SYNCHRONOUS ONLINE COLLABORATIONS:

DISCUSSING AND SOLVING PROBLEMS USING VIRTUAL TOOLS

A Dissertation Presented to the
Faculty of the College of Education
University of Houston

In Partial Fulfillment
of the Requirements for the Degree

Doctor of Philosophy in Education

By

Joshua Udy

December 2016

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Abstract

There is currently a lack of research regarding instances when individuals use virtual tools collaboratively to solve and discuss mathematical problems synchronously online (Francis & Jacobsen, 2013). This study investigated the patterns of communication and interaction that emerged during three, synchronous, online mathematics professional development sessions wherein three fifth-grade teachers used virtual tools to solve and discuss fraction multiplication and division problems. These online sessions, which were digitally archived, highlighted participating teachers' interactions while discussing problems solved independently and while using virtual tools and fraction manipulatives to solve problems collaboratively. Each of these sessions was analyzed using coding procedures outlined by Carspecken (1996), and this preliminary analysis was used as the primary record of data for this study. An interview protocol was designed and then used to facilitate semi-structured, one-on-one interviews with each participating teacher. Findings from the participant interviews substantiated the conclusions drawn from the analysis of the archived sessions and provided additional details about participants' experiences in using virtual tools to solve and discuss problems attempted either independently or collaboratively. Peer reviewers and member checks were used to attend to validity and reliability.

Findings were organized into three categories according to how participants communicated and interacted while 1) solving and discussing problems, 2) using virtual tools, and 3) interfacing with the course instructor while collaborating to solve problems.

Findings suggest differences in communications and interactions existed when participants used online tools to represent, solve, and discuss problems collaboratively versus when they discussed work completed individually. Participants reported advantages and disadvantages to both approaches to solving and discussing problems, and their communications and interactions seemed influenced by problem difficulty level and mode of engagement.

Implications of the findings of this study include designing online mathematics professional learning sessions while considering 1) the effects of asking participants to solve easy and hard problems using both independent and collaborative approaches, 2) modifications that might be required to engage a large group of participants in collaborating to solve problems using virtual tools, 3) how to minimize technological problems, and 4) the ramifications that facilitator interactions can have on groups as they collaborate to solve problems using virtual manipulations and drawings.

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Chapter I

Introduction

There is currently a lack of research regarding the nature of collaborative interactions when individuals utilize virtual tools to solve problems together synchronously online (Francis & Jacobsen, 2013). In light of new advances in technology that allow for classroom teachers to learn content and pedagogical best practices through the convenience of online trainings (Dede, Ketelhut, Whitehouse, Breit, & McCloskey, 2009), concerns about the integrity of such learning experiences are brought forward when teachers who attend such trainings are expected to later implement their learning in face-to-face classrooms with students (Cady & Rearden, 2009). An argument can be made that if teachers do not engage in learning during professional development sessions in the same way that they are expected to engage their own students in learning new content in the classroom, then transfer of learning will be low (Darling-Hammond & McLaughlin, 1995). In the context of elementary mathematics in particular, this concern is especially important to consider because teachers in this field are often expected to utilize concrete manipulatives and pictorial drawings during their face-to-face instruction in the classroom with students (Van de Walle, Karp, & Bay-Williams, 2013). To date, few research studies have explored ways to engage elementary mathematics teachers during online professional development sessions in interactive collaborations with manipulatives and pictorial models (Francis & Jacobsen, 2013). This is a concern when considering that the use of concrete and pictorial representations and collaborative problem-solving techniques are considered extremely important

components of content-based elementary mathematics professional-development learning opportunities (NCTM, 2000, 2014).

Statement of the Problem

Online professional development has the potential to reach a broad audience of teachers both effectively and efficiently (Dede, 2004; Dede et al., 2009), and it has been found to be a cost-effective alternative to multiple, face-to-face trainings (Ellis & Kisling, 2009). However, a key component of effective learning and professional development is the time participants spend together solving novel problems with their peers both interactively and collaboratively (Darling-Hammond & McLaughlin, 1995; Hiltz, 1998; Johnson, 1981). Very few studies have explored the nature of learning when teachers solve mathematics problems together synchronously online using interactive tools or virtual manipulatives (Francis & Jacobsen, 2013). Most synchronous online professional development sessions, regardless of the discipline of study in question, report about synchronous interactions that are limited to conversations among participants that rely solely on audio, visual, and/or the text-based sharing of ideas (Cady & Rearden, 2009; Hiltz, 1998; Marrero, Woodruff, Schuster, & Riccio, 2010; Meskill & Anthony, 2014), and not on collaborative interactions with virtual tools while solving problems in a particular context (Francis & Jacobsen, 2013).

In mathematics education, however, the interactive manipulation of concrete manipulatives and the use of pictorial models is an essential component to understanding many concepts (Van de Walle et al., 2013), including, in particular, the concepts of fraction multiplication and division (Li, 2008; Lo & Luo, 2012). Virtual manipulatives have been found to have a similar effect as concrete manipulatives on students' abilities

to develop conceptual understandings of fraction concepts (Reimer & Moyer, 2005; Suh, Moyer, & Heo, 2005), and teachers who learn to utilize manipulatives can successfully implement their use in the classroom with students (Bouck & Flanagan, 2010).

A quality synchronous online professional development for mathematics teachers, like any quality face-to-face professional development (Darling-Hammond & McLaughlin, 1995), should include opportunities where teachers engage in the same content and learning experiences that they are expected to deliver to their students (Desimone, 2009), especially when the professional development is focused on the concepts of fraction multiplication and division (Rhine & Bennett, 1998). This indicates that synchronous online mathematics professional development sessions should offer opportunities for teachers to not only collaborate in conversations about mathematical concepts and ideas (Hebert, 2007), but should afford them the opportunity to collaboratively and interactively employ virtual manipulatives and drawings to solve mathematical problems together in real time (Francis & Jacobsen, 2013). Additionally, in order for professional development instructors to monitor group collaborations and model reform-based pedagogical practices in an online environment, it is important for them to have the ability to interact with student groups while they solve problems together, including when those groups use drawings and/or virtual manipulatives to represent and derive a problem's solution (Cady & Rearden, 2009).

Need for the Study

Herrington, Oliver, and Reeves (2003) and Herrington, Reeves, Oliver, and Woo (2004) have called for online learning experiences for teachers that are authentic and allow participants to "work together online to solve complex problems and complete

authentic tasks" (Reeves, Herrington, & Oliver, 2004, p. 53). However, these same authors report that "there is little evidence that the developers of most online collaborative learning environments . . .have tried, much less attained, [this] vision" (p. 54). Herrington et al. (2003) describe how authentic tasks in an online learning environment should have, among many other qualities, "real-world relevance" (p. 55) and "opportunit[ies] to collaborate" (p. 56). This means that continued research about "outside of the box" methods for developing online environments where teachers engage collaboratively to solve problems online is needed (Twigg, 2003, p. 116). Cady and Rearden (2009) commented about the concerns expressed by both instructors and participants during a series of online mathematics professional development courses wherein they were "not . . .able to see others using their manipulatives in the online environment" and how the instructors found it "challenging to model . . .reform-based pedagogical approaches in the online environment" due to only being able to see or review participants' work on an individual basis (p. 295).

The current study investigated how participants in a series of synchronous online professional development sessions utilized interactive virtual tools, which were visible and manipulable by all, while collaborating online to solve problems. Understanding the nature of these online learning experiences is important because researchers in the field of synchronous online collaborations have highlighted the fact that genuine collaboration among participants, as well as immersed engagement with representations in an online platform, are important components, among others, of effective online task engagement (Herrington et al., 2003). This collaboration is especially important in teacher professional development where it is expected that learning experiences for educators

include “concrete tasks of teaching,” interaction through “the sharing of knowledge,” and connections to teachers’ “work with their students” (Darling-Hammond & McLaughlin, 1995, p. 598).

Purpose of the Study

The purpose of this study was to investigate the nature of the learning experiences of teachers who participated in a series of interactive, collaborative, synchronous online professional development sessions while solving fraction multiplication and division problems. This study explored data digitally archived in the online platform wherein these sessions took place, including video capturing the collaborative interactions of participating teachers as they utilized virtual manipulatives and drawings to solve problems. In addition, this study collected data through one-on-one interviews with the teachers who participated in these online sessions, and investigated through qualitative data analysis the patterns of communication and interaction that emerged during their learning experiences as they engaged synchronously and collaboratively online while solving problems.

Research Question

For teachers engaged in a synchronous, collaborative online mathematics professional development about fraction multiplication and division concepts, what patterns of communication and interaction emerged?

Significance of the Study

This research study is significant because there is currently a gap in the literature about the nature of learning experiences for individuals who engage collaboratively and synchronously online to solve mathematical problems using virtual tools (Francis &

Jacobsen, 2013). This project helps fill this gap because it adds an important piece to the research base regarding the patterns of communication and interaction that emerged during synchronous online professional development sessions when participants utilized virtual tools to discuss and solve problems collaboratively. The results of this study can help inform educational researchers, curriculum developers, and individuals who deliver online professional development about the nature of learning in synchronous online platforms when participants collaborate and utilize online tools to solve problems in real time.

Summary

Considering the difficulties teachers often have in attending after-hours professional development trainings, and the costs and travel time associated with meeting face to face (Ellis & Kisling, 2009), synchronous online professional development offers a convenient solution for teachers to engage in training they need while avoiding the difficulties and costs associated with face-to-face meetings. While large numbers of teachers can be accommodated in an online learning platform through the uses of synchronous online applications (Dede, 2004), there is a lack of research about the nature of learning in online sessions that engage teachers in problem-solving tasks while utilizing real-time drawings and virtual manipulatives. While it is well established in the literature that structuring online trainings according to researched best practices fosters collaboration and interaction (Dede et al., 2009), as well as authentic communities of practice (Hebert, 2007), little research exists that provides insight into the patterns of communication and interaction that emerge from collaborations that allow for real-time problem solving among participants.

Some researchers have expressed concerns because of their inability to see participants manipulate virtual tools in real time while problem solving during synchronous online professional development courses (Cady & Rearden, 2009). Other researchers have felt limited to a narrow range of possible mathematical questions to use during online professional development sessions due to the requirement for there to be, at best, minimal drawings on the screen of their online platform (Francis & Jacobsen, 2013). In light of these concerns, this study has merit in that it addresses the patterns of communication and interaction that emerged during online sessions where participants interactively collaborated to solve problems in real time utilizing various online drawings and virtual manipulatives.

Chapter II

Literature Review

In order to situate this study in the literature, several concepts related to collaboration and synchronous online learning have been reviewed. First, the nature of collaborative learning has been investigated and explored through the lens of best practices for teacher professional development, learning in mathematics, mathematics professional development, and the uses of models and pictures for understanding fraction multiplication and division concepts through problem-solving tasks. Second, research about collaboration in online environments has been considered in general and also in light of synchronous online interactions. This review of research included investigating the nature of collaborative learning and the development of communities of practice in online environments, online professional development for teachers, and online mathematics professional development courses, specifically those that include synchronous online collaborations and interactive problem-solving experiences for participants.

