btu content of low sulfur coal less coal must be burned to generate the same level of Btu and because the low sulfur coal has a lower ash content.

¹²See Robert U. Ayres and Allen V. Kneese, "Production, Consumption, and Externalities," The American Economic Review, LIX (June, 1969), 282-297, and Ayres and Kneese "Environmental Pollution," pp. 626-684 for a general equilibrium model of pollution control.

CHAPTER IV

OPTIMAL CONTROL POLICY CRITERIA

In this chapter optimal control policy criteria for sulfur dioxide emissions and concentrations are derived. The discussion is conducted within a static framework wherein supplies of fuels and, in turn, numbers of sources using the various fuels -- coal, residual fuel oil, and natural gas -- are fixed.

The basic model -- Model A -- is developed under the additional assumptions:

the combustion of fossil fuels constitutes the only source of sulfur dioxide emissions

the sulfur dioxide concentrations emitted per unit of time are the sulfur dioxide concentrations received per unit of time

given sulfur dioxide concentrations received per unit of time from the combustion of the various fuels entail the same social costs per unit of time

the source of sulfur dioxide emissions from the combustion of any one fuel is a representative source

total social cost avoided is linearly related to control

incremental cost of control at the source of sulfur dioxide emissions is an increasing function of control.

Consider the Lagrangean function

$$z = CC_c \cdot S_c + CC_o \cdot S_o + CC_g \cdot S_g + \lambda \left(\frac{1}{SO^E} - S_c \cdot \frac{1}{SO_c} - S_o \cdot \frac{1}{SO_o} - S_g \cdot \frac{1}{SO_g} \right)$$

where $CC_c \cdot S_c + CC_o \cdot S_o + CC_g \cdot S_g$ is total cost of control of sulfur dioxide emissions from the combustion of fossil fuels by all sources taken together, $\frac{1}{SO^E}$ is control per unit of time of sulfur dioxide emissions from the combustion of fossil fuels by all sources taken together and is equal to

$$S_c \cdot \frac{1}{SO_c} + S_o \cdot \frac{1}{SO_o} + S_g \cdot \frac{1}{SO_g},$$

CC_c is cost at the source of control of sulfur dioxide emissions from the combustion of coal, CC_o is cost at the source of control of sulfur dioxide emissions from the combustion of residual fuel oil, CC_g is cost at the source of control of sulfur dioxide emissions from the combustion of natural gas, S_c is the exogenously determined number of sources using coal, S_o is the exogenously determined number of sources using residual fuel oil, S_g is the exogenously determined number of sources using natural gas, $\frac{1}{SO_c}$ is control per unit of time at the source of sulfur emissions from the combustion of coal, $\frac{1}{SO_o}$ is control per unit of time at the source of sulfur dioxide emissions from the combustion of residual fuel oil, and $\frac{1}{SO_g}$ is control per unit of time at the source of sulfur dioxide emissions from the combustion of natural gas.

It is assumed that total cost of control at the source increases at an increasing rate as control at the source is increased. The rate of emission of sulfur dioxide at the source is the reciprocal of the rate of control. Total cost of control at the source is then a decreasing function of the rate of emission at the source. The total cost of control functions governing the sources of sulfur dioxide emissions from the combustion of coal, residual fuel oil, and natural gas are

$$CC_c = a \cdot \left(\frac{1}{SO_c}\right)^n$$

$$CC_o = b \cdot \left(\frac{1}{SO_o}\right)^s$$

$$CC_g = c \cdot \left(\frac{1}{SO_g}\right)^p$$

where a , b , c , n , s , and p are parameters, and CC_c , CC_o , CC_g , $\frac{1}{SO_c}$, $\frac{1}{SO_o}$ and $\frac{1}{SO_g}$ are endogenous variables.

Substituting the expressions above for CC_c , CC_o , and CC_g in the Lagrangean function yields

$$Z = a \cdot \left(\frac{1}{SO_c}\right)^n \cdot s_c + b \cdot \left(\frac{1}{SO_o}\right)^s \cdot s_o + c \cdot \left(\frac{1}{SO_g}\right)^p \cdot s_g \\ + \lambda \left(\frac{1}{SO^E} - s_c \cdot \frac{1}{SO_c} - s_o \cdot \frac{1}{SO_o} - s_g \cdot \frac{1}{SO_g} \right)$$

And setting the partials $\frac{\partial z}{\partial \text{SO}_c}$, $\frac{\partial z}{\partial \text{SO}_o}$ and $\frac{\partial z}{\partial \text{SO}_g}$ equal to zero

yields

$$\lambda = n \cdot a \cdot \left(\frac{1}{\text{SO}_c}\right)^{n-1} = s \cdot b \cdot \left(\frac{1}{\text{SO}_o}\right)^{s-1} = p \cdot c \cdot \left(\frac{1}{\text{SO}_g}\right)^{p-1}$$

the condition for minimizing the total cost of a given level of control per unit of time of sulfur dioxide emissions from the combustion of fossil fuels by all sources taken together. In Appendix 1 it is demonstrated that λ is the first derivative of the function governing the total cost of control of sulfur dioxide emissions from the combustion of fossil fuels by all sources taken together.

The total social cost avoided function is

$$\text{TSCA} = h \cdot \frac{1}{\text{SO}^E}$$

where h is a parameter, and TSCA and $\frac{1}{\text{SO}^E}$ are endogeneous variables. This function reflects the assumption that total social cost avoided is linearly related to control of sulfur dioxide emissions from the combustion of fossil fuels by all sources taken together. Setting λ equal to the first derivative of the total social cost avoided function yields

$$n \cdot a \cdot \left(\frac{1}{SO_c}\right)^{n-1} = h$$

$$s \cdot b \cdot \left(\frac{1}{SO_o}\right)^{s-1} = h$$

$$p \cdot c \cdot \left(\frac{1}{SO_g}\right)^{p-1} = h$$

which yield by solving for $\frac{1}{SO_c}$, $\frac{1}{SO_o}$ and $\frac{1}{SO_g}$ the optimal level of control per unit of time at the source of sulfur dioxide emissions from the combustion of each fuel. Specifically,

$$\frac{1}{SO_c^*} = \left(\frac{h}{n \cdot a}\right)^{\frac{1}{n-1}}$$

$$\frac{1}{SO_o^*} = \left(\frac{h}{s \cdot b}\right)^{\frac{1}{s-1}}$$

$$\frac{1}{SO_g^*} = \left(\frac{h}{p \cdot c}\right)^{\frac{1}{p-1}} .$$

The reciprocals of these expressions give the optimal rates of emission of sulfur dioxide from the sources using coal, residual fuel oil, and natural gas. These are, in turn,

$$SO_c^* = \left(\frac{n \cdot a}{h}\right)^{\frac{1}{n-1}}$$

$$SO_o^* = \left(\frac{s \cdot b}{h}\right)^{\frac{1}{s-1}}$$

$$SO_g^* = \left(\frac{p \cdot c}{h}\right)^{\frac{1}{p-1}} .$$

The optimal level of control per unit of time of sulfur dioxide emissions from the combustion of fossil fuels by all sources taken together is

$$\frac{1}{SO^{E^*}} = S_c \cdot \left(\frac{h}{n \cdot a}\right)^{\frac{1}{n-1}} + S_o \cdot \left(\frac{h}{s \cdot b}\right)^{\frac{1}{s-1}} + S_g \cdot \left(\frac{h}{p \cdot c}\right)^{\frac{1}{p-1}} .$$

The reciprocal of this expression is the optimal rate of emission of sulfur dioxide emissions from the combustion of fossil fuels by all sources taken together. Specifically,

$$SO^{E^*} = \frac{1}{S_c} \cdot \left(\frac{n \cdot a}{h}\right)^{\frac{1}{n-1}} + \frac{1}{S_o} \cdot \left(\frac{s \cdot b}{h}\right)^{\frac{1}{s-1}} + \frac{1}{S_g} \cdot \left(\frac{p \cdot c}{h}\right)^{\frac{1}{p-1}} .$$

Implicit on the benefit side of the analysis¹ are the statements

$$TSCA = SCA_c \cdot S_c + SCA_o \cdot S_o + SCA_g \cdot S_g$$

¹The implicit statements of the benefit side of the analysis yield the explicit statement of the benefit function. Assuming

$$TSCA = SCA_c \cdot S_c + SCA_o \cdot S_o + SCA_g \cdot S_g$$

and

$$SCA_c = h \cdot \frac{1}{SO_c}$$

$$SCA_o = h \cdot \frac{1}{SO_o}$$

$$SCA_g = h \cdot \frac{1}{SO_g}$$

$TSCA = h \cdot (S_c \cdot \frac{1}{SO_c} + S_o \cdot \frac{1}{SO_o} + S_g \cdot \frac{1}{SO_g})$ and since

$$\frac{1}{SO^E} = S_c \cdot \frac{1}{SO_c} + S_o \cdot \frac{1}{SO_o} + S_g \cdot \frac{1}{SO_g}$$

$$TSCA = h \cdot \frac{1}{SO^E} .$$

and .

$$SCA_c = h \cdot \frac{1}{SO_c}$$

$$SCA_o = h \cdot \frac{1}{SO_o}$$

$$SCA_g = h \cdot \frac{1}{SO_g}$$

where SCA_c is social cost avoided through control at the source of SO_c , SCA_o is social cost avoided through control at the source of SO_o , and SCA_g is social cost avoided through control at the source of SO_g . From the cost side of the analysis

$$CC_c = a \cdot \left(\frac{1}{SO_c}\right)^n$$

$$CC_o = b \cdot \left(\frac{1}{SO_o}\right)^s$$

$$CC_g = c \cdot \left(\frac{1}{SO_g}\right)^p \quad .$$

Setting

$$\frac{dCC_c}{d\frac{1}{SO_c}} = \frac{dSCA_c}{d\frac{1}{SO_c}}$$

$$\frac{dCC_o}{d\frac{1}{SO_o}} = \frac{dSCA_o}{d\frac{1}{SO_o}}$$

$$\frac{dCC_g}{d\frac{1}{SO_g}} = \frac{dSCA_g}{d\frac{1}{SO_g}}$$

yields the solutions derived above for $\frac{1}{SO_c^*}$, $\frac{1}{SO_o^*}$, and

$$\frac{1}{SO_g^*}.$$

The optimal level of control at the source is given, then, by equimarginal source control cost and benefit from control at the source. For the source using coal, for example, since

$$\frac{dCC_c}{d\frac{1}{SO_c}} = n \cdot a \cdot \left(\frac{1}{SO_c}\right)^{n-1}$$

and

$$\frac{dSCA_c}{d\frac{1}{SO_c}} = h$$

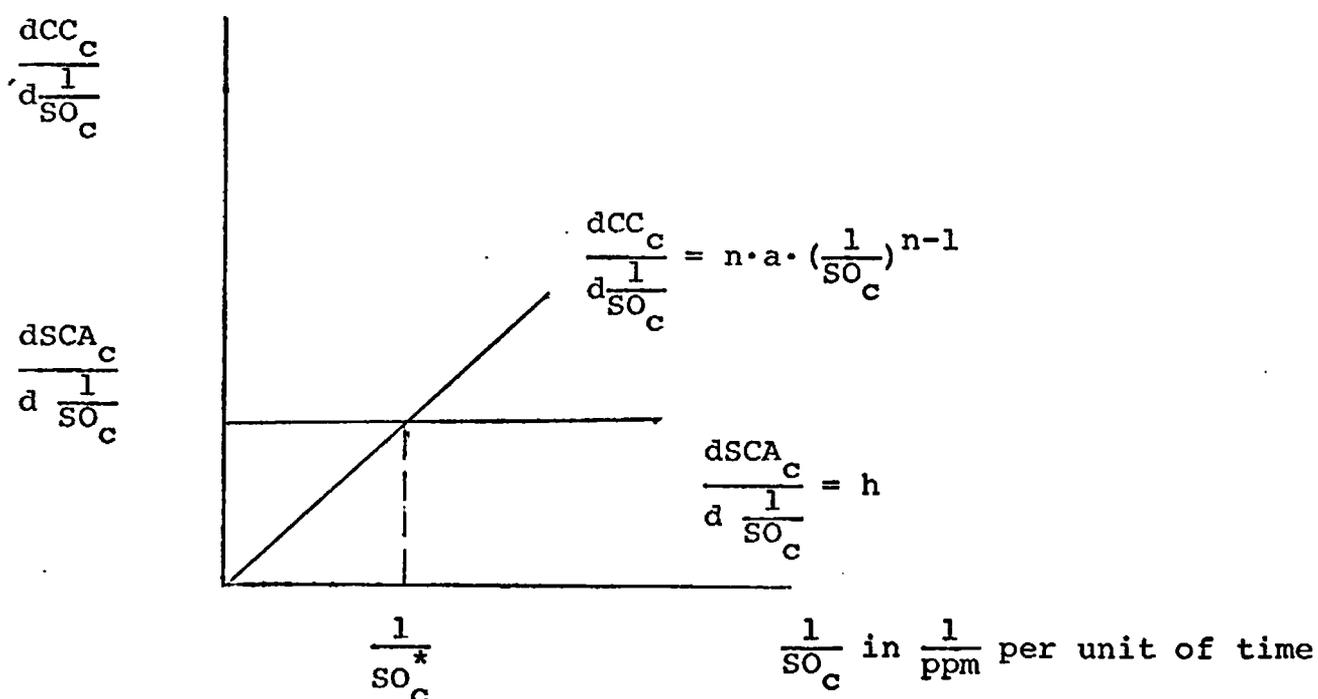
setting $\frac{dCC_c}{d\frac{1}{SO_c}} = \frac{dSCA_c}{d\frac{1}{SO_c}}$ yields

$$\frac{1}{SO_c^*} = \left(\frac{h}{n \cdot a}\right)^{\frac{1}{n-1}} .$$

Using the rule just derived that equimarginal source cost and benefit give the optimal level of control at the source the solution for the optimal level of control at the source is easily derived graphically. The solution for the optimal level of control at the source of sulfur dioxide emissions from, for example, the combustion of coal is graphically derived in Figure 4. In Figure 4 control is measured along the horizontal axis in units of $\frac{1}{\text{ppm}}$, where ppm is parts per million of sulfur dioxide by volume. A movement from the

Figure 4

THE OPTIMAL LEVEL OF CONTROL AT THE SOURCE OF SULFUR DIOXIDE EMISSIONS FROM THE COMBUSTION OF COAL



origin east along the horizontal axis is an increase in control -- a decrease in the sulfur dioxide emissions from the source using coal. Incremental source cost of control and incremental benefit from control at the source are measured in dollars along the vertical axis. Optimality is given by the intersection of the incremental control cost²

$$\left(\frac{dCC_c}{d\frac{1}{SO_c}}\right) \text{ and the incremental social cost avoided } \left(\frac{dSCA_c}{d\frac{1}{SO_c}}\right)$$

schedules.³

In Model B developed in Appendix II to this chapter two restrictive assumptions underlying Model A are relaxed. With the introduction of meteorological parameters the assumption that concentrations received per unit of time equal concentrations emitted per unit of time is relaxed. Secondly, the assumption that the source of sulfur dioxide emissions from the combustion of any one fuel is a representative source

²For convenience it is assumed that $\frac{d^2 CC_c}{d(\frac{1}{SO_c})^2} = 0$ when the more

reasonable assumption might be $\frac{d^2 CC_c}{d(\frac{1}{SO_c})^2} > 0$.