Collaborative Learning

Interactions among learners have long been considered essential components of effective learning in education (Johnson, 1981). Current learning theory outlines that people of any age are more likely to form strong neural connections as they learn new concepts when experiencing realistic tasks in a problem-based environment where familiar contexts are used as a platform for discussion and thinking (Schunk, 2012). In particular, engaging students in collaborative learning opportunities in mathematics is an expected standard of exemplary teaching (NCTM, 2000, 2014). In addition, facilitating

authentic collaborations among students engaged in solving rigorous mathematical tasks has been shown to promote student achievement and understanding, as well as reduce gaps in achievement between student groups (Boaler & Stapes, 2008; Treisman, 1992). Knowing that collaboration is an essential component to effective learning, it is understood that teacher professional development should include opportunities for participants to learn content and pedagogy through collaborative tasks (Desimone, 2009). This also includes providing opportunities for participants to engage with their peers in authentic professional learning communities (Darling-Hammond, 2005).

Teacher Professional Development

Darling-Hammond and McLaughlin (1995) established a list of practices and policies that support teachers in learning to deliver instruction through reform-based methods. They assert that teacher professional development must deepen teachers' understanding of how their students learn, as well as provide effective methods that can be used to support students' efforts in learning new content. More specifically, these authors assert that professional learning for teachers "must be collaborative" and include the "sharing of knowledge among educators that focus on teacher communities of practice" (p. 82).

On a related note, Desimone (2009) reported that if large amounts of money are going to continue to be spent on professional development services for teachers, then a call to truly understanding what makes professional development effective is in order. In her work, Desimone (2009) proposed a "core conceptual framework for studying professional development" built upon five agreed-upon characteristics for what is essential in a professional development course (p. 183). Two of these characteristics are

active learning experiences and opportunities for collective participant engagement. Desimone (2009) also explained how a professional development course should focus on the specific content that the teachers teach and should include opportunities for engaging in this content while considering specifically how students are most likely to learn the concepts at hand. These suggestions mean that for a mathematics professional development course, teachers should be given the opportunity to focus their learning on a specific concept, engage in the content themselves as learners, and consider how their students will think about and best learn the content.

Learning Mathematics

The National Council of Teachers of Mathematics (NCTM) set forth rigorous standards for effective teaching and learning mathematics in their publication entitled *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2000). Within this publication, it was explained that "an excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences" (p. 7). Moreover, NCTM (2000) described specific "principles of learning" that students of mathematics should experience in order to truly master mathematical concepts. These include—among others—opportunities to "engage with challenging tasks that involve active meaning making" and "construct knowledge socially, through discourse, activity, and interaction related to meaningful problems" (p. 9). These points clearly establish the expectation that for students to learn mathematics, they should be provided with numerous opportunities to collaborate to solve problems. When considering the call for professional development to include opportunities for teachers to engage in the activities that they are expected to provide to their own students

(Darling-Hammond & McLaughlin, 1995), it is thus reasonable to expect that professional development delivered to mathematics teachers engage participants in rigorous tasks that require collaboration, discourse, and opportunities to learn through inquiry.

Mathematics Professional Development

Rhine and Bennett (1998) described a project entitled *Integrating Mathematics Assessment* (IMA) that focused on addressing the problems prevalent in many mathematics professional development sessions, namely that such sessions often provide inadequate support in helping teachers effectively teach mathematics according to the standards set forth by NCTM. To address these concerns, IMA used a model that prepared teachers to learn new concepts in two ways. First, as students themselves, teachers learned "important math topics to provide them with the necessary depth to teach their students" (p. 18). Second, as teachers, they learned to "[observe] the use and implementation of various instructional practices and [consider] issues related to students' thinking" (p. 18). These authors argued that teachers need not only be engaged in the types of tasks that students must experience, while considering their students' thinking during the process, but that they must also be instructed through professional development sessions in ways that mirror this type of teaching. Assessing the quality of a mathematics professional development in terms of student achievement, then, would substantiate whether these types of practices involving collaboration and problem solving through rigorous tasks are positively affecting student achievement and are thus important to include in professional development sessions.

In their review of over 600 math professional development sessions reported in the literature, Gersten, Taylor, Keys, Rolfhus, and Newman-Gonchar (2014) focused their analysis on 32 studies aimed at evaluating the effectiveness of a professional development course in light of improvements in student achievement. Of the 32 studies analyzed in depth, five were identified as having met the What Works Clearinghouse standards for evidence. Of the five identified studies, two were found to have statistically significant positive effects with regards to student achievement. These were "intensive math content courses accompanied by follow-up workshops" and "lesson study focused on linear (measurement) model of fractions" (Gersten et al., 2014, p. 2). Both of these studies included opportunities for teacher collaboration, as explained below.

The first of these two studies that found significant student improvement in conjunction with mathematics teacher professional development was conducted by Sample McMeeking, Orsi, and Cobb (2012). These researchers conducted a series of content-focused math professional development workshops for middle grades mathematics teachers (N = 128). The teachers enrolled in either one or two summer courses (offered by a university) and attended subsequent follow-up workshops throughout the school year. Teachers engaged in challenging content-specific curricula and inquiry-focused experiences. The courses focused on mathematical content for 80% of the sessions and pedagogy for 20% of the sessions. After participants completed the initial two- or three-week summer courses, university researchers and school district personnel collaborated to offer follow-up workshops on select Saturdays throughout the school year. These researchers utilized a "quasi-experimental cohort control group design with an added pretest" to analyze measures of student achievement and growth (p.

166). Middle grades students (N = 1,002) of teachers who enrolled in both summer courses and attended the follow-up workshops throughout the school year had significantly improved results in mathematics achievement, as measured by a state-administered assessment, compared to a cohort comparison group of students (N = 1,317).

Sample McMeeking et al.'s (2012) findings indicate that mathematics teachers who receive intensive content-specific instruction and learn "how to use an inquiry-based approach to deliver that content" can positively and significantly affect their students' knowledge of, and success with, mathematics (p. 176). These authors asserted that "engaging in more practice with their own mathematical thinking and learning helps teachers to better understand the struggles of their students and the common misconceptions that they may need to address" (Sample McMeeking et al., 2012, p. 176). These authors further explained that they were not certain precisely what specific aspect of their research design contributed to the success of the students of teachers who completed both courses and engaged in the follow-up workshops. Though the teachers who participated did so voluntarily, thus creating a selective sample, the authors concluded that, in combination, this sustained and intensive professional development series was effective at increasing student achievement.

The second research study reporting positive student achievement gains connected to mathematics teacher professional development focused on the effectiveness of using lesson study with a resource kit for teaching elementary fraction concepts. Lesson study is a practice where teachers engage in regular small group collaborative planning sessions, observe and provide feedback on their peers' teaching, and analyze

and revise lessons accordingly (McGraw, Arbaugh, Lynch, and Brown, 2003). Perry and Lewis (2011) analyzed student achievement for a group of teachers (N = 213) who were randomly assigned to a group having one of these three conditions: 1) lesson study with a resource kit, 2) lesson study without a kit, and 3) "professional development as usual" (p. 1). The groups of teachers met 12 to 14 times over a period of five months during the school year. Utilizing a "two-level hierarchical linear modeling analysis" (p. 1), these authors found a statistically significant positive impact (effect size 0.50) on "students' fraction knowledge" (p. 3) for the students from the teachers' classes who were part of the lesson study group using the resource kit. In the group where teachers used lesson study with a resource kit, participating teachers observed and analyzed fraction lessons and engaged in collaborative planning. Unlike the research by Sample McMeeking et al. (2012), the lesson study participants did not receive direct training from university researchers; rather, participating teachers took turns leading group discussions following the lesson-study cycle outlined in their resource materials. In doing so, these teacher participants engaged in an authentic community of practice, or professional learning community (PLC), which is often a normal part of school and teacher planning (Gersten et al., 2014). Researchers McGraw et al. (2003) support the idea that lesson study can foster true community formation among group members. They recommend that lesson studies happen over "longer periods of time—multiple semesters—so that the groups have opportunities to move beyond the 'community forming' state of development" (p. 8).

The implications of this research suggest that teachers who learn the principles of effective lesson study with the necessary material resources can contribute to improved

student achievement in mathematics (Perry & Lewis, 2011). Considering the sustainability of this model and the similarity to which it mirrors already-occurring school practices is positive in that if teachers are supported and guided in designing and implementing effective lesson studies in their current professional communities, then student achievement can be improved. Understanding that teacher collaboration in solving rich mathematical problems is an important component of effective mathematical instruction (Sample McMeeking et al., 2012), and subsequently an important aspect of professional development for mathematics teacher (Rhine & Bennett, 1998), it is important to further analyze the types of problem tasks that teachers might engage in during professional development sessions.

Problem-Solving Tasks

Smith, Hughes, Engle, and Stein (2009) studied how to orchestrate rich problem-solving tasks with students and stated that once a teacher has determined her goals for instruction and has selected the problem-solving task to administer, she can engage in purposeful planning that will help guide her as students work on the task in class. The process for leading a rich classroom discussion about a given problem-based task (e.g., fraction multiplication or division) involves the teacher predicting what responses students will have as she engages in the problem-based task and then listing all expected responses to the task before the lesson begins. Next, through active monitoring, the teacher guides her students by using effective questions and prompts as they work to solve the problem, noting the strategies that particular students or student groups utilized when solving the task (specifically looking for pre-identified solution strategies, as well as noting when a strategy not predicted is used). Thereafter, the teacher selects specific

students to share their thinking and, purposefully, sequences the sharing of strategies in order to focus learners' attention on the mathematics of the task. The teacher then helps make connections between different student responses, as well as draws attention to the critical mathematics shared (Smith et al., 2009). Thus, by planning ahead for expected responses, monitoring carefully, selecting and sequencing responses in a purposeful manner, and, finally, drawing connections between various solution strategies and the mathematics at hand, teachers can effectively and efficiently structure their classroom problem-solving tasks to highlight students' ideas while ensuring that key mathematical concepts are made explicit.

NCTM (2000) explained that what is critical about mathematical tasks is that they "provide students with the opportunity to engage actively in reasoning, sense making, and problem solving" (p. 20), and it has been shown that when such tasks are used, students can learn and achieve at high levels (Boaler & Staples, 2008; Treisman, 1992). For the concepts of fraction multiplication and division, then, it is important to consider research about student learning and engagement for these concepts, as well as how findings from research about students' understanding of these concepts can transfer to mathematics professional development for classroom teachers.

Learning fraction multiplication and division concepts. Sharp and Adams (1995) conducted work with fifth-grade students on fraction division concepts by presenting them with realistic problems to solve. In their study, they built upon the idea that students best learn new mathematical concepts when teaching is grounded in problem situations that are both realistic and familiar, such as how fraction division relates to sharing tasks in problems about food, like partitioning out left over amounts of

pizza. These findings connect well with current learning theory (Schunk, 2012) and indicate that one of the first tasks teachers must consider is how to create "common experiences" to which students can refer as they connect new learning to familiar fraction elements in the world that they know (Sharp & Adams, 1995, p. 334). Next, according to these researchers, teachers must then consider why it is important that students develop a conceptual understanding of fraction multiplication and division as opposed to only learning the procedural techniques that many adults have only ever learned. These authors suggest that teachers consider their students' lives and what from their everyday life can be used "to relate fractional concepts to what they already know" (Sharp & Adams, 1995, p. 334).

Specific to the national and state standards of mathematics (NGA, 2010; TEA, 2012), students are to initially learn fraction multiplication and division using concrete and pictorial models. For teachers to understand the pedagogical best practices associated with employing such models, it is important for them to develop for themselves a sound understanding of what fraction multiplication and division mean conceptually (NCTM, 2014). According to Li (2008), such understanding is not likely to develop regarding fraction division unless the emphasis of instruction and teaching of such fraction concepts centers on problem solving. For students, introducing contextually meaningful problems encourages them to access what they know about whole numbers and apply similar techniques to solving fraction division problems, including skills like repeated subtraction.

These findings indicate that providers of teacher professional development must create, as did Rhine and Bennett (1998), a series of learning sessions that allows teachers

access to important mathematical ideas through carefully designed problem-solving tasks. Rhine and Bennett (1998) are quick to admit that professional development services cannot possibly provide teachers with all of the skills that they need to learn about fraction concepts and students' thinking, but they assert that well-designed professional development can help to change teachers' orientations and thinking about mathematics. These authors purport that this shift in teacher thinking is critical: math-confident teachers are possibly more inclined to change their teaching practices to challenge students with mathematically rich tasks, which can enhance students' abilities to problem solve and to acquire basic skills. Rhine and Bennett (1998) report that for teachers who are hesitant to change their teaching practices, engaging in reform-based educational experiences through the lens of the learner (one of the goals of the IMA program) can help them see and appreciate the benefit of teaching for conceptual understanding.

Ball (1990) reported in her study of prospective elementary and secondary mathematics teachers' understandings of division that nearly all participants (N = 19) experienced significant difficulties in their attempts to understand fraction division. Ball (1990) assessed prospective teachers' abilities to connect a model or pictorial representation to a non-contextually-based fraction division question. The findings revealed that while many of the pre-service teachers could accurately apply a procedural fraction algorithm to generate the correct answer, very few (N = 5) could actually explain fraction division concepts, even by using a model. The author emphasized that while explaining fraction concepts using a memorized, or previously determined, rule when justifying one's solution may be sufficient within a community of advanced mathematicians, quite the opposite is true in a classroom with students who are just

beginning to construct ideas related to multiplicative fraction operations. Ball (1990) explained that relying on pre-service teachers' mathematical background from college courses is not likely to provide them with deep-enough conceptual knowledge as they transition to the classroom and prepare to help students truly understand fraction division. Considering this, professional development about fraction division concepts should include rich experiences that allow participants, as learners themselves, to grapple with models that connect what they may have memorized about fraction operation procedures to actually understanding why fraction division works the way it does.

Lo and Luo (2012) found that prospective Taiwanese elementary mathematics (non-mathematics major) students performed much better on tasks related to fraction division than did their American counterparts. The one area that it was reported the American teachers were lacking was in their ability to "use word problems or pictorial diagrams appropriately" (Lo & Luo, 2012, p. 497). For prospective Taiwanese elementary mathematics teachers, their content knowledge and deeper understanding of fraction operations seemed to derive from their experiences during their K-12 education (Lo & Luo, 2012). For prospective elementary mathematics teachers from the USA, the authors hypothesized that their lack of understanding might have derived from poor K-12 preparation and that one possible remedy for this problem is a more focused attempt during elementary mathematics methods courses to bring said teacher candidates to the same deep level of understanding as their Taiwanese counterparts (Lo & Luo, 2012). These findings indicate the need for professional training regarding the development of more specialized fraction multiplication and division knowledge, including instruction that uses concept-based pictorial representations. Such experiences can help teachers

reason through students' attempts at solutions to problems and better understand their students' thinking regarding such concepts.

Using virtual manipulatives to learn fraction concepts. In their research about the effect of instruction about fraction concepts using online virtual manipulatives with a class of third grade students, Reimer and Moyer (2005) found that students (N = 19) who utilized virtual manipulatives had significant gains in their conceptual understanding of the fraction concepts of part-whole relationships and comparing fractions, as demonstrated through their ability to draw accurate pictorial representations. These researchers used a paired *t*-test to compare students' scores on a pre- and post-test of conceptual knowledge, noting significant differences in students' conceptual post-test scores, though they did not report effect sizes of these results. Similarly, Suh and Heo (2005) discussed how their use of virtual manipulatives in a fifth-grade classroom supported students (N = 46) in developing an understanding of fraction equivalence and fraction addition. These authors found that the use of virtual manipulatives allowed students to engage in discovery learning and helped to prevent common errors that students typically encounter when adding fractions with unlike denominators. Bouck and Flanagan (2010) described how teachers in the classroom can use virtual manipulatives and argued that continued research should look into the use of virtual manipulatives with students in the classroom.

While the amount of research on the uses of virtual manipulatives is limited, research seems to suggest that students can effectively learn mathematical concepts using virtual manipulatives in much the same way as they can by using concrete or pictorial representations (Reimer & Moyer, 2005). When considering the types of learning

experiences that teachers need to engage in themselves when learning about fraction concepts during mathematics professional development sessions then, it is reasonable to expect that virtual manipulatives could effectively replace concrete and pictorial representations during a professional development on fraction concepts that is offered online. For teachers to be engaged in collaborative problem solving to learn fraction concepts through problem tasks online, however, it would be important for them to interactively manipulate said virtual representations synchronously in real time. Real-time interactions and synchronous manipulation of virtual tools during an online professional development calls for a review of literature about collaborations in the online environment, including research about effective online professional development and, more specifically, synchronous online instruction.

Collaboration in the Online Environment

Herrington et al. (2003) described characteristics of "authentic activities," which they define as the types of tasks that should be the focus of interactions in online learning environments (p. 59). These authors provided a review of the literature about what makes certain activities authentic and offered a list of 10 characteristics defining the "broad design characteristics" of such activities (p. 61). In summary, these tasks are designed to be long-term, project-like units of study that students engage in over time as they investigate the solution to a problem that is ill-defined and that may have multiple and competing solutions. These tasks are complex and relevant to real-world problems and "provide the opportunity to collaborate," where collaboration is to be an "integral" part of the task (p. 61). In their analysis of how students engage in such tasks, these authors described how participants must be willing to suspend their disbelief in the

components of the tasks that are merely representations of other things and, instead, become "immersed in the learning context that has been created for them" in the online environment (p. 63). While the examples provided by these authors are not specific to problem-solving tasks in a mathematics classroom or professional development, they highlight the fact that genuine collaboration among participants, as well as immersed engagement with representations in the online platform, are important components, among others, of effective online task engagement.

On a similar note, Hiltz (1998) discussed the importance of student collaboration in online learning environments and sought to answer the question of whether or not collaborative online learning experiences are better than individual web-based activities where students simply interact with content in isolation. Hiltz (1998) provided a definition for collaborative learning, explaining that it "refers to instructional methods that encourage students to work together on academic tasks" and that it is "learner-centered rather than teacher centered," where "knowledge is viewed as a social construct, facilitated by peer interaction, evaluation and cooperation" (p. 4). After her review of relevant literature and studies investigating the effect of collaborative learning in online environments, Hiltz (1998) concluded that "the most basic premise from which all online teaching should begin is that the goal is to build a learning community and to facilitate the exchange of ideas, information, and feelings among the members of the community" (p. 7). Finally, in her analysis, Hiltz (1998) explained that this type of online learning environment is not necessarily conducive to large groups of students enrolling and completing a course of study simultaneously because the instructors need to interact with and among students, similar to how the students should interact with one another.

In support of online collaboration, Reeves et al. (2004) provided a description of the ideal online learning environment. In this ideal environment, collaborative interactions are an integral part of instruction. In conjunction with this ideal environment, these authors lamented how few online instructors are utilizing the technology available through online "course management systems [that] can support the collaborative engagement of students in solving complex problems or undertaking authentic tasks" (p. 60). These authors expounded upon the difference between students learning "from" technological tools and students learning "with" such tools (p. 61). They explained that when afforded the opportunity, learners can become "designers using media and technology as tools for analyzing the world, accessing and interpreting information, organizing their personal knowledge, and representing what they know to others" (p. 61).

Reeves et al. (2004) call for more research about the "learning-with pedagogy in online collaborative environments," expressing hope that such research will "yield guidance about online scaffolding and other support strategies for instructors seeking to foster collaborative learning" online (p. 61). In recognition that much research in the field of online instructional design has already been conducted, these authors further purported that online instructors and online course designers should consider whether they are utilizing the already-available research regarding the development of "online collaborative learning environments" (p. 62). Understanding that collaborative learning is a highly important component of effective online instruction, a look at research regarding online teacher professional development and the nature of synchronous online collaborations is in order.

Online Teacher Professional Development

A plethora of researchers have recently studied what makes online professional development for teachers effective (Chauvot & Lee, 2014; Dede et al., 2009; Ellis & Kisling, 2009; Hebert, 2007; Ostashewski, Moisey, & Reid, 2011; Signer, 2008; Surrette & Johnson, 2015). While each of these researchers provided different lists of essential or key components for what is recommended when designing an effective online teacher professional development course, several general themes are consistent among all of their suggestions, as outlined below.

- Teachers need to have opportunities for discourse in the online community, and the more opportunities they have to engage in discourse and the sharing of ideas via various media, the better (Hebert, 2007; Ostashewski et al., 2011). This discourse should include opportunities for participants to both reflect and collaborate as they learn new content (Signer, 2008).
- Teachers participating in an online professional development should feel a sense of trust with the other members of their course, as well as with their instructors (Hebert, 2007; see also Chauvot & Lee, 2014). This can be fostered by allowing for variations in the ways in which participants communicate (e.g., privately or publicly, in writing or with audio), as well as flexibility in how much or how little they are expected to engage (Hebert, 2007).
- Online professional development courses should offer dynamic and exciting content that stimulates and motivates participants, including tasks that require problem solving and promote higher-level thinking (Ellis & Kisling, 2009). This content should be focused specifically on teachers' classroom practice and should

allow participants the opportunity to test out ideas with their peers online before introducing new concepts to students in the classroom (Ostashewski et al., 2011).