³There is no effort to distinguish between incremental source control cost and benefit and incremental total control cost and benefit when, from the context, it is clear which set is intended.

is relaxed. Instead it is assumed that the source of sulfur dioxide emissions from the combustion of any one fuel in one of four uses is a representative source.

The assumptions underlying MODEL B are

the combustion of fossil fuels constitutes the only source of sulfur dioxide emissions

given sulfur dioxide concentrations received per unit of time from the combustion of the various fuels in the various uses entail the same social costs per unit of time

the source of sulfur dioxide emissions from the combustion of any one fuel in a given use is a representative source⁴

total social cost avoided is linearly related to control realized in the area of reception

incremental cost of control at the source of sulfur dioxide emissions is an increasing function of control

Twelve sectors constitute the basic sources of sulfur dioxide emissions. These are

1. Combustion of coal by industrial sources
2. Combustion of residual fuel oil by industrial sources
3. Combustion of natural gas by industrial sources
4. Combustion of coal by power plants

⁴Teller relaxes assumptions 2 and 3. He allows for differential control costs for different sources using a given fuel in a given use and for differential impacts upon the same receptor of different sources using a given fuel in a given use.

5. Combustion of residual fuel oil by power plants
6. Combustion of natural gas by power plants
7. Combustion of coal by residences
8. Combustion of residual fuel oil by residences
9. Combustion of natural gas by residences
10. Combustion of natural gas by commercial sources
11. Combustion of residual fuel oil by commercial sources
12. Combustion of natural gas by commercial sources

The analytic solution of the model yields a set of twelve reduced form expressions -- one for each representative source -- giving optimality in control efforts. The reduced form expression for the optimal level of control per unit of time at the source of sulfur dioxide emissions from the industrial combustion of coal is

$$\frac{1}{SO_{cI}^*} = \left(\frac{\psi \cdot M_4}{m \cdot b_1} \right)^{\frac{1}{m-1}}$$

where the parameter ψ gives the effect upon total social cost avoided of a change in the rate of control of the sulfur dioxide concentration from all sources in the area of reception, M_4 is a meteorological parameter which relates control realized in the area of reception of sulfur dioxide concentrations from the industrial source using coal to control at the source of sulfur dioxide emissions from the industrial source using coal, and m and b are parameters which reflect the cost of control at the source of sulfur dioxide

emissions from the industrial combustion of coal. The reduced form expression for $\frac{1}{SO_{cI}}$ is derived in Appendix 2.

The functions governing incremental cost of control at the source of sulfur dioxide emissions from the industrial combustion of coal and incremental benefit from control at this source are:

$$\frac{dCC_{cI}}{d \frac{1}{SO_{cI}}} = b_1 \cdot m \cdot \left(\frac{1}{SO_{cI}} \right)^{m-1}$$

and

$$\frac{dSCA_{cI}}{d \frac{1}{SO_{cI}}} = \psi \cdot M_4$$

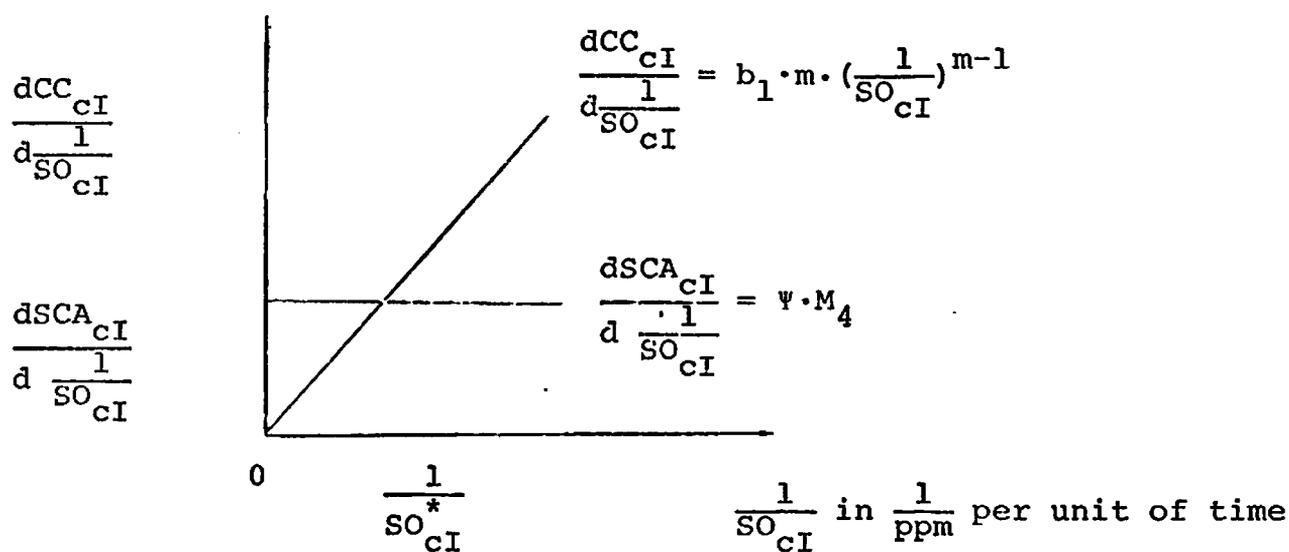
where CC_{cI} is total cost of control at the source of sulfur dioxide emissions from the industrial combustion of coal, and SCA_{cI} is social cost avoided through control at the source of sulfur dioxide emissions from the industrial combustion of coal. These expressions are derived in Appendix 3.

The optimal level of control per unit of time at the source of sulfur dioxide emissions from the industrial combustion of coal is graphically derived in Figure 5. Optimality is given by the intersection of the incremental cost

of control and the incremental social cost avoided functions.⁵

Figure 5

THE OPTIMAL LEVEL OF CONTROL AT THE SOURCE OF SULFUR DIOXIDE EMISSIONS FROM THE INDUSTRIAL COMBUSTION OF COAL



⁵It is assumed, for convenience, that $\frac{d^2 CC_{CI}}{d\left(\frac{1}{SO_{CI}}\right)^2} = 0$.

APPENDIX 1

THE INTERPRETATION OF λ

Let TCC be total cost of control of sulfur dioxide emissions from the combustion of fossil fuels by all sources taken together. Since

$$dTCC = \frac{\partial CC_c \cdot S_c}{\partial \frac{1}{SO_c}} \cdot d \frac{1}{SO_c} + \frac{\partial CC_o \cdot S_o}{\partial \frac{1}{SO_o}} \cdot d \frac{1}{SO_o} + \frac{\partial CC_g \cdot S_g}{\partial \frac{1}{SO_g}} \cdot d \frac{1}{SO_g}$$

and

$$\frac{\partial CC_c \cdot S_c}{\partial \frac{1}{SO_c}} = \lambda \cdot S_c$$

$$\frac{\partial CC_o \cdot S_o}{\partial \frac{1}{SO_o}} = \lambda \cdot S_o$$

$$\frac{\partial CC_g \cdot S_g}{\partial \frac{1}{SO_g}} = \lambda \cdot S_g$$

$$dTCC = \lambda (S_c \cdot d \frac{1}{SO_c} + S_o \cdot d \frac{1}{SO_o} + S_g \cdot d \frac{1}{SO_g})$$

Now, since

$$d \frac{1}{SO^E} = \frac{\partial \frac{1}{SO^E}}{\partial \frac{1}{SO_c}} \cdot d \frac{1}{SO_c} + \frac{\partial \frac{1}{SO^E}}{\partial \frac{1}{SO_o}} \cdot d \frac{1}{SO_o} + \frac{\partial \frac{1}{SO^E}}{\partial \frac{1}{SO_g}} \cdot d \frac{1}{SO_g}$$

and

$$\frac{\partial \frac{1}{SO^E}}{\partial \frac{1}{SO_c}} = s_c$$

$$\frac{\partial \frac{1}{SO^E}}{\partial \frac{1}{SO_o}} = s_o$$

$$\frac{\partial \frac{1}{SO^E}}{\partial \frac{1}{SO_g}} = s_g$$

$$d \frac{1}{SO^E} = s_c \cdot d \frac{1}{SO_c} + s_o \cdot d \frac{1}{SO_o} + s_g \cdot d \frac{1}{SO_g} \text{ and}$$

$$\frac{d \text{ TCC}}{d \frac{1}{SO^E}} = \lambda .$$

APPENDIX 2

MODEL B

Consider the Lagrangean function

$$L = \sum_{jN} CC_{jN} \cdot S_{jN} + \lambda \left(\frac{1}{SO^R} - \sum_{jN} \frac{1}{SO_{jN}^R} \cdot S_{jN} \right) \quad \begin{matrix} (j=1, \dots, m) \\ (N=1, \dots, t) \end{matrix}$$

where $\sum_{jN} CC_{jN} \cdot S_{jN}$ is total cost of control per unit of time at all sources taken together of sulfur dioxide concentrations from the combustion of fossil fuels, CC_{jN} is total cost of control at the source of sulfur dioxide emissions from the combustion of the j^{th} fuel in the N^{th} use, S_{jN} is the exogenously determined number of sources of sulfur dioxide emissions from the combustion of the j^{th} fuel in the N^{th} use, $\sum_{jN} \frac{1}{SO_{jN}^R} \cdot S_{jN}$ is control realized per unit of time in the area of reception of sulfur dioxide concentrations from the combustion of fossil fuels by all sources taken together and sums to $\frac{1}{SO^R}$, and $\frac{1}{SO_{jN}^R}$ is control realized per unit of time in the area of reception of sulfur dioxide concentrations from the source using the j^{th} fuel in the N^{th} use.

CC_{jN} is a function of $\frac{1}{SO_{jN}^R}$, the rate of control at the

source of sulfur dioxide emissions from the combustion of the j^{th} fuel in the N^{th} use. $\frac{1}{SO_{jN}^R}$ is a function of $\frac{1}{SO_{jN}}$ and

the meteorological variable governing the source using the j^{th} fuel in the N^{th} use.

The total cost of a given level of control of sulfur dioxide concentrations in the area of reception from the combustion of fossil fuels by all sources taken together is minimized by setting

$$\frac{\partial L}{\partial \frac{1}{SO_{jN}^R}} = 0$$

for each fuel in each use.

CC_{cI} is total cost at the source of control of sulfur dioxide emissions from the industrial combustion of coal, S_{cI} is the exogenously determined number of sources of sulfur dioxide emissions from the industrial combustion of coal, and $\frac{1}{SO_{cI}^R}$ is control realized per unit of time in the area of reception of sulfur dioxide concentrations from the industrial source using coal.

The function governing total cost of control at the industrial source using coal is

$$CC_{cI} = b_1 \cdot \left(\frac{1}{SO_{cI}^R}\right)^m$$

where $\frac{1}{SO_{CI}}$ is control per unit of time at the source of sulfur dioxide emissions from the industrial combustion of coal, and b_1 and m are parameters.

It is assumed that control realized in the area of reception of concentrations from the source is a function of control at the source. The relationship governing the source of sulfur dioxide emissions from the industrial combustion of coal is

$$\frac{1}{SO_{CI}^R} = M_4 \cdot \frac{1}{SO_{CI}}$$

where M_4 is a meteorological parameter.

Substituting the relationships developed above for the source of sulfur dioxide emissions from the industrial combustion of coal into the Lagrangean function for CC_{CI} yields

$$L = \frac{S_{CI} \cdot b_1}{M_4^m} \cdot \left(\frac{1}{SO_{CI}^R}\right)^m + \dots + \lambda \left(\frac{1}{SO^R} - \frac{1}{SO_{CI}^R} \cdot S_{CI} - \dots\right)$$

Setting the partial $\frac{\partial L}{\partial \frac{1}{SO_{CI}^R}}$ equal to zero yields

$$\frac{m \cdot b_1}{M_4^m} \cdot \left(\frac{1}{SO_{CI}^R}\right)^{m-1} = \lambda$$

where λ is the incremental cost of control realized of the sulfur dioxide concentration in the area of reception from the combustion of fossil fuels by all sources taken together.

The TSCA function is

$$\text{TSCA} = \psi \cdot \frac{1}{\text{SO}^R}$$

where ψ is a parameter. This function reflects the assumption that total social cost avoided is linearly related to the sulfur dioxide concentration in the area of reception.

$$\text{Since } \frac{d \text{TSCA}}{d \frac{1}{\text{SO}^R}} = \psi, \text{ setting } \lambda = \frac{d \text{TSCA}}{d \frac{1}{\text{SO}^R}} \text{ yields}$$

$$\frac{m \cdot b_1}{M_4^m} \cdot \left(\frac{1}{\text{SO}_{cI}^R} \right)^{m-1} = \psi. \text{ Substituting } M_4 \cdot \frac{1}{\text{SO}_{cI}} \text{ for } \frac{1}{\text{SO}_{cI}^R} \text{ yields}$$

$$\frac{m \cdot b_1}{M_4} \cdot \left(\frac{1}{\text{SO}_{cI}} \right)^{m-1} = \psi$$

and, then,

$$\frac{1}{\text{SO}_{cI}^*} = \left(\frac{M_4 \cdot \psi}{m \cdot b_1} \right)^{\frac{1}{m-1}}$$

and

$$SO_{cI}^* = \left(\frac{m \cdot b_1}{M_4 \cdot \psi} \right)^{\frac{1}{m-1}}$$

the optimal rate of control at the source of sulfur dioxide emissions from the industrial combustion of coal, and the optimal rate of emission of sulfur dioxide from the industrial source using coal.

APPENDIX 3

THE DERIVATION OF THE EXPRESSIONS FOR

$$\frac{d SCA_{CI}}{d \frac{1}{SO_{CI}}} \text{ and } \frac{d CC_{CI}}{d \frac{1}{SO_{CI}}}$$

Implicit in the TSCA function of Model B are the statements

$$TSCA = \sum_{jN} SCA_{jN} \cdot S_{jN}$$

and

$$SCA_{jN} = \psi \cdot \frac{1}{SO_{jN}^R}$$

where SCA_{jN} is social cost avoided through control at the source of sulfur dioxide emissions from the combustion of the j^{th} fuel in the N^{th} use.

SCA_{CI} is social cost avoided through control at the source of sulfur dioxide emissions from the industrial combustion of coal and is equal to $\psi \cdot \frac{1}{SO_{CI}^R}$.

Substituting from Appendix 2 the expression $M_4 \cdot \frac{1}{SO_{CI}}$ for $\frac{1}{SO_{CI}^R}$ in the expression $\psi \cdot \frac{1}{SO_{CI}^R}$ yields

$$SCA_{cI} = \psi \cdot M_4 \cdot \frac{1}{SO_{cI}} \cdot$$

From Appendix 2 the cost function governing the source of sulfur dioxide emissions from the industrial combustion of coal is

$$CC_{cI} = b_1 \cdot \left(\frac{1}{SO_{cI}}\right)^m \cdot$$

Now, setting $\frac{dSCA_{cI}}{d \frac{1}{SO_{cI}}} = \frac{dCC_{cI}}{d \frac{1}{SO_{cI}}}$ yields

$$\frac{1}{SO_{cI}^*} = \left(\frac{\psi \cdot M_4}{m \cdot b_1}\right)^{\frac{1}{m-1}}$$

the expression derived in Appendix 2 for the optimal level of control at source of sulfur dioxide emissions from the industrial combustion of coal.

CHAPTER V

A PRIORI DECISION RULES

The analysis of Chapter IV suggests rules which are useful in making a priori judgements about optimal levels of control of sulfur dioxide emissions and concentrations. These rules are now derived and their use demonstrated in a priori decision making. First, rules governing absolute optimal levels of control at the source are developed. The focus here is initially under Model A where it was earlier assumed that the source of sulfur dioxide emissions from the combustion of any one fuel is a representative source and that sulfur dioxide concentrations emitted per unit of time are the sulfur dioxide concentrations received per unit of time. Next, rules governing absolute optimal levels of control at the source for Model B are developed. Under Model B it was earlier assumed that the source of sulfur dioxide emissions from the combustion of any one fuel in a given use is a representative source and that sulfur dioxide concentrations received from the source are related by way of meteorological parameters to sulfur dioxide concentrations emitted at the source. A discussion of relative optimality then follows. This discussion is conducted first under the simplifying assumptions of Model A and then under the con-

ditions of Model B. The rules developed in this chapter are then applied in Appendix 4 in the analysis of a set of regulations governing emissions of sulfur dioxide.

From Chapter IV the optimal level of control at the source of sulfur dioxide emissions is that level of control which entails equimarginal source control cost and social cost avoided through control at the source. For example, the expression

$$\frac{dCC_c}{d\frac{1}{SO_c}} = \frac{dSCA_c}{d\frac{1}{SO_c}}$$

gives the optimal level of control at the source of sulfur dioxide emissions from the combustion of coal. Now, dividing

both sides of this expression by $\frac{dSCA_c}{d\frac{1}{SO_c}}$ yields

$$\frac{\frac{dCC_c}{d\frac{1}{SO_c}}}{\frac{dSCA_c}{d\frac{1}{SO_c}}} = 1$$

a rule which is useful in making a priori judgements about optimal levels of control at the source of SO_c . From Chapter IV, since

the ratio $\frac{\frac{d CC_c}{d \frac{1}{SO_c}}}{\frac{d SCA_c}{d \frac{1}{SO_c}}}$ is equal to $\frac{n \cdot a}{h} \cdot \left(\frac{1}{SO_c}\right)^{n-1}$, the optimal level

of control at the source of SO_c is, then, that level of control which entails $\frac{n \cdot a}{h} \cdot \left(\frac{1}{SO_c}\right)^{n-1}$ equal to unity.

Four cases are examined. In Case I--given h -- n and a are large. Here the ratio of incremental cost of control to incremental social cost avoided is constrained to unity through lax control. In Case II--given h -- n and a are small. Here the ratio of incremental cost of control to incremental social cost avoided is forced to unity through strict control. In Case III--given n and a -- h is large. Here the ratio of incremental cost of control to incremental social cost avoided is forced to unity through strict control. In Case IV--given n and a -- h is small. Here the ratio of incremental cost of control to incremental social cost avoided is constrained to unity through lax control.

The conclusion is that if incremental source cost of

control per unit of incremental social cost avoided through control at the source is large over a relatively large range of control (Cases I and IV) optimality requires that control at the source be lax and that if incremental source cost of control per unit of incremental social cost avoided through control at the source is small over a relatively large range of control (Cases I and III) optimality requires that control at the source be strict.

The inverse relationship, given the benefit function, between incremental cost of control at every level of control at the source and the optimal level of control at the source is apparent from an inspection of Figure 6 wherein the cost of control at the source of SO_c is examined. In Figure 6

$(\frac{dCC_c}{d\frac{1}{SO_c}})'$ is the incremental cost of control function for large

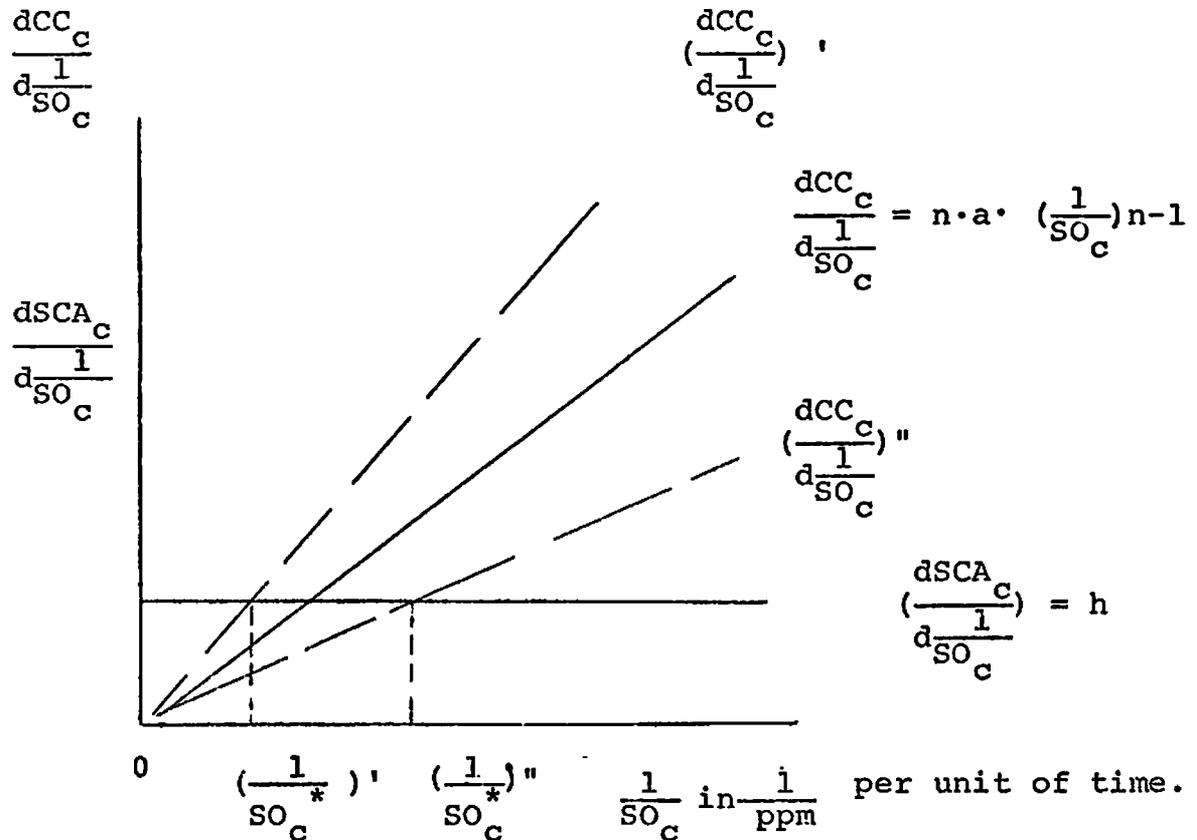
values of n and a and $(\frac{dCC_c}{d\frac{1}{SO_c}})''$ is the incremental cost of con-

trol function for small values of these parameters. In the former case optimality requires lax, $(\frac{1}{SO_c})'$, control. In the

latter case optimality requires strict, $(\frac{1}{SO_c})''$, control.

Figure 6

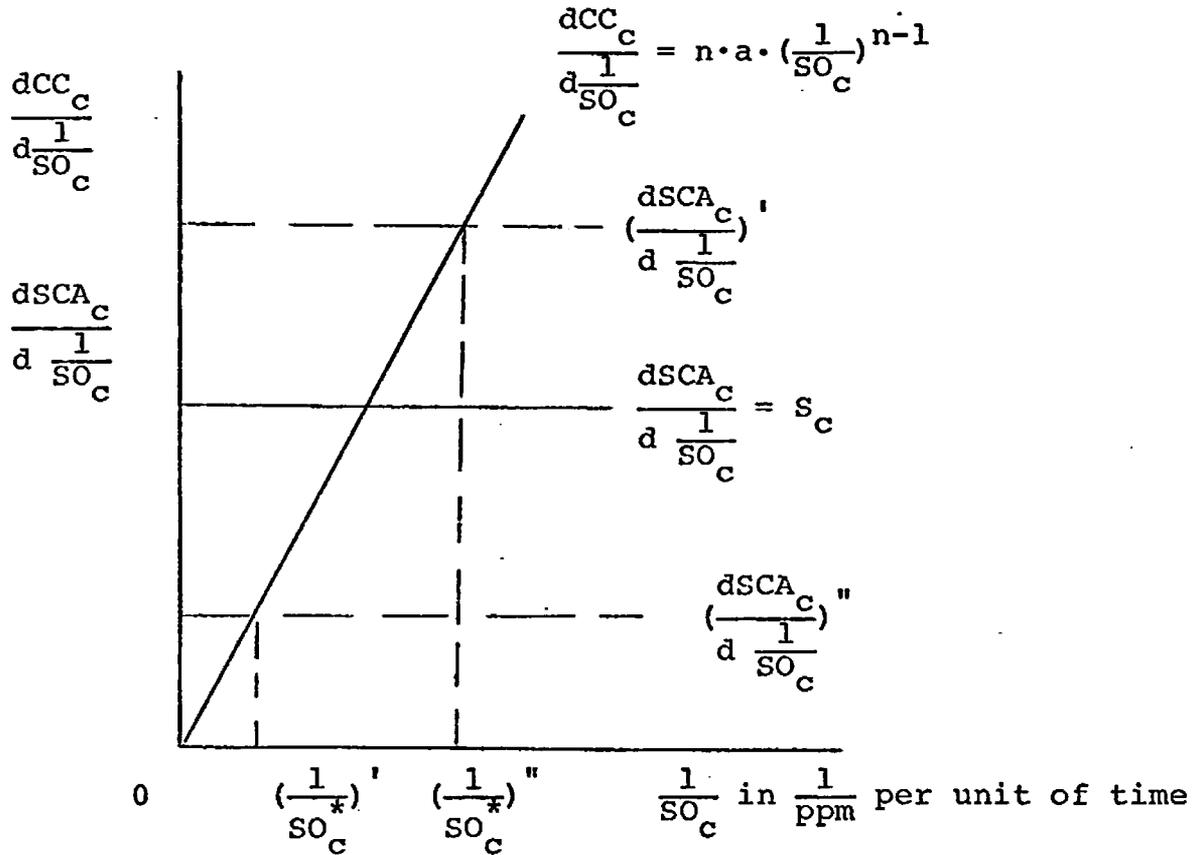
THE OPTIMAL LEVEL OF CONTROL AT THE SOURCE OF SULFUR DIOXIDE EMISSIONS FROM THE COMBUSTION OF COAL FOR n AND a LARGE, AND FOR n AND a SMALL



The direct relationship, given the cost function, between the optimal level of control at the source and incremental benefit at every level of control at the source is apparent from an inspection of Figure 7, wherein the case of control at the source of SO_c is, again, examined. In Figure 7 $\frac{dSCA_c}{d\frac{1}{SO_c}}$ is the incremental benefit function for one case entailing a large and constant impact upon social cost avoided

Figure 7

THE OPTIMAL LEVEL OF CONTROL AT THE SOURCE OF SULFUR DIOXIDE EMISSIONS FROM THE COMBUSTION OF COAL FOR h LARGE, AND FOR h SMALL



of a change in the level of control and $(\frac{dSCA_c}{d \frac{1}{SO_c}})''$ is the incre-

mental benefit function for a second case entailing a small and constant impact upon social cost avoided

of a change in the level of control. In the former case there is a relatively large range of control over which the incremental social cost avoided is large relative to the incremental cost of control. Here, optimality requires strict control at the source in order to force equality between incremental control cost and incremental social cost avoided. In the latter case equality between incremental social cost avoided and incremental control cost and, therefore, optimality are given by lax control.