- The course facilitator should guide the participants in the online community through direct interaction and active monitoring (Signer, 2008). This should include making technical support available as needed (Hebert, 2007) and providing assistance or redirection to participants through private emails and conversations, whereby addressing individual concerns and answering questions (Signer, 2008).
- Online professional development courses should be designed with flexibility in mind, understanding that adaptations and modifications will likely occur as the course proceeds (Hebert, 2007). These changes might include variations in how participants and instructors meet together (e.g., asynchronously, synchronously, in small groups, or as a large group), understanding that ever-changing online technologies can support the types of interactions that are often valued and considered key to successful face-to-face professional development trainings (Chauvot & Lee, 2014).

Synchronous Online Interactions

Marrero et al. (2010) discussed their use of one-hour long science courses delivered synchronously online across the United States where a variety of educators (N = 248, mostly science teachers) interacted through conversations and questions with science experts in the field. In their study, these researchers used qualitative data collected from a survey that was completed by part of the teachers (N = 59) who attended the synchronous sessions. The most significant finding from their qualitative analysis of

the data was that teachers highly valued the opportunity to interact authentically with other educators through discussions about their learning, lives, and experiences. Teachers also valued that they could implement their learning from the online courses in their classroom practices. Other important findings included that teachers valued the immediate feedback they could receive during the synchronous sessions from the instructors and guest lecturers, as well as the flexibility the course provided in that each session was offered twice on the same day. These authors reflected that the next step to evaluating their online courses for effectiveness would be investigating teachers' content knowledge gains and subsequent changes in teaching practices (Marrero et al., 2010).

In a similar way, Woodcock, Sisco, and Eady (2015) utilized a synchronous online learning platform that allowed for talk, text, video and small-group interactions while instructing a group of 53 undergraduate students in their education program. These researchers administered a survey to participants in order to investigate the extent to which the synchronous environment could facilitate learning opportunities. The survey results indicated that these participating pre-service teachers found that the online learning experience was beneficial and offered them authentic learning opportunities.

Considering another example from higher education, Yamagata-Lynch (2014) conducted a self-study of her experiences as a facilitator of a higher-education course about online learning. She conducted the course in a 50/50 model that blended asynchronous activities with synchronous meetings online. Through her study and reflections of her blended course, she learned that participating students (N = 8) came with varied levels of expertise regarding online technologies and took time to develop their new identities as online learners. Further, she learned that the synchronous

components of the course contributed to the students' abilities to build a strong sense of connection among themselves, developing what she termed a "stronger sense of social presence" (p. 203). Last, she learned that the experiences of the participants while engaging in the online course were affected by her "ability to bring a sense of cohesion and structure in the synchronous learning environments" (p. 204). She indicated that this means instructors in synchronous online courses must be deliberate about how they engage participants and conscientious about what structures they put in place to moderate participants' learning. Yamagata-Lynch (2014) indicated that future research might benefit from investigating how structured learning experiences can be provided in meaningful ways within flexible, synchronous online learning platforms.

Synchronous Online Mathematics Professional Development

Cady and Rearden (2009) provided professional development to mathematics teachers (N = 8) in rural schools by setting up four semesters of courses using an online, synchronous platform. Groups of teachers met in person at individual school locations and then collaborated synchronously with other groups of teachers in the program who were located at distant geographic locations. Participating teachers utilized both physical and virtual tools while engaging in the sessions, including the chat features of the online, synchronous platform, a shared whiteboard, web cameras, agree/disagree buttons, and app-sharing technologies. Participants also had access to asynchronous features of the platform, including a password protected course website where documents were posted and discussed, and assignments and discussion-board threads were housed.

In designing their professional development courses, Cady and Rearden (2009) elected to follow best practices from the literature. The researchers designed the course

following constructivist principles, which included opportunities for the teachers to interact with both the environment and other teachers in their construction of knowledge. Because teachers signed into the online platform in cohorts of three to five, and then connected synchronously with other groups of teachers (number of cohort groups not reported), participants were able to "interact with the material and with each other" (Cady & Rearden, 2009, p. 283). These researchers explained that "an online environment has the potential for teachers and mathematics educators to exchange ideas," which they assert can lead to true communities of practice (p. 285). These researchers expressed belief that this type of community of practice was essential in that it "provide[d] an ongoing forum for teacher learning" (p. 285).

In order to ground their online courses in practice, Cady and Rearden (2009) used 1) a standards-based curriculum to help focus teachers' attention on conceptual understanding versus procedural understanding, 2) videos to showcase mathematics instruction and help "expand teachers' understanding of the link between instruction and students' mathematical understandings", and 3) student work samples that focused on helping teachers think more about their students' math thinking (p. 283). Course assignments required participating teachers to choose between either a cognitively-guided-instruction (CGI) framework or a lesson-study framework. Through these frameworks, teachers investigated how students learn a given math concept and utilized key findings from research when teaching the concept to their own students. Through course assignments, participating teachers were encouraged to focus all decisions about teaching and pedagogy on student learning. Course participants planned and taught lessons and then met to reflect on what their students really understood from each lesson

after it was delivered. Using these reflections as a driving force, these teachers then refined their lessons for future use.

While these researchers had hoped to see an increase in teacher content knowledge, quantitatively no such improvement was significant when measured by teachers' pre- and post-test scores on the "Diagnostic Teacher Assessments in Mathematics and Science (DTAMS) for middle school" (p. 286). Through qualitative analysis of interactions and statements made during the courses by participants, these researchers report that teachers' pedagogical content knowledge did increase. One example provided was with participants' ability to teach and explain fraction and proportional reasoning concepts, which was evidenced in one session as "teachers drew pictures to solve word problems involving division" coming to better understand the "importance of identifying the whole in a fraction problem" (p. 289). Even still, the researchers speculate that actual classroom observations would have been a better tool to measure whether or not teachers' pedagogical content knowledge truly did increase (Cady & Rearden, 2009).

Finally, Cady and Rearden (2009) explained how the online course instructors and participating teachers felt frustrated about the limitation imposed upon them in "not being able to see others using their manipulatives in the online environment" (p. 295). In particular, the course instructor commented about the difficulty faced in attempting to "model the reform-based pedagogical approaches in the online environment" explaining that interactions were limited to one-on-one exchanges with the instructor and a student, and that it was not possible to quickly "tour the room" to see the students working on the problems (p. 295).

During a more recent study, Francis and Jacobsen (2013) engaged 13 teachers in a collaborative online teacher professional development over a series of four sessions where participants were given mathematical problems to solve together online. During the first session, all participants and facilitators experienced technology problems and learning of the prepared math problem was greatly limited. Their second session was better in terms of technology, but the task they chose did not facilitate the type of collaborations for which they had hoped. As a result, the solution to the procedurally based problem presented online was simply *told* to the participants by an instructor. During the third session, the instructors designed a mathematical task that required minimal writing and participants simply had to draw lines on the whiteboard of the course to contribute to solving the problem. During this session, the course instructors discovered that by assigning different colors to each participant, identity of who was contributing to the solution of the problem was established. During this third session, participants actually solved the problem together and a rich mathematical conversation ensued. The authors reported that during session three the use of technology focused on learning mathematics, as opposed to merely learning how to use the online technologies. In the fourth session, the instructors pre-loaded an image onto the screen that acted as a template where participants could play a game with a partner. During the game, participants used lines to strike through each circle in a given set of circles in an effort to be the last person to eliminate a circle from the set. This specific game, identified as "the task of Nim" by these researchers (p. 16), introduced concepts of logic, strategy, and problem solving. This game facilitated collaborative problem solving, and the instructors deemed this session successful because the participants collaborated to solve problems.

Francis and Jacobsen (2013) reflected upon the conditions they believed existed that helped to optimize the online learning of mathematics during their course. These included 1) rich, "non-routine" tasks that allowed for the easy use of drawn manipulatives (i.e., colored lines), 2) established identities among participants in a trusting environment where they knew it was okay to make mistakes in their attempt to solve problems, 3) easy navigation of the online technologies, and 4) ample time for participants and instructors to converse and interact with one another, which led to the development of an online learning community (p. 329). These authors assert that the most important finding from their work was the effects that the chosen mathematical task had on the success of each session. They concluded that a well-chosen task affected 1) how participants used the online technologies, 2) the subsequent learning that resulted from the use of the online technologies, and 3) the nature of the participants' collaborations as they worked together to solve mathematical problems. These authors found that presenting problems where easy drawings could be used to solve the problem led to more interaction in the online platform, and that by using non-routine tasks, authentic collaborations could take place as participants developed problem-solving strategies together online. Francis and Jacobsen (2013) recommended that future research include follow-up interviews with participating teachers investigating "whether the professional learning transferred to the classroom," as well as "how online teacher professional learning can inform new understandings and change mathematics teaching" (p. 338).

While recognizing that these authors were able to facilitate online interactions among participants as they solved novel problems, it could be considered concerning to some that they recommended that future studies interested in utilizing online tools to

teach mathematical concepts consider how to pose problems that require only limited drawings or manipulations. While this advice may seem prudent so that technology problems are less likely to occur, it may also not be sufficient for online courses where the goal is to help participants collaborate in order to learn mathematics interactively using virtual manipulatives and pictorial models. Francis and Jacobsen (2013) created new problems for participants to solve together online after they began experiencing technological difficulties, like when they struggled to write neatly on the online whiteboard. While this may have been acceptable for their particular sessions, changing the types of problems presented to participants is not a luxury that many creators of professional development have, especially when a series of online courses needs to be developed for a specific concept.

Conclusion

Collaborative learning is a critical part of effective professional development for teachers (Darling-Hammond & McLaughlin, 1995; Desimone, 2009; Hiltz, 1998; Johnson, 1981). Mathematics teachers especially benefit from this type of learning because they are expected to use problem tasks that require collaboration (NCTM, 2000, 2014; Van de Walle et al., 2013), which is especially advantageous when representing fraction multiplication and division concepts with concrete and pictorial models (Li, 2008; Lo & Luo, 2012; Rhine & Bennett, 1998; Sharp & Adams, 1995). Through collaborations and intensive study of content at a conceptual level during professional development courses, teachers can positively affect their students' achievement in mathematics (Perry & Lewis, 2011; Sample McMeeking et al., 2012).

Researchers in the field of online learning believe that participants need genuine opportunities to interact and collaborate while solving problems (Herrington et al., 2003; Herrington et al., 2004). An ever-increasing number of new and emerging technologies is available to deliver online professional development to teachers (Dede, 2004; Dede et al., 2009), and these technologies make professional learning opportunities more accessible (Desimone, 2009). A synchronous online learning platform is an example of one such technology that can effectively and efficiently allow teachers to access content-specific materials and collaborate in discourse with other educators (Ellis & Kisling, 2009; Ostashewski et al., 2011). Teacher interactions that are facilitated through a synchronous online platform can lead to the formation of true communities of practice wherein authentic learning can take place (Hebert, 2007; Signer, 2008). Researchers have used synchronous online platforms to conduct teacher professional development for a variety of topics (Marrero et al., 2010; Woodcock et al., 2015; Yamagata-Lynch, 2014). Some mathematics researchers, however, have felt limited by their inability to see participants interact with virtual manipulatives while solving problems during synchronous online sessions (Cady & Rearden, 2009). Still others have felt restrained to use only mathematical problems requiring minimal drawings or virtual manipulations, thus limiting the types of problems they presented to teachers during their professional development sessions (Francis & Jacobsen, 2013). These limitations highlight that, for mathematics educators engaged in solving problems during an online professional development, new ways of facilitating collaborative interactions using virtual tools need to be explored.

Chapter III

Methodology

This study was designed to explore the nature of learning, specifically the patterns of communication and interaction between and among participants and their course instructors, during a series of synchronous online professional development sessions where these individuals discussed and solved fraction multiplication and division problems using virtual tools. This chapter 1) provides background information related to this study, 2) lists the primary and secondary sources of data that were used to answer this study's research question, 3) describes the data collection methods used during this study, 4) summarizes the qualitative methodological techniques used to analyze these sources of data, 5) reviews the validity procedures used to address researcher bias, 6) outlines the procedures followed to establish trustworthiness of this study's findings, and 7) lists the limitations of this study's research design.

Background of the Study

Through a series of five synchronous, online professional development sessions, three fifth-grade mathematics teachers learned about fraction multiplication and division concepts by solving and discussing problems using virtual tools. These five sessions focused on both content and pedagogical best practices and took place over a period of about two months, totaling about 15 hours of professional development. During these synchronous online sessions, participating teachers collaborated using virtual tools to develop their own knowledge of fraction concepts specific to multiplication and division. They also planned for, delivered, reflected upon, and revised their own classroom lessons. Course participants took a pre- and post-test on fraction multiplication and

division content; results suggested that their ability to solve and represent these fraction concepts increased, though this data was only analyzed descriptively and not statistically (Udy, 2016). In addition, results from qualitative analysis suggested that teacher implementation of the practices taught in the online professional development occurred to some degree for each participating individual. Moreover, teachers attributed their use of a particular practice to their involvement in the online professional development (Udy, 2016).

The primary researcher of this study led all five sessions, and an assistant course designer collaborated with the researcher to design the online sessions and the presentation of course materials. The assistant course designer is a mathematics teacher development specialist from the participants' school district with expertise both in fraction multiplication and division as well as online course design. He participated in the first two online sessions where teachers solved problems. I, as the primary researcher of this study, was employed as the elementary mathematics curriculum manager of the participants' school district during the time when this research project was implemented.

The current study investigated the nature of learning during three of the aforementioned synchronous online sessions. These three sessions focused specifically on opportunities for the participants to solve and discuss fraction multiplication and division problems in the online environment. The teachers' efforts in solving and discussing such problems differentiated these sessions from the other two sessions in that the other two sessions focused primarily on lesson planning and pedagogical best practices.

Multiple online applications were used during these professional development sessions to support teacher learning and engagement. First, a secure online course page in the blended-learning platform available from the teachers' school district was used to house course materials, asynchronous discussion boards, and links to live session presentations (see Figure 1).

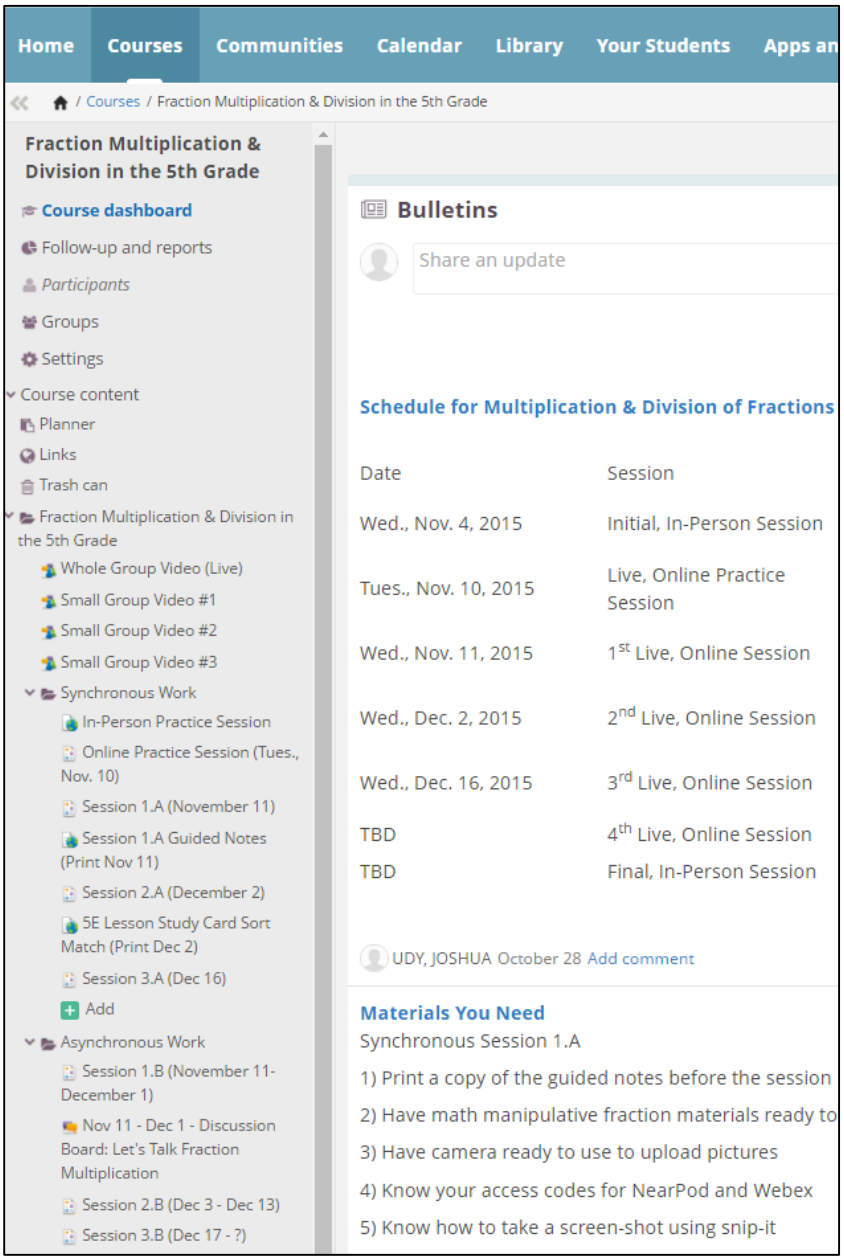


Figure 1. Screenshot of secure online course page.

Second, an online application entitled *nearpod* was used during the synchronous sessions, wherein content-specific presentations were uploaded and participants could "join" to "follow along" as informational slides, quizzes, videos, and response screens were displayed to them by the course instructors in real time (see Figure 2).

Figure 2. Screenshot of interactive *nearpod* response board.

Third, *Google Presentation* was used as a platform to collaboratively and interactively move virtual manipulatives in real time during the online sessions. Images of virtual manipulatives were loaded into slides in *Google Presentation*, participants would virtually "enter" the document through the sharing features of *Google*, and then participants and instructors would manipulate the picture images in real time to solve different problems (see Figure 3).

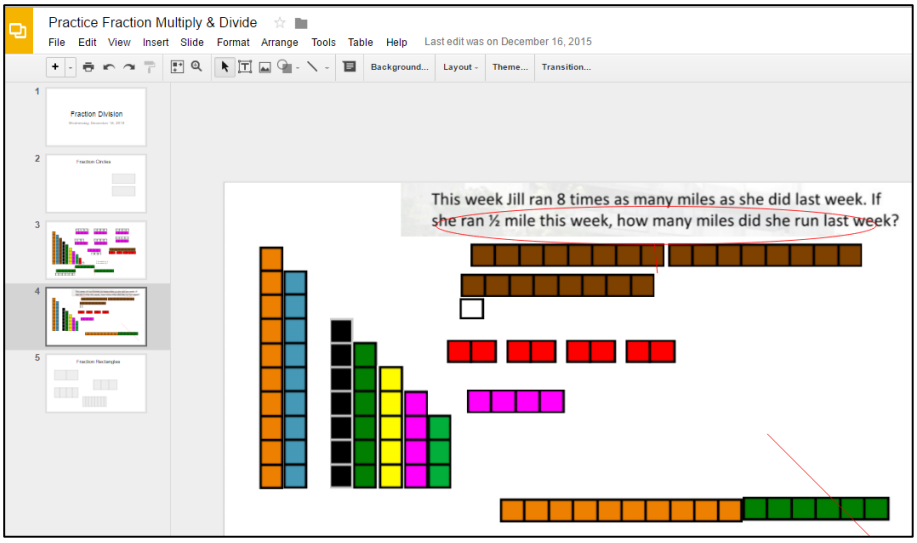


Figure 3. Screenshot of interactive *Google Presentation* slide.

Within the *Google Presentation*, three types of virtual manipulatives were embedded for participants to use while representing problems: fraction circles, fraction rectangles, and Cuisenaire rods. Cuisenaire rods are colored rods of graduated lengths. There are 10 different rods, the shortest of which is one unit in length (the white rod), and the longest of which is ten units in length (the orange rod). These colored rods can be used to model a variety of different fraction problems, in that different of the rods can be identified as the "whole" and then, when comparing other rods to the identified whole, fractional amounts can be modeled.

Fourth, *Cisco WebEx*, a webinar platform primarily used in the teachers' school district by the Informational Technology (IT) department, was used to host the actual synchronous online sessions. This webinar platform allowed the teachers and course instructors to use web cameras, microphones, speakers, and chat boxes to communicate together in real time, and the webinar was used at the same time as the three aforementioned applications. The webinar platform allowed for interactive annotations, image and file uploads, and screen sharing of its "whiteboard." Each webinar was

digitally archived, meaning that all audio and visual components of each session were video recorded and saved electronically as video files in a secure online server. These recordings included the virtual interactions in which the teachers engaged while using the platform's whiteboard, as well as the screen-captured interactions of the movements of the virtual manipulatives that were loaded into the *Google Presentation* slides. Because *Google Presentation* displays a "name tag" when someone else in the presentation manipulates an element or draws, these "name tags" were also digitally archived in the webinar recording and provided evidence during later analysis regarding who was drawing or manipulating elements during the time when problems were being solved collaboratively (see Figures 4 & 5).