In Chapter IV the expression

$$\frac{dCC_{cI}}{d\frac{1}{SO_{cI}}} = \frac{dSCA_{cI}}{d\frac{1}{SO_{cI}}}$$

which gives the optimal level of control at the source of sulfur dioxide emissions from the industrial combustion of coal is derived. Dividing both sides of this expression by

$$\frac{dSCA_{cI}}{d\frac{1}{SO_{cI}}} \text{ yields}$$

$$\frac{\frac{dCC_{cI}}{d\frac{1}{SO_{cI}}}}{\frac{dSCA_{cI}}{d\frac{1}{SO_{cI}}}} = 1$$

a rule which is useful in making a priori judgements about optimal levels of control at the source of SO_{cI} . From

Chapter IV, since the ratio $\frac{\frac{dCC_{cI}}{d\frac{1}{SO_{cI}}}}{\frac{dSCA_{cI}}{d\frac{1}{SO_{cI}}}}$ is equal to

$\frac{b_{1,m}}{\psi \cdot M_4} \cdot \left(\frac{1}{SO_{cI}}\right)^{m-1}$, the optimal level of control at the source of SO_{cI} is, then, that level of control which entails

$\frac{b_{1,m}}{\psi \cdot M_4} \cdot \left(\frac{1}{SO_{cI}}\right)^{m-1}$ equal to unity.

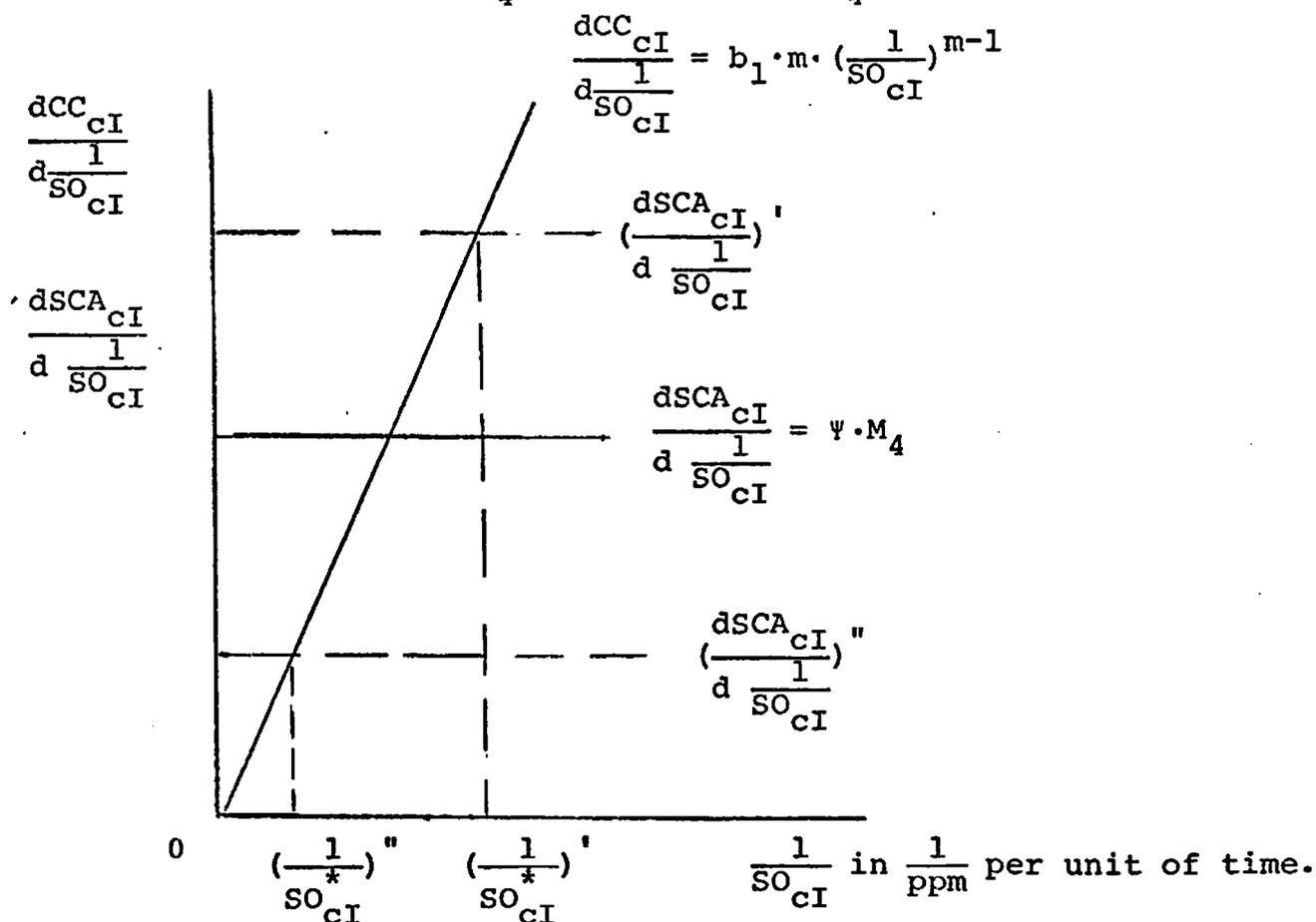
Two cases are examined. In Case I--given $b_{1,m}$, and $\psi \rightarrow M_4$, the meteorological parameter, is small. Here, given the incremental cost of control function, incremental social cost avoided over a large range of control is, given ψ , small relative to incremental cost of control and the ratio of incremental control cost to incremental social cost avoided is constrained to unity through lax control at the source. In Case II--given $b_{1,m}$, and $\psi \rightarrow M_4$ is large. Here, given the incremental cost of control function, incremental social cost avoided over a large range of control at the source is, given

Ψ , large relative to incremental cost of control and strict control at the source forces equality between incremental control cost and incremental social cost avoided.

The two cases reflecting the direct relationship, given the incremental cost of control function and Ψ , between the optimal level of control of SO_{cI} and incremental social cost avoided at every level of control are graphically derived in Figure 8,

Figure 8

THE OPTIMAL LEVEL OF CONTROL AT THE SOURCE OF SULFUR DIOXIDE EMISSIONS FROM THE INDUSTRIAL COMBUSTION OF COAL FOR M_4 LARGE, AND FOR M_4 SMALL



wherein $\left(\frac{dSCA_{cI}}{d \frac{1}{SO_{cI}}}\right)'$ is the incremental social cost avoided

function for the case of large M_4 , and $\left(\frac{dSCA_{cI}}{d \frac{1}{SO_{cI}}}\right)''$ is the in-

cremental social cost avoided function for the case of small M_4 . In the former case optimality requires strict, $\left(\frac{1}{SO_{cI}^*}\right)'$,

control. In the latter case optimality requires lax,

$\left(\frac{1}{SO_{cI}^*}\right)''$, control.

The discussion has focused on absolute optimal levels of control. Relative optimal levels of control are now considered. First, relative optimal levels of control of SO_c , SO_o , and SO_g within the framework of Model A are examined.

Recalling that for Model A optimality requires that

$$n \cdot a \cdot \left(\frac{1}{SO_c}\right)^{n-1} = s \cdot b \cdot \left(\frac{1}{SO_o}\right)^{s-1} = p \cdot c \cdot \left(\frac{1}{SO_g}\right)^{p-1} = \lambda$$

and that, in turn, $\lambda = h$, optimality requires, then, that

$$\frac{n \cdot a}{h} \cdot \left(\frac{1}{SO_c}\right)^{n-1} = \frac{s \cdot b}{h} \cdot \left(\frac{1}{SO_o}\right)^{s-1} = \frac{p \cdot c}{h} \cdot \left(\frac{1}{SO_g}\right)^{p-1} = 1.$$

Alternatively, optimality requires that

$$\frac{\frac{d CC_c}{d \frac{1}{SO_c}}}{\frac{d SCA_c}{d \frac{1}{SO_c}}} = \frac{n \cdot a}{h} \cdot \left(\frac{1}{SO_c}\right)^{n-1} = 1$$

$$\frac{\frac{d CC_o}{d \frac{1}{SO_o}}}{\frac{d SCA_o}{d \frac{1}{SO_o}}} = \frac{s \cdot b}{h} \cdot \left(\frac{1}{SO_o}\right)^{s-1} = 1$$

and

$$\frac{\frac{d CC_g}{d \frac{1}{SO_g}}}{\frac{d SCA_g}{d \frac{1}{SO_g}}} = \frac{p \cdot c}{h} \cdot \left(\frac{1}{SO_g}\right)^{p-1} = 1$$

Relative optimal levels of control at the sources of SO_c , SO_o , and SO_g depend, then, upon the relative values of $n \cdot a$, $s \cdot b$, and $p \cdot c$, and n , s , and p . The assumption, for example, that $n \cdot a > s \cdot b > p \cdot c$ means that, for $\frac{1}{SO_c} = \frac{1}{SO_o} = \frac{1}{SO_g} = \phi$,

$$\frac{\frac{d CC_c}{d \frac{1}{SO_c}}}{\frac{d SCA_c}{d \frac{1}{SO_c}}} > \frac{\frac{d CC_o}{d \frac{1}{SO_o}}}{\frac{d SCA_o}{d \frac{1}{SO_o}}} > \frac{\frac{d CC_g}{d \frac{1}{SO_g}}}{\frac{d SCA_g}{d \frac{1}{SO_g}}}$$

and optimality requires $\frac{1}{SO_g} > \frac{1}{SO_o} > \frac{1}{SO_c}$. This means that given the relatively large incremental control cost per unit of incremental social cost avoided over a relatively large range of control at the source of sulfur dioxide emissions from the combustion of coal, the optimal level of control at the source of sulfur dioxide emissions from the combustion of coal is less strict than the optimal level of control at the source of sulfur dioxide emissions from the combustion of residual fuel oil, for which optimality requires, in turn, a less strict level of control than the optimal level of control at the source of sulfur dioxide emissions from the combustion of gas. Over a relatively large range of control at the source of SO_g incremental control costs per unit of incremental social costs avoided are small relative to incremental costs of control per unit of incremental social costs avoided over relatively large ranges of control at the sources of sulfur dioxide emissions from the combustion of coal and residual fuel oil.

Collapsing MODEL B into three sectors -- each one of three fuels in one use -- relative optimal levels of control of SO_c , SO_o , and SO_g are examined for a set of values of a and n , s and b , p and c , and M_c , M_o , and M_g , where M_c is a meteorological parameter which relates control realized in

the area of reception of sulfur dioxide concentrations from the source using coal to control at the source of sulfur dioxide emissions from the combustion of coal, M_o is a meteorological parameter which relates control realized in the area of reception of sulfur dioxide concentrations from the source using residual fuel oil to control at the source of sulfur dioxide emissions from the combustion of residual fuel oil, and M_g is a meteorological parameter which relates control realized in the area of reception of sulfur dioxide concentrations from the source using natural gas to control at the source of sulfur dioxide emissions from the combustion of natural gas. By an argument similar to that of Appendix 2 the conditions for optimality for the present model -- MODEL C -- require that

$$\frac{n \cdot a}{M_c} \cdot \left(\frac{1}{SO_c}\right)^{n-1} = \frac{s \cdot b}{M_o} \cdot \left(\frac{1}{SO_o}\right)^{s-1} = \frac{p \cdot c}{M_g} \cdot \left(\frac{1}{SO_g}\right)^{p-1} = \lambda$$

and that, in turn, $\lambda = h$. Optimality requires, then, that

$$\frac{n \cdot a}{M_c \cdot h} \cdot \left(\frac{1}{SO_c}\right)^{n-1} = \frac{s \cdot b}{M_o \cdot h} \cdot \left(\frac{1}{SO_o}\right)^{s-1} = \frac{p \cdot c}{M_g \cdot h} \cdot \left(\frac{1}{SO_g}\right)^{p-1} = 1$$

and alternatively that

$$\frac{\frac{d CC_c}{d \frac{1}{SO_c}}}{\frac{d SCA_c}{d \frac{1}{SO_c}}} = \frac{n \cdot a}{M_c \cdot h} \cdot \left(\frac{1}{SO_c}\right)^{n-1} = 1$$

$$\frac{\frac{d CC_o}{d \frac{1}{SO_o}}}{\frac{d SCA_o}{d \frac{1}{SO_o}}} = \frac{s \cdot b}{M_o \cdot h} \cdot \left(\frac{1}{SO_o}\right)^{s-1} = 1$$

and

$$\frac{\frac{d CC_g}{d \frac{1}{SO_g}}}{\frac{d SCA_g}{d \frac{1}{SO_g}}} = \frac{p \cdot c}{M_g \cdot h} \cdot \left(\frac{1}{SO_g}\right)^{p-1} = 1 .$$

The relative optimal levels of control of SO_c , SO_o , and SO_g depend upon the relative values of $n \cdot a$, $s \cdot b$, and $p \cdot c$, M_c , M_o , and M_g , and n , s and p . The assumptions, for example, that $n, a > s, b > p, c$, and $M_c > M_o > M_g$ and for

$$\frac{1}{SO_c} = \frac{1}{SO_o} = \frac{1}{SO_g} = \phi$$

$$\frac{\frac{d CC_c}{d \frac{1}{SO_c}}}{\frac{d SCA_c}{d \frac{1}{SO_c}}} > \frac{\frac{d CC_o}{d \frac{1}{SO_o}}}{\frac{d SCA_o}{d \frac{1}{SO_o}}} > \frac{\frac{d CC_g}{d \frac{1}{SO_g}}}{\frac{d SCA_g}{d \frac{1}{SO_g}}}$$

optimality requires

$$\frac{1}{SO_g} > \frac{1}{SO_o} > \frac{1}{SO_c}$$

that is, that control at the source of sulfur dioxide emissions from the combustion of gas be strict relative to control at the sources of sulfur dioxide emissions from the combustion of coal and residual fuel oil, and that control at the source of sulfur dioxide emissions from the combustion of oil be strict relative to control at the source of sulfur dioxide emissions from the combustion of coal. This configuration of control entails more strict control at the source with which is associated over a relatively large range of control relatively small incremental costs of control per unit of incremental social costs avoided. This configuration of control is optimal.

The analytic solutions of the models provide useful frameworks for making a priori judgments about optimality in control efforts. The rules developed in this chapter are

applied in Appendix 4 in an analysis of the New York City regulations governing the sulfur content of fuels.

APPENDIX 4

AN ANALYSIS OF THE NEW YORK CITY REGULATIONS GOVERNING
THE SULFUR CONTENT OF FUELS

Consider the New York City regulations governing the sulfur content of coal and residual fuel oil consumed in that jurisdiction. These regulations prohibit the sale or use after May, 1971, of coal or residual fuel oil containing one percent sulfur by weight.

One condition for this regulation to entail economic optimal levels of control at the sources of sulfur dioxide emissions from the combustion of residual fuel oil and coal in the different uses is that for

$$\frac{1}{SO_{cI}} = \frac{1}{SO_{oI}} = \dots = \theta$$

and

$$\frac{1}{SO_{cP}} = \frac{1}{SO_{oP}} = \dots = \mu$$

$$\frac{\frac{dCC_{cI}}{d \frac{1}{SO_{cI}}}}{\frac{dSCA_{cI}}{d \frac{1}{SO_{cI}}}} = \frac{\frac{dCC_{oI}}{d \frac{1}{SO_{oI}}}}{\frac{dSCA_{oI}}{d \frac{1}{SO_{oI}}}} = \dots = 1$$

and

$$\frac{\frac{dCC_{CP}}{d\frac{1}{SO_{CP}}}}{\frac{dSCA_{CP}}{d\frac{1}{SO_{CP}}}} = \frac{\frac{dCC_{OP}}{d\frac{1}{SO_{OP}}}}{\frac{dSCA_{OP}}{d\frac{1}{SO_{OP}}}} = \dots = 1$$

where SO_{CI} is the rate of emission at the source of sulfur dioxide from the industrial combustion of coal, SO_{CP} is the rate of emission at the source of sulfur dioxide from the combustion of coal by the power plant, SO_{OI} is the rate of emission at the source of sulfur dioxide from the industrial combustion of oil, SO_{OP} is the rate of emission at the source of sulfur dioxide from the combustion of oil by the power plant, CC_{CI} is cost of control at the source of SO_{CI} , CC_{OI} is cost of control at the source of SO_{OI} , CC_{CP} is cost of control at the source of SO_{CP} , CC_{OP} is cost of control at the source of SO_{OP} , SCA_{CI} is social cost avoided through control at the source of SO_{CI} , SCA_{OI} is social cost avoided through control at the source of SO_{OI} , SCA_{CP} is social cost avoided through control at the source of SO_{CP} , and SCA_{OP} is social cost avoided through control at the source of SO_{OP} . This is Condition I.

The regulation calls for more strict control at small-

er fuel burning installations and, therefore, a second condition for the regulation to entail economic optimal levels of control of sulfur dioxide emissions from the combustion of residual fuel oil and coal in the different uses is that the ratio of incremental control cost to incremental social cost avoided for a relatively large range of control at the source with a relatively large Btu requirement per unit of time be relatively large and that the ratio of incremental control cost to incremental social cost avoided for a relatively large range of control at the source with a relatively small Btu requirement per unit of time be relatively small.

Abstracting from residential and commercial combustion of coal and residual fuel oil and assuming that the Btu requirement per unit of time is large at the power plant and small at the industrial source. The regulation results in

$$\frac{1}{SO_{CP}} < \frac{1}{SO_{CI}} \quad \text{and} \quad \frac{1}{SO_{OP}} < \frac{1}{SO_{OI}}$$

and optimality requires that

$$\frac{\frac{d CC_{OP}}{d \frac{1}{SO_{OP}}}}{\frac{d SCA_{OP}}{d \frac{1}{SO_{OP}}}} > \frac{\frac{d CC_{OI}}{d \frac{1}{SO_{OI}}}}{\frac{d SCA_{OI}}{d \frac{1}{SO_{OI}}}}$$

and

$$\frac{\frac{d CC_{CP}}{d \frac{1}{SO_{CP}}}}{\frac{d SCA_{CP}}{d \frac{1}{SO_{CP}}}} > \frac{\frac{d CC_{CI}}{d \frac{1}{SO_{CI}}}}{\frac{d SCA_{CI}}{d \frac{1}{SO_{CI}}}}$$

for $\frac{1}{SO_{CP}} = \frac{1}{SO_{CI}}$ and $\frac{1}{SO_{OP}} = \frac{1}{SO_{OI}}$. This is Condition II.

In order to simplify the analysis of the regulation assume that all fuel burning installations to which the regulation is applicable have the same BTU requirements per unit of time and that the source of sulfur dioxide emissions from the combustion of a fuel is a representative source. Condition II is now not applicable and Condition I becomes,

for $\frac{1}{SO_c} = \frac{1}{SO_o} = \Delta$,

$$\frac{\frac{d CC_c}{d \frac{1}{SO_c}}}{\frac{d SCA_c}{d \frac{1}{SO_c}}} = \frac{\frac{d CC_o}{d \frac{1}{SO_o}}}{\frac{d SCA_o}{d \frac{1}{SO_o}}} = 1$$

The application of MODEL C to the problem is appropriate and the question of whether or not the regulation en-

tails optimal levels of control turns on the values of the parameters governing the incremental cost of control functions and the relationships between concentrations of sulfur dioxide emitted and concentrations of sulfur dioxide received. Two cases are considered.

For $n, a > s, b$ and $M_c > M_o$, the ratios of incremental control cost to incremental social cost avoided may for

$$\frac{1}{SO_c} = \frac{1}{SO_o} = \Delta$$

be equal to unity thereby justifying a regulation which sets

$\frac{1}{SO_c} = \frac{1}{SO_o}$. Assume that for $\frac{1}{SO_c} = \frac{1}{SO_o} = \Delta$ the ratios of in-

cremental control cost to incremental social cost avoided are equal to unity. This is Case I.

For $n, a > s, b$ and $M_c < M_o$ the regulation is non-optimal. Setting $\frac{1}{SO_c} = \frac{1}{SO_o}$ under the conditions of this

case--Case II--entails

$$\frac{\frac{d CC_c}{d \frac{1}{SO_c}}}{\frac{d SCA_c}{d \frac{1}{SO_c}}} > \frac{\frac{d CC_o}{d \frac{1}{SO_o}}}{\frac{d SCA_o}{d \frac{1}{SO_o}}}$$

Optimality under Case II requires $\frac{1}{SO_o} > \frac{1}{SO_c}$, that is, more strict control at the source of sulfur dioxide emissions from the combustion of residual fuel oil, the fuel with which is associated at the source over a relatively large range of control at the source a relatively small incremental control cost per unit of incremental social cost avoided.

The question remains whether optimality requires under the two cases absolutely strict or lax control of SO_c and SO_o . Recalling that optimality requires that the ratios of incremental control cost to incremental social cost avoided equal unity, the question turns on the value of each ratio relative to unity. Assuming that for every level of

control of SO_c the ratio $\frac{\frac{d CC_c}{d \frac{1}{SO_c}}}{\frac{d SCA_c}{d \frac{1}{SO_c}}}$ is large and constrained to

unity only through lax control, and that for every level of

control of SO_o the ratio $\frac{\frac{d CC_o}{d \frac{1}{SO_o}}}{\frac{d SCA_o}{d \frac{1}{SO_o}}}$ is small and forced to unity

only through strict control, then under Case II optimality

requires that control of SO_o be strict relative to control of SO_c and that control of SO_o be absolutely strict and that control of SO_c be absolutely lax.

Under Case I where for $\frac{1}{SO_o} = \frac{1}{SO_c} = \Delta$.

$$\frac{\frac{d CC_c}{d \frac{1}{SO_c}}}{\frac{d SCA_c}{d \frac{1}{SO_c}}} = \frac{\frac{d CC_o}{d \frac{1}{SO_o}}}{\frac{d SCA_o}{d \frac{1}{SO_o}}}$$

control of SO_c and SO_o are absolutely strict or lax depending upon whether for every level of control of SO_c and SO_o the ratios are large and constrained to unity only through lax control or are small and forced to unity through strict control. The assumption that for every level of control of SO_c and SO_o the ratios are large, optimality requires absolutely lax control of SO_c and SO_o .

CHAPTER VI

THE NUMERICAL SOLUTION

A variation of MODEL B was employed to determine optimal rates of emission in the 400 square miles in and surrounding St. Louis, Missouri at the sources of sulfur dioxide emissions from the industrial combustion of coal, the industrial combustion of residual fuel oil, the combustion of coal by power plants, and the combustion of residual fuel oil by power plants.

Among the assumptions underlying the model are that:

concentrations emitted are the concentrations received

sources of SO_{OI} , SO_{CI} , SO_{OP} , and SO_{CP} are representative sources

sulfur dioxide emissions per unit of time entail given social costs

total social cost avoided is linearly related to concentrations of sulfur dioxide

the one option for the control of sulfur dioxide emissions from the combustion of coal is the substitution in the combustion process of low sulfur (.85%) coal, the cost of which increases as greater and greater quantities of this fuel are employed

the one option for the control of sulfur dioxide emissions from the combustion of residual fuel oil is the substitution in the combustion process of low sulfur (1.0%) residual fuel oil, the cost of which increases as greater and greater quantities of this fuel are employed.

Equations 1-4 give cost at the source of control of sul-

$$1. \quad CC_{OI} = b_1 \cdot (Q_O^L)^m$$

$$2. \quad CC_{CI} = b_2 \cdot (Q_C^L)^r$$

$$3. \quad CC_{OP} = b_3 \cdot (Q_O^L)^s$$

$$4. \quad CC_{CP} = b_4 \cdot (Q_C^L)^t$$

fur dioxide emissions from the combustion of coal and residual fuel oil in the two uses. Equation 1 is the cost per 24 hours of control at the source of sulfur dioxide emissions from the industrial combustion of oil in terms of barrels of low sulfur oil (Q_O^L) substituted per 24 hours in the combustion process for high sulfur oil. Equation 2 is the cost per 24 hours of control at the source of sulfur dioxide emissions from the industrial combustion of coal in terms of tons of low sulfur coal (Q_C^L) substituted per 24 hours in the combustion process for high sulfur coal. Equation 3 is the cost per 24 hours of control at the source of sulfur dioxide emissions from the combustion of oil at the power plant in terms of barrels of low sulfur oil substituted per 24 hours in the combustion process for high sulfur oil. Equation 4 is the cost per 24 hours of control at the source of sulfur dioxide

emissions from the combustion of coal at the power plant in terms of tons of low sulfur coal substituted per 24 hours in the combustion process for high sulfur coal.

Equations 5 - 8 are the technical relationships between

$$5. \quad SO_{OI} = L - a_1 \cdot Q_O^L$$

$$6. \quad SO_{CI} = M - a_2 \cdot Q_C^L$$

$$7. \quad SO_{OP} = N - a_3 \cdot Q_O^L$$

$$8. \quad SO_{CP} = T - a_4 \cdot Q_C^L$$

emissions from the source of sulfur dioxide in milligrams per 100 square centimeters per day ($\text{mg./100cm}^2/\text{day}$) and quantity of low sulfur fuel substituted in the combustion process per 24 hours. Equation 5 is the technical relationship governing the industrial source burning oil and substituting low sulfur oil. Equation 6 is the technical relationship governing the industrial source burning coal and substituting low sulfur coal. Equation 7 is the technical relationship governing the power plant burning oil and substituting low sulfur oil. Equation 8 is the technical relationship governing the power plant burning coal and substituting low sulfur coal.

Equations 9 - 12 are the incremental social cost

$$9. \quad \frac{dSCA_{OI}}{d SO_{OI}} = -h$$

$$10. \quad \frac{dSCA_{CI}}{d SO_{CI}} = -h$$

$$11. \quad \frac{dSCA_{OP}}{d SO_{OP}} = -h$$

$$12. \quad \frac{dSCA_{CP}}{d SO_{CP}} = -h$$

avoided functions governing emissions per 24 hours at the source of sulfur dioxide from the combustion of the two fuels in the two uses . h is a parameter. These functions reflect the assumptions that sulfur dioxide emissions per unit of time entail, regardless of source, given social costs, and that social cost avoided is linearly related to emissions.