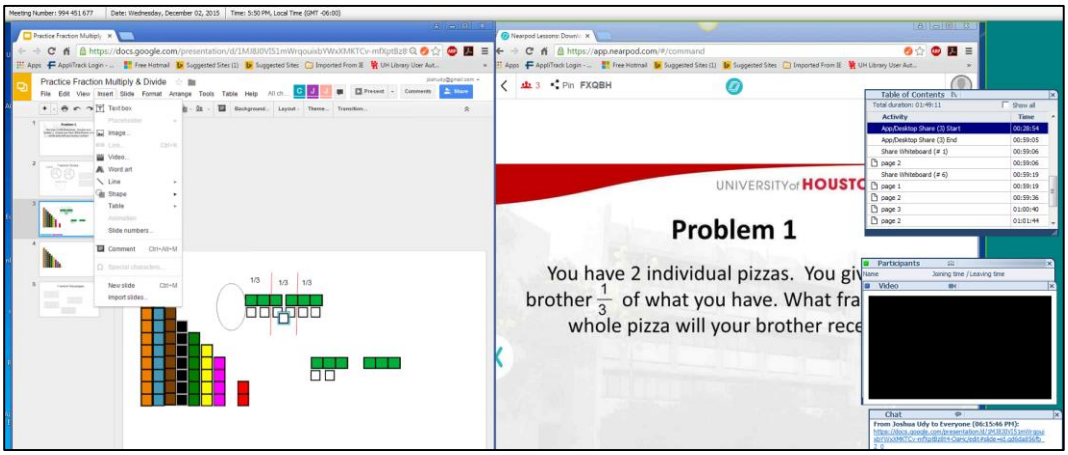


Figure 4. Screenshot of webinar platform, *nearpod*, and *Google Presentation* slides.

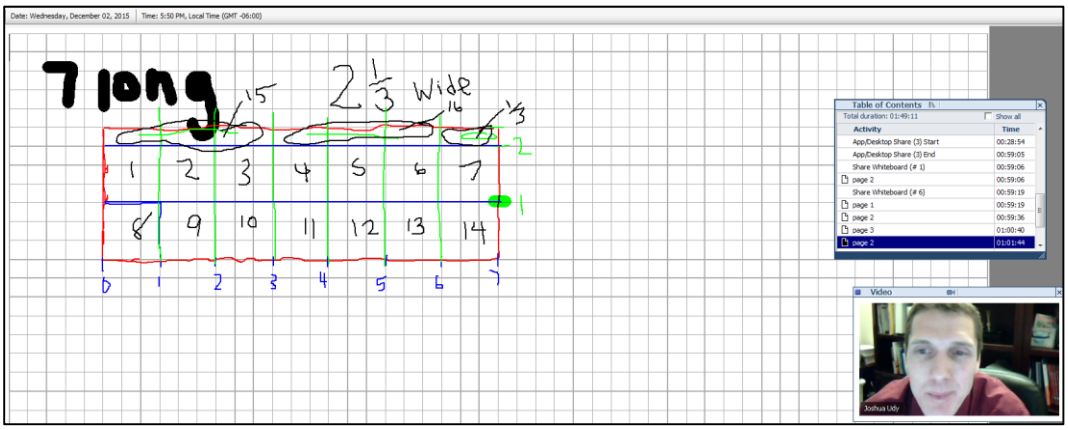


Figure 5. Screenshot of webinar platform and interactive drawings on whiteboard.

As the instructor of the course, I had initially planned for the teachers to solve problems independently and then upload and share pictures of their work for whole-group synchronous discussion using *nearpod*. During the first session, we followed that format for sharing, and participants were successful in uploading pictures of their work. During the time in the first session when participants discussed their solution strategies, we either viewed their work through *nearpod* or we pasted images of their work on the webinar's whiteboard and utilized the webinar's annotation tools to highlight certain aspects of their different representations. These real-time drawings seemed to enhance our ability to understand one another.

While the conversation among course participants seemed authentic during the sharing of their work in the first session, I was dissatisfied that the teachers were not working together to solve the problems being presented online. As a result of this dissatisfaction, I discussed with the assistant course designer—during an audio-recorded call using *Google Voice*—what we could do to facilitate interactive collaborations among participants so that they could solve problems together. I felt that creating a venue for this type of interaction to occur was an important problem to resolve because collaborating while solving problems as a group was one of the strategies participating teachers were learning about during the pedagogical aspects of the course. In short, I was asking teachers to utilize collaborative groupings in their own classrooms and, as such, I felt like our online sessions needed to model this practice, as is recommended by researchers in the field of professional development (Darling-Hammond & McLaughlin, 1995).

The discussion held between the assistant course designer and I eventually led to the creation of the aforementioned *Google Presentation* where virtual manipulatives were inserted on presentation slides that could be manipulated collaboratively, in real time, by members of the group. We also determined how to allow all participants the ability to annotate on the whiteboard of the webinar platform at the same time. Additionally, we learned how to paste images, such as grid paper, into the online whiteboard, thus allowing the participants to more easily solve fraction area problems involving square units. Through screen sharing capabilities, the interactions in both the webinar platform and in the *Google Presentation* document were audio and video recorded and archived.

Summary of Background

This study was designed to explore the nature of participating teachers' learning experiences while engaged in three of five online professional development sessions, specifically the three sessions that focused on problem solving to learn new content. The last two of these three sessions provided an opportunity for participants to collaborate synchronously online to solve problems using virtual manipulatives and interactive annotation tools in the webinar platform. This research project was designed to investigate the nature of these two learning experiences and the learning experiences of the first session where synchronous collaborative problem solving did not occur. This study was part of a larger study that investigated changes in participants' mathematical content knowledge regarding fraction multiplication and division concepts and their uses of recommended pedagogical strategies (Udy, 2016). A report providing information about the other components of the larger study is available online at this website: http://joshudy.weebly.com/scholarship.html.

In order to provide a summary for the reader, Figure 6 shows an overview of what the teachers experienced as they engaged in this professional learning opportunity. This overview includes information about the change in course design, which occurred after the first session that allowed for collaborative, synchronous online problem solving in subsequent sessions.

November 4, 2015
- In-person overview of online course webpage, *nearpod*, and *Cisco WebEx*
- In-person paper/pencil, pre-test assessment

November 10, 2015
- 1-hour online practice session using course webpage, *nearpod*, and *Cisco Webex*

November 11, 2015
- Archived 2-hour online session, meeting #1
- Independently solved (or attempted) three fraction multiplication problems

November 11, 2015
- Post-session, recorded phone call between researcher and assistant course designer
- Generated idea to use *Google Presentation* and annotation in *Webex* to allow for synchronous online collaborative problem solving

November 25, 2015
- Archived 1-hour meeting between researcher and assistant course designer
- Practiced using *Google Presentation* slides and *WebEx* to manipulate pieces and draw together online

December 2, 2015
- Archived 2-hour online session, meeting #2
- Collaboratively solved two fraction multiplication problems

December 9, 2015
- Archived 1-hour online planning session, meeting #3
- Planned fraction multiplication lesson for classroom use

December 16, 2015
- Archived 2-hour online session, meeting #4
- Collaboratively solved seven fraction division problems

January 4, 2016
- Archived 1-hour online planning session, meeting #5
- Planned fraction division lesson for classroom use

January 26, 2016
- In-person paper/pencil, post-test assessment

March 31, 2016
- Recorded interview between researcher and assistant course designer

August – September 2016
- Recorded interviews between researcher and each course participant

Figure 6. Overview chart of online course and interviews.

Research Question

For teachers engaged in a synchronous, collaborative online mathematics professional development about fraction multiplication and division concepts, what patterns of communication and interaction emerged?

Sources of Data

Primary Sources

Three primary sources of data were used to investigate the answer to the research question of this study. First, video-archived online sessions served as the primary record for this study (Carspecken, 1996). These archived sessions showed interactions between myself, as the course instructor, teacher participants, and—on occasion—the assistance course designer. Second, one-on-one interviews that I conducted with each of the participating teachers were used as the primary source of dialogical data (Carspecken, 1996), which is data derived from interpersonal dialogue generated during participant interviews. Third, a one-on-one interview I conducted with the assistant course designer was used as a dialogical data source to triangulate the findings from the participant interviews.

Video-archived online sessions. Three, two-hour audio- and video-recorded online professional development sessions served as one of the primary sources of data to answer the research question of this study. These video-archived sessions provided data about how participating teachers communicated and interacted in real time while solving problems. These archived sessions also provided information about how the assistant course designer and I, as the course instructor, communicated and interacted with participants. These data sources provided information about 1) similarities across all

three synchronous, problem-solving based sessions, 2) similarities between the final two sessions where interactive collaborations and the use of virtual online tools and drawings were supported by the online environment, and 3) differences between the first session, where participants solved problems independently and then discussed their work in as a group, and the final two sessions, where participants used virtual tools to collaboratively solve the problems presented.

Participant interviews. Approximately eight months after the post-test assessment, I conducted a semi-structured, 90-minute interview with each participating teacher. These three interviews constituted the second primary source of data used in answering the research question of this study (see Appendix A for interview protocol). Following Carspecken's (1996) structure for generating data based on dialogue through interviews, each interview was audio recorded and included questions designed to elicit reflection from each interviewee about the nature of her experiences during the online course, specifically her interactive collaborations with the other course participants and instructors while using virtual tools to solve and/or discuss problems. At the beginning of her interview, each interviewee was asked to share 1) what she remembered about the time when we were together online and we solved and discussed problems, 2) whether she thought that it was possible that a change could have taken place in how we discussed and solved problems over the course of the three problem-solving sessions, and also her reflections on the idea of change related to our time spent solving problems, 3) anything that she liked or disliked about the time when we solved and/or discussed problems, and 4) what she remembered about the different types of online tools that were used during the course while solving problems. Connected to this fourth query, each interviewee was

also asked to share about what she liked/disliked about these tools, her opinion about the advantages and disadvantages of these tools, and whether she would have liked for anything to have been different regarding the ways in which we used these tools.

Each interview also included interpersonal process recall (Kagan, 1984, see also Carspecken, 1996), wherein each interviewee viewed specific sections of video-recorded interactions from the online course and was then asked to provide further reflection regarding the nature of her experiences during those specific moments in time. The primary purpose of using interpersonal process recall was to develop additional dialogical data from the interviewees regarding the nature of their online communications and interactions during moments when they were discussing and/or solving problems using virtual tools (see Carspecken, 1996). In her report, Rowe (2009) reflected upon her use of a similar methodological process entitled video-stimulated recall when conducting semi-structured interviews with participants where there were possible issues with power relations among the interviewees. In her interviews with teachers and students, Rowe (2009) utilized stimulated recall with videos in an effort to "involve teachers and pupils on equal terms" so that "they felt they had some ownership in their contributions" (p. 426).

As I was the course instructor of the online sessions for this study, I felt as though utilizing interpersonal process recall could also help to equalize issues related to power that might have existed during my interviews with participating teachers. My intent in showing video clips of our interactions during the online course was similar to Rowe's (2009) intents, in that I wanted to stimulate the teachers' ability to "recall and develop their ideas" with the hope that the video clips would act "as a springboard to further

discussion" about the times when they used virtual tools to solve and discuss problems (p. 426). Rowe (2009) found that by using video-stimulated recall, interviewees were able to provide "a valuable 'insider' perspective on the recordings" (p. 434) and have "an opportunity to raise their own ideas and concerns" (p. 435) about the topic in question—as opposed to just talking about what the researcher had planned to discuss. In her final remarks, Rowe (2009) revealed that video-stimulated recall allowed for "the sharing of control over the topics discussed," which generated a "feeling of empowerment" for the interviewees (p. 436). This information resonated with me as I considered that the goal of my interviews with participating teachers was to allow them to share their thinking about the patterns of communication and interactions that took place during the times when we used virtual tools to solve and discuss math problems while engaging online together. Thus, interpersonal process recall was used in my interviews with participants because I wanted them to have a feeling of shared control and empowerment over what we discussed.

When conducting the participant interviews, each teacher was initially asked to recall, on her own, a specific problem that was solved and/or discussed online. After discussing her thoughts about this recalled problem, each interviewee was then shown components of up to five pre-selected video clips. The variations in the number of pre-selected clips shown to each participant depended on the time remaining during the interview, and fewer clips were shown when time was running short. During each instance of interpersonal process recall, each interviewee was asked to share 1) her classification of the given problem as either easy or hard and why, 2) her thoughts about how the problem was solved and/or discussed, 3) her thoughts about the online tools and

technology that was used to facilitate how the problem was solved and/or discussed, and 4) her classification or ranking of the value of the time that was spent solving and/or discussing the problem.

The first four pre-selected clips were chosen based on whether the particular problem was considered by myself as easy or hard. In order to determine problem difficulty, I analyzed the participating teachers' pre-test assessment scores to see if they had been successful at solving and representing the problem before our online sessions began. I also considered my personal experiences in having used these problems in prior professional development trainings and the typical reactions I had seen over the years when working with teachers on representing, solving, and discussing these problems. The purpose of my asking each participant whether she classified each problem we viewed together during the interview as easy or hard was to triangulate whether or not my classification of each problem was accurate.

In addition to selecting problems based on whether I considered them easy or hard, I also determined which easy and hard problems to select based on the way in which I had asked the teachers to engage in solving it. For each type of problem (i.e., easy and hard), I found 1) a clip where I had asked the teachers to solve the problem independently and then share and discuss their work after taking time to work on the problem alone, and 2) a clip where I had asked them to use interactive virtual tools to collaborate and solve the problem together in real time. This resulted in my selection of four different clips: an easy problem solved independently, an easy problem solved collaboratively, a difficult problem attempted independently, and a difficult problem solved collaboratively. As a point of clarification, there was only one difficult problem

presented to the teachers to solve independently. This problem was presented during the first session, and each participating teacher was, respectively, unable to determine the problem's solution by using a concrete or pictorial representation (Udy, 2016).

As was explained above, during the interview, before being shown any one of the pre-selected sections of the video archive, participants were asked to reflect and then share about a specific problem that they remembered solving. Two participants reflected upon a difficult problem that had been solved collaboratively using virtual graph paper, which had been pasted onto the webinar's whiteboard screen. When the teachers solved this problem during the online professional development, they used colored annotations on the webinar's whiteboard to represent and solve it. During the interviews, I happened to have this video clip available for viewing, and, consequently, showed it to the two participants who described it during their initial reflection. As a result, during their respective interviews, these two participants viewed an additional clip that showed the teachers collaborating to solve a second difficult problem.

The fifth video clip I selected showed me interacting with the teachers when they had been asked to work collaboratively to solve a problem by using virtual drawings on the webinar's whiteboard. I selected this clip because it was one of several instances where I had created a majority of the drawings on the whiteboard for the teachers. I showed this clip during only one of the three interviews and used it to ask the questions in the interview protocol related to my role as a course instructor (see Appendix A). In the two interviews where I did not show this clip, I referred to times when I had interacted with the teachers as they solved problems together and had participated in drawing parts of a given problem's representation on the whiteboard screen for them.

After referencing these instances, I then asked the other two participants the questions in the interview protocol related to my role as instructor in the course (see Appendix A).

Interview with assistant course designer. Approximately three months after the conclusion of the online professional development, I conducted a 40-minute audio-recorded interview with the assistant course designer. This interview was initially conducted as part of a class assignment for a qualitative methods research course in which I was enrolled. Upon later reflection, I realized that certain parts of this interview could be helpful in answering the research question of this study. As a result, I asked for and obtained consent from the assistant course designer to use the transcription of the interview as a data source for this study. During this interview, interpersonal process recall was utilized with the interviewee (Kagan, 1984; see also Carspecken, 1996), who was asked to reflect on his experiences observing the teachers as they utilized virtual tools to solve problems collaboratively. The video clip that I selected for use in this interview showed a segment of the first time the teachers collaborated online to solve a difficult problem together using virtual tools. This video clip was not one that I later selected for use during the interviews with participating teachers, but it was the same problem that two of the teachers recalled on their own when asked if there was a specific problem that they remembered.

The protocol used during my interview with the assistant course designer differed from the interview protocol that I used with participating teachers (see Appendix B). This interview provided additional perspective into the patterns of communication and interaction between and among the teachers and course instructor during the times when problems were solved and discussed using virtual tools. The purpose of using this

interview as a primary source of data was to triangulate findings from the interviews with the participating teachers.

Secondary Sources

Three secondary sources of data were used to provide additional context to the analysis of the primary data of this study. First, an audio-recorded phone conversation between the assistant course designer and me was analyzed to provide insight into the change in design that occurred after the first session of the professional development course. Second, a video-recorded online practice session between the assistant course designer and myself was analyzed to provide additional context to patterns of interaction and communication that emerged during the online synchronous collaborations. This archived practice session was also reviewed in order to provide context about the change in the structure of the course. Third, written reflections provided by the participants throughout the duration of the course were analyzed and used to add to the trustworthiness of the study.

Audio-recorded phone conversation. Immediately following the conclusion of the first online session, I engaged in a 25-minute, audio-recorded phone conversation with the assistant course designer who had attended the first online session. During the phone call, we debriefed about the session, and I expressed concern about the fact that the participants did not collaborate to solve problems, as they would have been able to do if sitting face to face. As the conversation progressed, we discussed how the whiteboard in the webinar platform could be used to facilitate collaborative interactions, and we generated the idea of using *Google Presentation* to support interactive, real time use of virtual manipulatives. The content of this phone call was analyzed to provide insight into

the change in course design that occurred after the first session of the professional development.

Video-recorded online practice session. After the phone conversation described above, I conducted a one-hour online practice session with the assistant course designer. During the practice session, we utilized the whiteboard in the online webinar platform and *Google Presentation*, and we practiced collaborating synchronously in an attempt to simulate how the teachers would interact while solving problems together in real time during the next online session. This video-recorded practice session was analyzed to provide additional context about the patterns of interaction and communication that emerged during the online synchronous collaborations. The analysis of this practice session also provided additional context regarding the decisions made to change the structure of the course after the first online session in order to allow for collaborative problem solving in the two subsequent sessions.

Written reflections by course participants. Throughout the online professional development, opportunities were provided for participants to reflect upon their experiences engaging in the synchronous and asynchronous components of the online course, as well as in their attempts to apply what they were learning in the classroom with students. These written reflections were reviewed for content related to the participants' experiences interacting online to solve problems during the synchronous sessions. Data from these reflections related to the research question of this study were used to illuminate the context of the situation and to triangulate, where possible, findings from the analysis of the archived online sessions and the audio-recorded participant interviews. These data were analyzed in an effort to add to the trustworthiness of the study.

Data Collection Methods

Primary and secondary data sources were collected according to the University of Houston's Institutional Review Board (IRB), Committee for the Protection of Human Subjects (CPHS) requirements as outlined in this research study's approval letter 15611-01, which was updated with approved revisions for participant interviews on March 23, 2016 (see Appendix C). Data from the video-archived online sessions was collected from the secure server whereon they were saved at the time of recording. Participant interviews were conducted approximately eight months after the post-assessment. I, the primary researcher of this study and the course instructor of the online professional development sessions, conducted the participant interviews. Each semi-structured interview was conducted in person in a one-on-one setting following an interview protocol (see Appendix A) that was constructed according to the structure recommended by Carspecken (1996). Data from the interview with the assistant course designer was retrieved from the secure online server where it was electronically archived.

Data Analysis Procedures

Primary Sources

In an effort to triangulate findings, each of the primary sources—that is, the archived online sessions, the participant interviews, and the interview with the assistant course designer—was analyzed separately and findings were, respectively, compared to the codes and emergent categories from the other primary sources. The data analysis procedures that were used to analyze each primary source are described below.

Video-archived online sessions. A rich description of what happened during each of the three online sessions where participants interacted to discuss and/or solve

problems was created by re-watching each session and recording, in a word-processing file, a near transcription of all of the activities and dialogue that took place. The sections of these sessions that included participant collaboration and/or discussion as they engaged to solve problems were transcribed verbatim. As part of this rich description, all speech acts made by participants or instructors were color-coded, and each person had a different, assigned color that was used to highlight her/his speech acts for all three sessions. In addition, screenshots of the webinar's whiteboard were taken and inserted throughout the thick description to show a visualization of those things to which participants were referring as they conversed and solved problems. These screenshots showed still images of how the participants used the virtual tools to solve and/or discuss each problem.

This thick description constituted what Carspecken (1996) referred to as a primary record. While creating this primary record, observer comments and low-level inferences were recorded using comment bubbles in a word processing document, and these comments and inferences were digitally connected to certain sections of the actual text of the primary record. A low-level inference is one that "falls close to the primary record and requires little abstraction" (Carspecken, 1996, p. 146). By recording notes, observations, and these low-level inferences, I engaged in a process similar to what Merriam (2009) described as category construction. Because these sessions were video-recorded archives, elements of interaction were watched multiple times. These archives allowed for the creation of a "more complete" record of exactly what happened during each of these three sessions, and this thick description made it "easier . . .to analyze the data" (Merriam, 2009, p. 128). The complete primary record provided specificity as to

the patterns of interaction and communication that emerged during these online sessions (see Figure 7).