Equations 13 - 16 are the conditions for optimality. The optimal rate of emission of sulfur dioxide from each source is that rate which entails equimarginal control costs and

social costs avoided.

$$13. \quad \frac{dCC_{OI}}{dSO_{OI}} = -h$$

$$14. \quad \frac{dCC_{CI}}{dSO_{CI}} = -h$$

$$15. \quad \frac{dCC_{OP}}{dSO_{OP}} = -h$$

$$16. \quad \frac{dCC_{CP}}{dSO_{CP}} = -h$$

Substituting equations 5 - 8 into equations 1 - 4 yield expressions for cost at the source of control of sulfur dioxide emissions in terms of rates of emission of sulfur dioxide. Hence,

$$CC_{OI} = b_1 \cdot \left(\frac{L}{a_1} - \frac{1}{a_1} \cdot SO_{OI} \right)^m$$

$$CC_{CI} = b_2 \cdot \left(\frac{M}{a_2} - \frac{1}{a_2} \cdot SO_{CI} \right)^r$$

$$CC_{OP} = b_3 \cdot \left(\frac{N}{a_3} - \frac{1}{a_3} \cdot SO_{OP} \right)^s$$

$$CC_{CP} = b_4 \cdot \left(\frac{T}{a_4} - \frac{1}{a_4} \cdot SO_{CP} \right)^t \quad .$$

The reduced form expressions for SO_{OI}^* , SO_{CI}^* , SO_{OP}^* , and SO_{CP}^* provide the framework for the empirical investigation. Setting the first derivatives of the expressions above equal to $-h$, in accordance with equations 13 -16, yield expressions for the optimal level of emission at the source of sulfur dioxide per 24 hours in milligrams per 100 square centimeters for each fuel in each use. Specifically,

$$SO_{OI}^* = L - a_1 \cdot \left(\frac{a_1 \cdot h}{m \cdot b_1} \right)^{\frac{1}{m-1}}$$

$$SO_{CI}^* = M - a_2 \cdot \left(\frac{a_2 \cdot h}{r \cdot b_2} \right)^{\frac{1}{r-1}}$$

$$SO_{OP}^* = N - a_3 \cdot \left(\frac{a_3 \cdot h}{s \cdot b_3} \right)^{\frac{1}{s-1}}$$

$$SO_{CP}^* = T - a_4 \cdot \left(\frac{a_4 \cdot h}{t \cdot b_4} \right)^{\frac{1}{t-1}}$$

Control costs per 24 hours incurred in meeting these standards are given by the following expressions:

$$CC_{OI}^* = b_1 \cdot \left(\frac{L}{a_1} - \frac{1}{a_1} \cdot SO_{OI}^* \right)^m$$

$$CC_{CI}^* = b_2 \cdot \left(\frac{M}{a_2} - \frac{1}{a_2} \cdot SO_{CI}^* \right)^r$$

$$CC_{OP}^* = b_3 \cdot \left(\frac{N}{a_3} - \frac{1}{a_3} \cdot SO_{OP}^* \right)^s$$

$$CC_{CP}^* = b_4 \cdot \left(\frac{T}{a_4} - \frac{1}{a_4} \cdot SO_{CP}^* \right)^t .$$

The parameters L, M, N, and T are, for zero control, the emissions from the source per day of sulfur dioxide in mg./100 cm². The parameters a₁, a₂, a₃ and a₄ give the incremental impact upon the emissions at the source per 24 hours of sulfur dioxide in mg./100 cm² of substituting at the source low sulfur fuel for high sulfur fuel. The parameters b₁, b₂, b₃, b₄, m, r, s, and t reflect the cost per 24 hours at the source of substituting low sulfur fuel for high sulfur fuel. The parameter h gives the constant incremental social cost avoided per 24 hours through reduction at the source per 24 hours of sulfur dioxide emissions in mg./100 cm².

The parameters of the total cost functions were estimated by fitting the least square curves to the data of Table 5. In Table 5 there is reported for each representative source the total cost of control per 24 hours incurred in substituting in the combustion process given amounts of low sulfur fuel for high sulfur fuel. The upper limit upon substitution per 24 hours of low sulfur fuel at the source is

Table 5

TWENTY-FOUR HOUR SOURCE TOTAL CONTROL COSTS FOR GIVEN RATES OF
 SUBSTITUTION IN THE COMBUSTION PROCESS OF LOW SULFUR FUEL FOR
 HIGH SULFUR FUEL

	Number of Tons or Barrels of Low Sulfur Fuel Substi- tuted per 24 hours at the source.	Total Cost of Control Incurred at the source per 24 hours.
Industrial Source	40	4.40
Using Oil	160	113.60
	282	566.82
Power Plant	150	100.50
Using Oil	450	630.00
	752	1812.32
Power Plant	60	1413.00
Using Coal	120	5946.00
	185	17998.65
Industrial Source	20	110.00
Using Coal	40	500.00
	70	2135.00

Source: Appendix 5.

given by boiler capacity as reported by Ernst and Ernst¹ for 1.9% sulfur coal and Number 6 residual fuel oil.

The data of Table 5 were transformed into log linear form and the parameters of equations 1 - 4 were estimated using simple linear regression. From the regression analysis²

$$b_1 = 0.0005$$

$$m = 2.45$$

$$b_2 = 0.0869$$

$$r = 2.37$$

$$b_3 = 0.0130$$

$$s = 1.78$$

$$b_4 = 0.1498$$

$$t = 2.23$$

and, then

$$CC_{OI} = 0.0005 (Q_O^L)^{2.45}$$

$$CC_{CI} = 0.0869 (Q_C^L)^{2.37}$$

¹Ernst and Ernst, The Fuel of Fifty Cities (Washington, D. C.: Ernst and Ernst, 1968), p. VI-10.

²The data is, in large part, hypothetical. In view of this no statistical tests for reliability were made.

$$CC_{OP} = 0.0130 (Q_O^L)^{1.78}$$

$$CC_{CP} = 0.1498 (Q_C^L)^{2.23} .$$

Estimation of the parameters of the technical relationships of equations 5 - 8 is straightforward. Assume that the representative source consumes fuel per 24 hours at boiler capacity. The boiler capacity of the industrial source using residual fuel oil is 282 barrels per 24 hours. The boiler capacity of the power plant using residual fuel oil is 751.68 barrels per 24 hours. The boiler capacity of the industrial source using coal is 69.6 tons per 24 hours, and the boiler capacity of the power plant using coal is 184.8 tons per 24 hours.

Emissions of sulfur dioxide from the combustion of 2.2% sulfur residual fuel oil are 14.5 pounds per 24 hours per barrel of this fuel consumed per 24 hours.³ Assume that emissions per 24 hours are evenly distributed over the 400 square mile area of reception. On the strength of this assumption emissions of sulfur dioxide per barrel of 2.2% sulfur residual fuel oil consumed per 24 hours are

³All emission rates in pounds are from Ernst and Ernst, The Fuel of Fifty Cities, p. VI-8.

.0000638 mg./100 cm² per 24 hours.⁴ Emissions of sulfur dioxide from the combustion of 1% sulfur residual fuel oil are 6.6 pounds or .0000290 mg./100 cm² per day per barrel of this fuel consumed per 24 hours. Emissions of sulfur dioxide from the combustion of 3.3% sulfur coal are 125.4 pounds or .000551 mg./100 cm² per day per ton of this fuel consumed per 24 hours. Emissions of sulfur dioxide from the combustion of .85% sulfur coal are 32.3 pounds or .000142 mg./100 cm² per day per ton of this fuel consumed per 24 hours.

The above yields

a ₁	=	.000035
a ₂	=	.0004
a ₃	=	.000035
a ₄	=	.0004
L	=	.0179
M	=	.0383
N	=	.0479
T	=	.1020

⁴One pound of sulfur dioxide per day evenly distributed over 400 square miles is equal to .0000044 mg./100 cm²/day of sulfur dioxide.

and; then

$$SO_{CI} = .0179 - .000035 Q_O^L$$

$$SO_{CI} = .0383 - .0004 Q_C^L$$

$$SO_{OP} = .0479 - .000035 Q_O^L$$

$$SO_{CP} = .1020 - .0004 Q_C^L$$

The parameter (h) of the incremental social cost avoided functions (equations 9 - 12) was estimated from data reported in the literature. Ridker and Henning conclude that for a drop of 0.25 mg./100 cm²/day in the sulfation level to which any single-family dwelling unit is exposed in the St. Louis, Missouri area "the value of that property could be expected to rise by at least \$83.00 and more likely closer to \$245.00."⁵ Anderson and Crocker conclude that for an additional 10 mg./m³/day of suspended particulates and an additional 0.10 mg./100 cm²/day of sulfation the marginal capitalized loss in the St. Louis and some other areas due to pollution ranges from about \$300.00 to \$700.00 per pro-

⁵Ronald G. Ridker and John A. Henning, "The Determinants of Residential Property Values with Special Reference to Air Pollution," The Review of Economics and Statistics, XLIX (May, 1967), 254.

perty.⁶

It is assumed that property values significantly reflect the social costs of air pollution. Ayres concludes that property values reflect the costs of more rapid deterioration, extra cleaning and maintenance, the costs of the milder medical symptoms and the costs of aesthetic effects.⁷ He argues that the costs of the more acute medical effects of air pollution are not reflected in property values.⁸

It is further assumed that the total social cost avoided function is linear. There is some evidence, however, that this is not the case.⁹

Using Ridker and Henning's higher figure of \$245.00 and applying a rate of interest of 10%, it is concluded that, for a decline of 1.0 mg./100 cm²/day in the sulfation level in the St. Louis area, total social cost avoided increases

⁶Robert J. Anderson, Jr. and Thomas D. Crocker, Air Pollution and Housing: Some Findings (Lafayette, Indiana: Purdue University, 1970), p. 21.

⁷Ayres, "Air Pollution in Cities," 17.

⁸Ibid., 16-17.

⁹See Anderson and Crocker, Air Pollution and Housing: Some Findings, pp. 25-26, and Thomas D. Crocker, "Urban Air Pollution Damage Functions: Theory and Measurement," pp. 69-73. (Mimeographed.)

by \$90,455 per day.¹⁰ This yields then

$$h = 90,455$$

and

$$\frac{dSCA_{OI}}{d SO_{OI}} = -90,455$$

$$\frac{dSCA_{CI}}{d SO_{CI}} = -90,455$$

$$\frac{dSCA_{OP}}{d SO_{OP}} = -90,455$$

$$\frac{dSCA_{CP}}{d SO_{CP}} = -90,455$$

Substituting the values of the parameters into the reduced form expressions for optimal rates of emission of sulfur dioxide at the representative sources yield the following numerical solutions:

¹⁰Ridker and Henning conclude using their higher figure of \$245 that for a drop in the sulfation level of 0.25 mg./100 cm²/day the total increase in property values for the St. Louis area could be as much as \$82,790,000 and that this sum invested at 10% would yield \$8 million annually. Ridker and Henning, "The Determinants of Residential Property Values with Special Reference to Air Pollution;" 254.

$$SO_{OI}^* = .0179 - .000035 \left[\frac{(.000035)(90,455)}{(2.45)(.0005)} \right]^{\frac{1}{1.45}}$$

$$= .0099 \text{ mg./100 cm}^2/\text{day}$$

$$SO_{OP}^* = .0479 - .000035 \left[\frac{(.000035)(90,455)}{(1.78)(.0130)} \right]^{\frac{1}{.78}}$$

$$= .0269 \text{ mg./100 cm}^2/\text{day}$$

$$SO_{CP}^* = .1020 - .0004 \left[\frac{(.0004)(90,455)}{(2.23)(.1498)} \right]^{\frac{1}{1.23}}$$

$$= .0841 \text{ mg./100 cm}^2/\text{day}$$

$$SO_{CI}^* = .0383 - .0004 \left[\frac{(.0004)(90,455)}{(2.37)(.0869)} \right]^{\frac{1}{1.37}}$$

$$= .0206 \text{ mg./100 cm}^2/\text{day} .$$

Substituting the values of the parameters into the reduced form expressions for 24-hour control costs incurred at the sources in reducing emission rates to the optimal emission rates yield the following numerical solutions:

$$CC_{OI}^* = .0005 \left(\frac{.0179}{.000035} - \frac{1}{.000035} \cdot 0.0099 \right)^{2.45}$$

$$= \$301.00/\text{day}$$

$$CC_{OP}^* = .0130 \left(\frac{.0479}{.000035} - \frac{1}{.000035} \cdot 0.0269 \right)^{1.78}$$

$$= \$1145.00/\text{day}$$

$$\begin{aligned} CC_{CP}^* &= .1498 \left(\frac{.1020}{.0004} - \frac{1}{.0004} .0841 \right)^{2.23} \\ &= \$721.00/\text{day} \end{aligned}$$

$$\begin{aligned} CC_{CI}^* &= .0869 \left(\frac{.0383}{.0004} - \frac{1}{.0004} .0206 \right)^{2.37} \\ &= \$691.00/\text{day} \end{aligned}$$

The models developed in this study are useful in generating numerical solutions for optimality in control efforts. In addition, the study points up the need for and some directions that data collection might usefully take. Under the present technology control of sulfur dioxide emissions from the combustion of fossil fuels is effected through the substitution in the combustion process of desulfurized fuels and naturally occurring low sulfur fuels. Reliable numerical solutions for optimal levels of control must await the collection and availability of data on fuel prices at various rates of substitution.

APPENDIX 5

SOURCES OF DATA FOR SOURCE TOTAL CONTROL COST

Total control cost incurred per 24 hours by the representative source for a given rate of substitution in the combustion process of low sulfur fuel in tons or barrels for high sulfur fuel is the product of the rate of substitution reported in the first column of Table 6 and the control cost per unit of the substitute fuel reported in the fourth column of Table 6. Control cost per ton or barrel of low sulfur fuel is the difference between the second and third columns of Table 6. The data reported in the third column (price per ton or barrel of high sulfur fuel) is, with the exception of the data for the power plant using oil, from the Ernst and Ernst study of 1968 fuel prices.¹ The prices per unit reported in this column for high sulfur fuel are the 1968 unit prices for 2.9 - 3.7% sulfur coal consumed by the industrial source, 2.2% sulfur residual fuel oil consumed by the industrial source, and 3.5% sulfur coal consumed by the power plant.² Because of an absence of data the price

¹Ernst and Ernst, The Fuel of Fifty Cities, pp. III-II, II-18, and V-4.

²Ernst and Ernst did not correlate price per ton and sulfur content for coal consumed by the power plant. It is assumed that relatively high sulfur coal was traded at relatively low prices in 1968.

Table 6

DATA FROM WHICH THE RELATIONSHIPS OF TABLE 5 ARE DERIVED

	Number of Tons or Barrels of Low Sulfur Fuel Substi- tuted per 24 hours at the source.	Price per ton or barrel of low sulfur fuel.	Price per ton or barrel of high sulfur fuel.	Control cost per ton or barrel of low sulfur fuel.	Total control cost incurred at the source per 24 hours.
Industrial Source Using Oil	40	2.90	2.79	.11	4.40
	160	3.50	2.79	.71	113.60
	282	4.80	2.79	2.01	566.82
Power Plant Using Oil	150	3.17	2.50	.67	100.50
	450	3.90	2.50	1.40	630.00
	752	4.91	2.50	2.41	1812.32
Power Plant Using Coal	60	28.00	4.45	23.55	1413.00
	120	54.00	4.45	49.55	5946.00
	185	101.74	4.45	97.29	17998.65
Industrial Source Using Coal	20	12.00	6.50	5.50	110.00
	40	19.00	6.50	12.50	500.00
	70	37.00	6.50	30.50	2135.00

per ton of high sulfur (2.2%) residual fuel oil consumed by the representative power plant is arbitrarily set at about 90% of the price per ton of high sulfur residual fuel oil consumed by the representative industrial source using oil.

In the second column there are reported prices per unit of 1.0% sulfur residual fuel oil consumed at various levels per 24 hours by industrial sources and power plants, and prices per unit of .85% sulfur coal consumed by these sources per 24 hours at various levels. Price per unit of low sulfur fuel oil consumed in the smallest quantity noted for industrial fuel oil consumption is from the Ernst and Ernst study.³ It is assumed that the price per unit reported by the Ernst and Ernst study for low sulfur fuel oil consumed by the industrial source is consistent only with relatively low rates of substitution at the industrial source of low sulfur fuel oil for high sulfur fuel oil. Prices per unit of low sulfur fuel oil consumed at the industrial source at other, higher rates of substitution, are arbitrary. The use of these arbitrary numbers, which reflect the assumptions that the price of low sulfur fuel oil increases at an increasing rate as the rate of substitution increases, is necessitated by a lack of data on low sulfur fuel oil prices for

³Ernst and Ernst, The Fuel of Fifty Cities, p. III-II.

various rates of substitution in the combustion process of low sulfur fuel oil for high sulfur fuel oil.

Coal prices reported by the Ernst and Ernst Study are prices of coal for metallurgical uses "... because, according to suppliers, low sulfur coal ... for non metallurgical uses is unobtainable at present in most cities."⁴ Prices per unit of low sulfur coal consumed by the industrial source at all rates of substitution are arbitrary.

Fuel prices for all rates of substitution in the combustion process of low sulfur fuel for high sulfur fuel at the power plant are arbitrary. This is necessitated by a lack of data for all levels of substitution of low sulfur fuel for high sulfur fuel. The prices of low sulfur fuel substituted at the lowest rates at the power plant are taken to be about 90% of the prices of low sulfur fuel substituted at about the same rates in the combustion process at the industrial source. Fuel prices for low sulfur fuel substituted at other, higher rates of substitution reflect the assumptions that low sulfur fuel prices increase and at an increasing rate as the rate of substitution at the power plant increases.

⁴Ibid., p. II-5.

CHAPTER VII

THE STATIC SOLUTION AND OPTIMIZING OVER TIME

In Chapter IV the criteria for static optimality are derived. In this chapter the conditions under which the static solution for a series of instantaneous optima approaches the dynamic solution for the optimal path of control is investigated.

Minimizing the cost of sulfur dioxide pollution over a given planning horizon requires that over the period the sum of the cost of control at the source and the cost of uncontrolled emissions from the source be minimized.¹ The objective is to minimize the functional

$$TC_s = \int_{t_0}^{t_n} f(t)e^{-rt} dt$$

where e^{-rt} is a rate of discount, TC_s is source total cost over the planning horizon, t_0 is the initial unit of time of the planning horizon, t_n is the terminal unit of time of the period and $f(t)$ is the sum over the period of source total

¹The following paragraphs are based on R. G. D. Allen, Mathematical Analysis for Economists (New York: St. Martin's Press, 1964), pp. 521-541.

cost of control (CC_s) and total damage (TD_s) from uncontrolled emissions from the source and is dependent upon the variable control function governing the source.

Given $\frac{1}{SO_s}(t)$, the control function governing the source, CC_s and TD_s and, then, TC_s at each moment of time t are known. TC_s over the period is the sum of the total costs in the successive units of time from $t = t_0$ to $t = t_n$.

Therefore, for a given path of $\frac{1}{SO_s}$, TC_s at each time t , and, then, over the planning horizon, is determined. Since TC_s over the period is given by the course of control, TC_s is minimized over the period by that function $\frac{1}{SO_s}(t)$ that minimizes the integral

$$TC_s = \int_{t_0}^t f(t) e^{-rt} dt .$$

The determination of the function which minimizes TC_s over a planning horizon is a problem in the calculus of variations. In what follows the dynamic conditions for optimality are not derived rather the underlying economics of this solution are considered.²

² Wright derives the conditions for the optimal path of control of all air pollution using the calculus of variation. Wright, "Economic and Political Aspects of Dynamic Pollution Control," 39-55.

Assume that the combustion of coal by representative sources and the combustion of natural gas by representative sources are the only sources of sulfur dioxide emissions. Assume further that in the initial time unit the options facing each source of sulfur dioxide emissions from the combustion of coal for the control of SO_c is, because of fixed supplies of fuels, the substitution in the combustion process of a large quantity of mechanically cleaned coal and a small quantity of natural gas and that over time the options facing the source for the control of SO_c are technologically limited to the substitution in the combustion process of natural gas and mechanically cleaned coal for high sulfur coal. Assume, too, that the source of sulfur dioxide emissions (SO_g) from the combustion of natural gas is constrained to employ natural gas only over time. Assume finally that all sources have the same energy requirements and that energy requirements at each source and in the aggregate are fixed over the period.

Consider the source of SO_c . Total cost (TC_c) over the period at the source of SO_c is the sum over the period of source total control cost (CC_c) and total cost of uncontrolled emissions (TD_c) from the source.

CC_c over the period is a function of $\sigma(t)$, $Q_c^C(t)$, $P_g(t)$, $Q_g^C(t)$, and $\Delta P_c(t)$, where σ is exogenously determined

and, it is assumed, the constant cost per Mbtu of cleaning coal mechanically, Q_c^C is the quantity of mechanically cleaned coal in Mbtu substituted per unit of time at the source of SO_c , P_g is the price of natural gas per Mbtu, ΔP_c is the change in the price of coal per Mbtu, and Q_g^C is the quantity of natural gas in Mbtu substituted at the source of SO_c per unit of time.

The paths of the variables upon which CC_c over the period is dependent are, in turn, functions of the course of the control variable, $\frac{1}{SO_c}(t)$. The relationship between the variables in one period upon which CC_c in that period is dependent and the control variable in a prior period is through fuel demand responses to the configuration of source total control cost per unit of each fuel and, then, fuel supply responses. The relationship between source total control cost in one period and control in a prior period is the subject of Appendix 6.

Given the relationship between $TC_c(t)$ and $\frac{1}{SO_c}(t)$, then, since $TD_c(t)$ and $CC_c(t)$ and, in turn, $TC_c(t)$ are functions of the path of $\frac{1}{SO_c}$, TC_c over the period is minimized by that course of control at the source of SO_c which induces those paths of TD_c , Q_c^C , P_g , and ΔP_c which minimize the sum over the period of CC_c and TD_c .

The question of the condition under which the static solution for $\frac{1}{SO_c}(t_0)$ coincides with the solution for the initial time unit from a dynamic analysis is considered now and under the assumptions that the parameters of the functions governing the demands for fuels are constant over time, that fuel supply responses per unit of time to fuel price changes are constant over time, and that source total control cost incurred in meeting $\frac{1}{SO_c}(t_0)$ is large relative to source total control cost incurred in meeting $\frac{1}{SO_g}(t_0)$ with the result that, where prior to t_0 total cost per unit of natural gas is greater than total cost per unit of high sulfur coal, with the imposition of control in t_0 total cost per unit of coal is greater than total cost per unit of natural gas.

Two cases are examined. In Case I there is a large and positive demand response per unit of time for natural gas on the part of users of coal to control, with the demand response greater the larger the ratio of total cost of coal per unit to total cost of natural gas per unit. By prior assumption energy requirements at the source and in the aggregate are fixed, and the source of SO_g is constrained over time to use only natural gas in the combustion process, therefore, the demand response per unit of time for coal on

the part of users of coal to the imposition in the initial time unit of control is also large but opposite in sign. In the initial time unit source total control cost incurred in meeting the standard governing $SO_c(t_0)$ is large relative to source total control cost incurred in meeting the standard governing SO_g and the source of SO_c under Case I seeks to minimize the cost per Mbtu of energy consumption by substituting in the combustion process the now relatively low cost fuel, natural gas, for the now relatively high cost fuel, coal.

In Case II the fuel demand responses to the configuration of control and the configuration of source total control costs to which this gives rise are small. This set of responses is more difficult to rationalize but might be expected to reflect the behavior of those sources which administer prices or whose revenue is regulated and there is no strong inducement to minimize the cost of fuel consumption. The coal-fired power plant may fall under the latter, while the large industrial users of fuel may fall under the former.

Under Case I, where it is assumed that source total control costs in the initial time unit bear important relations to the demands over time for natural gas and coal, the price over time of natural gas and the quantity of natural gas traded over time as well as the price over time of coal

and the quantity of coal traded over time bear, in turn, important relations to source total control costs and, in turn, the configuration of control in the initial time unit. The specific price and quantity effects on each fuel depend upon the strengths of the demand responses per unit of time, the price elasticities of fuel supplies and fuel demands, the cost effects per unit of time upon fuel production of entry and exit, and the rate of entry and exit per unit of time.

Under Case II, on the other hand, where it is assumed that source total costs of control in the initial time unit bear no important relations to the demands for fuels, fuel prices over time and the quantities of fuels traded over time are, for the most part, independent of the configuration of control in t_0 .

Under Case I, there is associated with each level of control of SO_c and SO_g in t_0 a set of fuel demand responses which bear upon the cost over time of control at the source. TC_c over the planning horizon is minimized by a level of control in the initial time unit at the source of SO_c which is greater than, equal to, or less than the static optimum depending, in part, upon the fuel demand responses and the fuel supply responses to price changes. Under Case II the static solution for $\frac{1}{SO_c}(t_0)$ approaches the dynamic solution. This

is because the factors that bear upon the cost over time of control of SO_c are independent of the level of control of SO_c in the initial time unit.

Setting standards in the initial time period consistent with optimizing over the planning horizon requires estimation of the parameters governing fuel demand responses to control and fuel supply responses. Depending upon the values of these parameters, depends whether minimizing the cost of sulfur dioxide pollution over time requires pushing control in the initial time period up to, short of, or beyond that level of control indicated by a static analysis.

APPENDIX 6

THE RELATIONSHIP BETWEEN THE CONFIGURATION OF CONTROL IN ONE PERIOD AND SOURCE TOTAL CONTROL COST IN A SUBSEQUENT PERIOD

The analysis is conducted under, among other assumptions, the following:

the combustion of coal and natural gas by representative sources are the only sources of sulfur dioxide emissions

the options into which confronting each source of SO_c for the control of SO_c are the substitutions of a large quantity of mechanically cleared coal and a small quantity of natural gas for high sulfur coal and over time the options facing each source of SO_c for the control of SO_c are technologically limited to the substitution of natural gas and mechanically cleaned coal for high sulfur coal

the source of SO_g is constrained to employ natural gas only over time

energy requirements are the same for all sources and are fixed over time at the source and in the aggregate

control of sulfur dioxide emissions is imposed in t_0 .

Total cost per unit¹ of a fuel to the user of the fuel prior to t_0 is given by the price per unit of the fuel. Since the price per unit of gas is large relative to the price per

¹The unit of a fuel is million Btu (Mbtu) consumed.

unit of high sulfur coal the source of SO_c seeking to minimize the cost of producing the given level of energy employs prior to t_0 a relatively large quantity of coal. Given the imposition and configuration of control in t_0 and the options facing each source for the control of sulfur dioxide emissions and, then, the configuration of source total cost of control per unit of each fuel and assuming that source total control cost per unit of coal is large relative to source total control cost per unit of natural gas, and assuming further that, total cost per unit of coal -- the sum of price per unit of coal and source total control cost per unit of coal -- is now large relative to total cost per unit of natural gas the source of SO_c seeks in t_0 , in order to minimize the cost of fuel consumption, to substitute, within limits, natural gas for coal.

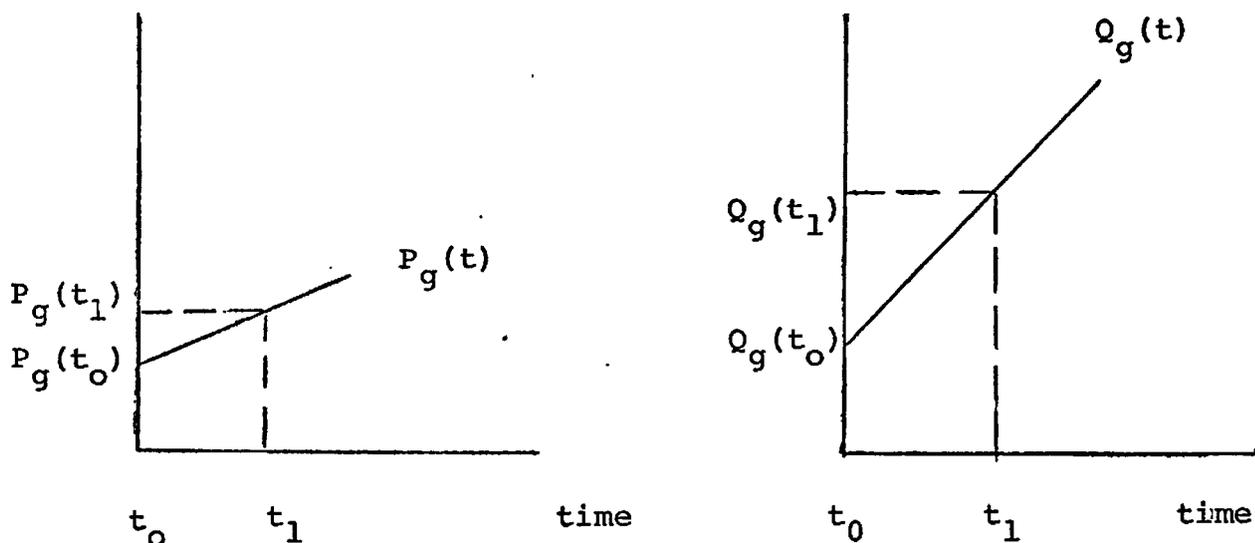
In t_0 the source of SO_c is constrained, however, to a very limited use of natural gas for pollution abatement. The standard governing SO_c is met largely through the substitution in the combustion process of mechanically cleaned coal, which, by assumption, is a relatively high cost technique of control of sulfur dioxide emissions.

Among the parameters of the function governing the demand for natural gas are the source total control costs incurred in t_0 per unit of each fuel, with the imposition of

control in t_0 inducing a positive change in the demand for natural gas. The price and quantity effects in t_1 in the natural gas market of the imposition of control in t_0 are given in Figure 9, where time is measured along the horizontal axes and the price of natural gas (P_g) per Mbtu and the quantity of natural gas traded (Q_g) per Mbtu are measured along the vertical axes.

Figure 9

THE PRICE AND QUANTITY EFFECTS IN t_1 IN THE NATURAL GAS MARKET OF THE IMPOSITION OF CONTROL IN t_0



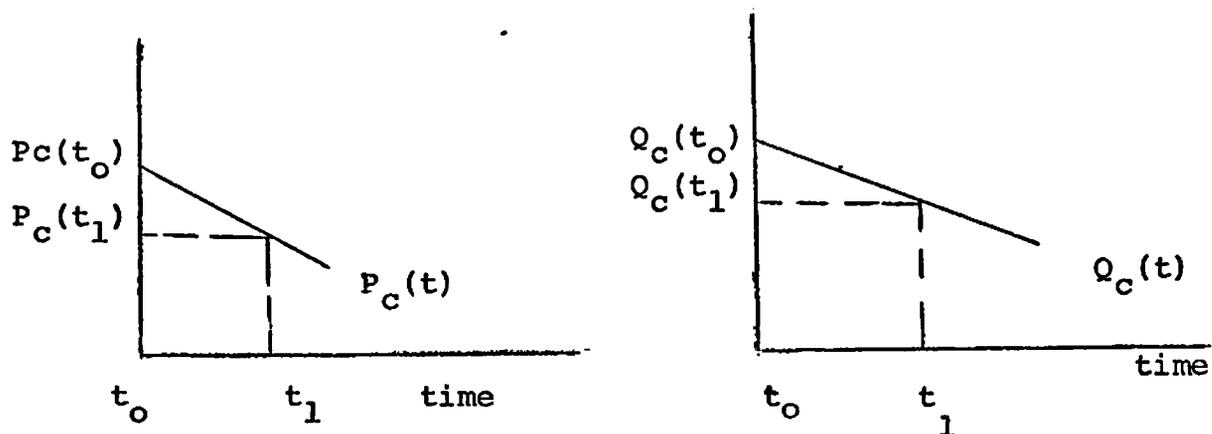
The elasticities of the $P_g(t)$ and the $Q_g(t)$ functions are functions of the strength of the demand response to the imposition of control in t_0 and the strength of the supply re-

response to the price change induced by the change in demand. The supply response is dependent, in part, upon the rate of entry into the industry and the cost effects of entry upon production. The impact of control imposed in t_0 is to force p_g and Q_g up in t_1 from $P_g(t_0)$ and $Q_g(t_0)$ to $P_g(t_1)$ and $Q_g(t_1)$.

Among the parameters of the function governing the demand for coal are the source total control costs in t_0 per unit of each fuel, with the imposition of control in t_0 inducing a negative change in the demand for coal. The price (P_c) and quantity (Q_c) effects in t_1 in the coal market of the imposition of control in t_0 are given in Figure 10.

Figure 10

THE PRICE AND QUANTITY EFFECTS IN t_1 IN THE COAL MARKET OF THE IMPOSITION OF CONTROL IN t_0



The elasticities of the $P_c(t)$ and the $Q_c(t)$ functions are functions of the strength of the demand response to control and the strength of the supply response to the price change induced by the change in demand. The impact of control in t_0 is to force P_c and Q_c down in t_1 from $P_c(t_0)$ and $Q_c(t_0)$ to $P_c(t_1)$ and $Q_c(t_1)$.

The function governing cost of control in t_1 at the source of SO_c is

$$CC_c(t_1) = P_g(t_1) \cdot Q_g^C(t_1) + \Delta P_c \cdot Q_c^C(t_1) + \sigma \cdot Q_c^C(t_1)$$

where ΔP_c is the change in the price of coal per Mbtu, $Q_g^C(t_1)$ is the quantity of natural gas in Mbtu substituted per unit of time in t_1 at the source of SO_c for sulfur dioxide abatement, σ is exogenously determined and is the constant cost per Mbtu of cleaning coal mechanically, and $Q_c^C(t_1)$ is the quantity of mechanically cleaned coal substituted per unit of time in t_1 at the source of SO_c for sulfur dioxide abatement.²

The function governing the cost of control in t_0 at

²In order to simplify the analysis two components of control cost are abstracted from. These are $\Delta P_c \cdot Q_c^{NC}(t_1)$ and $P_c(t_1) \cdot Q_c^C(t_1)$ where $Q_c^{NC}(t_1)$ is the quantity of raw coal employed by the source in t_1 . These two components of control cost like $\Delta P_c \cdot Q_c^C(t_1)$ are negative.

the source of SO_c is

$$CC_c(t_0) = \sigma \cdot Q_c^C(t_0) + P_g(t_0) \cdot Q_g^C(t_0)$$

where $Q_c^C(t_0)$ is the quantity of mechanically cleaned coal substituted at the source of SO_c per unit of time in t_0 for sulfur dioxide abatement, and $Q_g^C(t_0)$ is the quantity of natural gas substituted at the source of SO_c per unit of time in t_0 for sulfur dioxide abatement. By prior assumption $Q_g^C(t_0)$ is very small.

It is assumed that the function governing the production of control at the source of SO_c takes the form of the Cobb-Douglas.³ The function governing production of control in t_0 is

$$\frac{1}{SO_c}(t_0) = \alpha [Q_c^C(t_0)]^b \cdot \beta [Q_g^C(t_0)]^a .$$

The function governing production of control in t_1 is

$$\frac{1}{SO_c}(t_1) = \alpha [Q_c^C(t_1)]^b \cdot \beta [Q_g^C(t_1)]^a .$$

³The Cobb-Douglas is used to simplify the analysis, but in doing so the notion that the incremental control product associated with the employment of a technique of control is probably independent of the quantities employed of other techniques of control is violated.

$CC_c(t_0)$ and $CC_c(t_1)$ in terms of $\frac{1}{SO_c}(t_0)$ and $\frac{1}{SO_c}(t_1)$, respectively, are, then,

$$CC_c(t_0) = \frac{\sigma}{\alpha^{b-1} \cdot \beta^{b-1} [Q_g^c(t_0)]^{ab-1}} \cdot \left[\frac{1}{SO_c}(t_0)\right]^{\frac{1}{b}}$$

$$+ \frac{P_g(t_0)}{\beta^{a-1} \cdot \alpha^{a-1} [Q_c^c(t_0)]^{ba-1}} \cdot \left[\frac{1}{SO_c}(t_0)\right]^{\frac{1}{a}}$$

$$CC(t_1) = \frac{\sigma + \Delta P_c}{\alpha^{b-1} \cdot \beta^{b-1} [Q_g^c(t_1)]^{ab-1}} \cdot \left[\frac{1}{SO_c}(t_1)\right]^{\frac{1}{b}}$$

$$+ \frac{P_g(t_1)}{\beta^{a-1} \cdot \alpha^{a-1} [Q_c^c(t_1)]^{ba-1}} \cdot \left[\frac{1}{SO_c}(t_1)\right]^{\frac{1}{a}} .$$

The relative positions of the total cost of control functions in t_0 and t_1 depend upon the values of α , β , σ , ΔP_c , b , a , $Q_g^c(t_1)$, $Q_g^c(t_0)$, $P_g(t_1)$, $P_g(t_0)$, $Q_c^c(t_1)$, and $Q_c^c(t_0)$.

For $\frac{1}{SO_c}(t_0) = \frac{1}{SO_c}(t_1)$, that is, for a given level of control in t_0 and t_1 and assuming that $\alpha = \beta = a = b = 1$,⁴

⁴This assumption is introduced in order to simplify the analysis but in so doing the fact that the substitution of natural gas in the combustion process is efficient per Mbtu relative to the substitution of mechanically cleaned coal is violated.

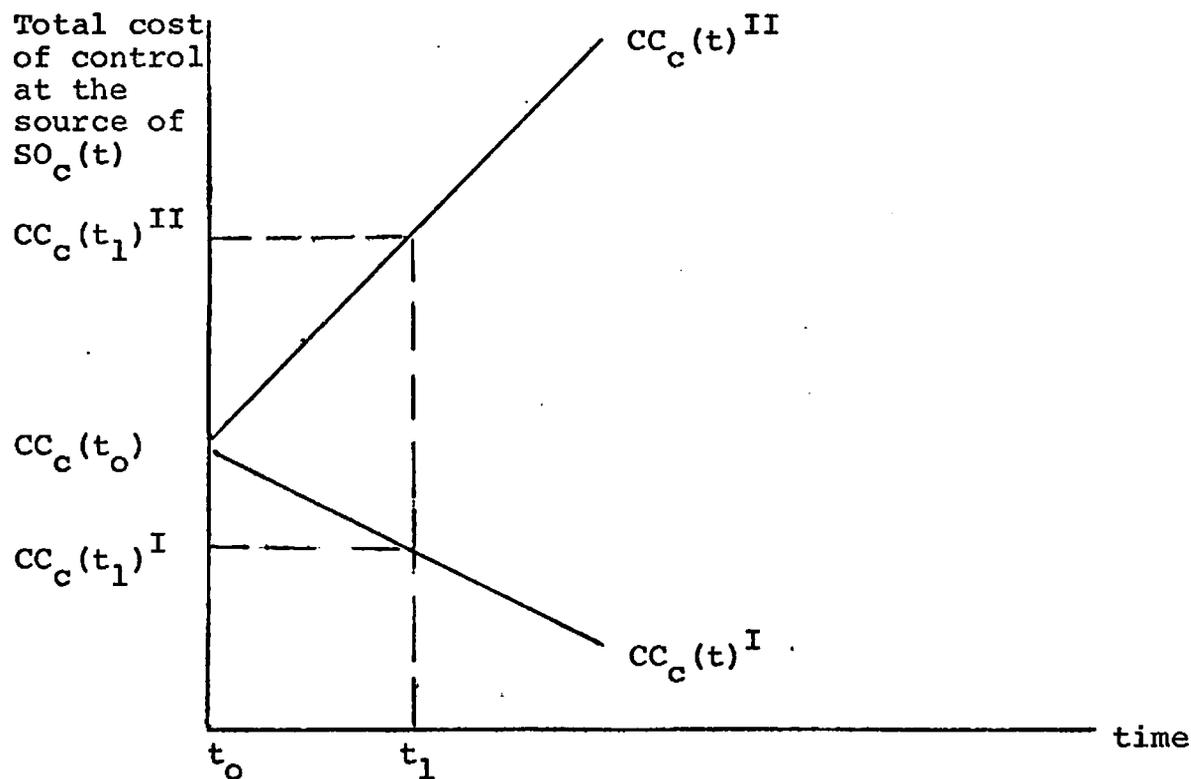
$CC_c(t_1) \underset{>}{<} CC_c(t_0)$ as

$$\left[\frac{P_g(t_1)}{Q_c^C(t_1)} - \frac{P_g(t_0)}{Q_c^C(t_0)} + \frac{\sigma + \Delta P_c}{Q_g^C(t_1)} - \frac{\sigma}{Q_g^C(t_0)} \right] \cdot \left(\frac{1}{SO_c} \right) \underset{>}{<} 0 .$$

The course over time of the source total cost of control variable depends upon the relative elasticity of the fuel price and fuel quantity functions. The course of source total control cost follows course I or course II in Figure 11

Figure 11

THE COURSE OVER TIME OF THE SOURCE TOTAL COST OF CONTROL VARIABLE



or some other course depending upon the courses over time of fuel prices and quantities of fuels available for substitution. The latter, in turn, depend upon the fuel demand responses and the fuel supply responses to control imposed in the prior period.

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