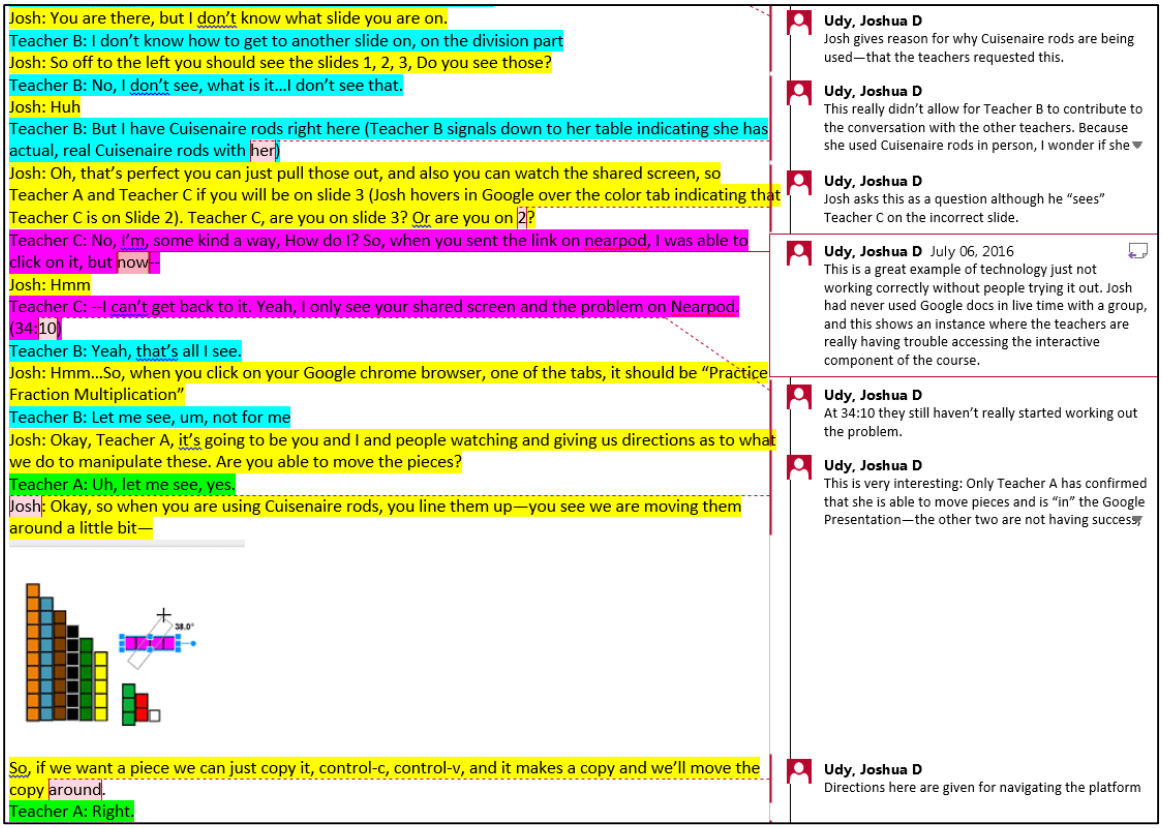


Figure 7. Screenshot of thick description notes in primary record.

After recording the thick description for each archived session, an open-coding process was used to conduct a preliminary analysis of emergent categories about the nature of the participants' interactions and communications. Glaser and Laudel (2013) explained that this type of open-coding procedure is appropriate for "investigations that are purely descriptive or highly explorative" (p. 21). This procedure correlates to six steps outlined by Carspecken (1996) as a method for effectively coding data. In following this process, I 1) opened a new word processing document, 2) split the computer screen to see both the primary record and the new document, 3) recorded in the new document details from the primary record that seemed important for analysis, 4)

created sub-codes under already-created codes that were related but important in different ways, 5) continued reading the primary document and constructing new codes as appropriate, or returned to previously-created codes and added additional examples from later sections of the primary record, and 6) used reconstructive analysis on what I deemed important parts of the primary record, thus forming high-inference codes based on a complete analysis of the low-inference codes previously identified.

Carspecken (1996) explained that high-inference codes are "dependent on greater amounts of abstraction" but are useful because they can help one to "generalize findings that emerge from various forms of qualitative data analysis" (p. 148). The codes and categories that were derived from the analysis of the primary record were used to develop the questions included in the interview protocol, as previously explained (see Appendix A). Similarly, these categories informed which video-recorded archive segments were selected and subsequently shown to each interviewee during the parts of the interviews that included interpersonal process recall. After completing the participant interviews, I returned to each of the thick descriptions from the video-recorded archives and re-engaged in steps 5 and 6 of the aforementioned process by connecting low-inference codes from the interviews with low-inference codes from the thick descriptions to generate higher-inference codes in an effort to conceptualize more fully the findings from the study.

Participant interviews. Each of the 90-minute interviews with participating teachers was audio-recorded and then sent securely to a professional transcription service, where the audio was fully transcribed to text. After receiving these transcriptions, I re-listened to each of the audio-recorded interviews several times and made any necessary

corrections to the transcriptions. In an effort to identify the answer to the research question of this study, each interview was analyzed first for low-level codes and then for high-level codes, in the same way as described previously for the video-recorded sessions. A coding comparison was conducted between each of the three participant interviews and each of the other participant interviews in order to identify codes and categories that existed across each of these three data sources (Carspecken, 1996). Connections between coded data and emergent categories from the thick descriptions of the video-archived online sessions and the participant interviews were used to justify higher-inference codes, which were used in part to more fully answer the research question of this study.

Three peer debriefers, each of whom is a doctoral candidate with a working understanding of qualitative research techniques, provided feedback on the codes and analyses that were constructed both before and after the completion of the initial coding comparison between the primary record and the participant interviews. These peer debriefers first analyzed the codes from the sections of the thick description of the primary record for the five video-recorded clips that were used during the participant interviews, as well as the video-recorded clip that was referred to by two participants when they were asked to recall a question on their own. The purpose of this initial peer debriefing was for them to "check biases or absences in reconstructions" from the initial coding and analysis of the primary record (Carspecken, 1996, p. 141). These peer debriefers next analyzed the codes constructed from the analysis of the interview transcriptions, as well as the connections made between the codes from the interviews and the codes from the primary record. The purpose of having the peer debriefers

analyze the codes from the interview was first to check for possible leading on my part as the interviewer, and, second, to examine the legitimacy of the inferences and analysis provided in the comparative analysis.

In addition, strip analysis was used to verify the validity of reconstructions developed from analysis of the participant interviews. Strip analysis is a process where specific segments of the primary record are reanalyzed to check for alignment to categories formed during coding and analysis (Carspecken, 1996). After the peer debriefing of the analysis of participant interviews was completed, select sections of the primary record where participants engaged to discuss or solve problems were re-examined using strip analysis. Moreover, negative case analysis (Carspecken, 1996) was used to explain inconsistencies between overall findings from the participant interviews and instances where these findings seemed to contradict a particular section of the primary record. Negative case analysis was used to provide reasons for why a particular reconstruction was either inaccurate or why a particular section of the primary record did not fit with the overall analysis.

Interview with assistant course designer. The audio-recorded interview with the assistant course designer was analyzed in the same way as the participant interviews. This interview, in particular, was used as another source to triangulate the data from the audio- and video-recorded online sessions and the participant interviews. A coding comparison was conducted between the coding scheme developed from the interview with the assistant course designer and the coding schemes developed from each of the three participant interviews, and also from the analysis of the primary record of the online sessions.

Secondary Sources

In order to provide additional context to the emergent patterns of communication and interaction that occurred among participants and the course instructors, each of the secondary data sources—that is, the hour-long online practice session, the audio-recorded phone conversation, and the written reflections from course participants—was analyzed for connections to the codes and categories that emerged during the analysis of the primary sources of data. The data analysis techniques that were used to analyze each secondary source are explained in more detail below.

Audio-recorded phone conversation. This phone conversation was transcribed and coded following the techniques used to code and analyze each of the aforementioned interviews. The transcription and analysis of the audio-recorded phone conversation between the assistant course designer and myself provided context to the initial design, and the subsequent re-design, of the online course. This phone conversation took place immediately after the first online session, and a coded comparison of this conversation and the analysis of interactions that occurred during the first online session was conducted to provide additional trustworthiness to the analysis of the primary record.

Video-recorded online practice session. A detailed description of what took place during the one-hour online practice session was created to provide context about the process undertaken to redesign the online learning environment. The video-recorded archive of this session was reviewed to triangulate findings from the analysis of the primary sources of data. This included instances of interaction that validated or invalidated codes and categories regarding patterns of communication and interaction in

which participants engaged as they discussed and/or solved problems, which had been identified during analysis of primary data sources.

Written reflections by course participants. All written reflections submitted by course participants during the asynchronous components of the online course were reviewed with the intention of identifying possible points of data that substantiated or disproved the codes and categories that emerged during the analysis of the primary sources of data. These points of data were used to triangulate findings from the primary sources of data and were used to establish additional trustworthiness in answering the research question of this study.

Addressing Researcher Bias

Prior to the start of this study, where I began by asking a question about what patterns of communication or interaction emerged through our online collaborations, I was convinced that solving problems collaboratively was "better" than solving problems independently, even though the time spent by participants solving problems independently was followed by rich group discussion of strategies. My doctorate advisor was not convinced that conducting a comparative analysis of what made the interactive collaborations better than the independent work was a valid approach to follow in my research efforts (J. Chauvot, personal communication, May 23, 2016). She suggested that I instead look at what emerged from the online group's interactions while solving and discussing problems using virtual tools.

In recognizing my personal bias about the two approaches in which teachers engaged to solve problems during the sessions—that is, either independently with subsequent group discussion or collaboratively using virtual tools—I designed this study

to include methods to control for this bias. The next section will address the methodological techniques I enacted to control for this bias and in other ways substantiate the trustworthiness of my research design.

Trustworthiness

In order to establish trustworthiness in the analysis of the primary data sources mentioned above, peer debriefing was used to determine whether biases existed and whether unsubstantiated, high-inference vocabulary was used in the analysis of the participants' interactions during the online sessions or interviews. Peer debriefing also allowed for verification about whether leading questions were used during the interviews (see Carspecken, 1996). For each peer debriefing, salient sections of the archived course or the transcribed interviews were provided to a colleague familiar with qualitative research, and each was asked to verify if the analysis showed signs of bias or one-sidedness.

To substantiate further the trustworthiness of the analysis of data from this study, member checks (see Carspecken, 1996) were used with each participating teacher and with the assistant course designer. These individual member checks, which occurred before the final write up of the results of this study, included the sharing of the analysis of interview data with each respective interviewee along with a transcription of her/his interview. For each member check, interviewees were asked whether they agreed or disagreed with the record, codes, and analyses provided to them. They were also asked to provide further clarification to specific sections of the analysis that were still unclear to the researcher.

In addition to these opportunities for peer debriefing and member checks, the trustworthiness of this study was strengthened using triangulation of the multiple primary and secondary sources of data available. By using cross comparisons of primary data—that is, the recorded online sessions and the transcribed interviews—as well as connections to secondary data, emergent categories were verified and confirmed across multiple, different sources. Finally, strip analysis and negative case analysis (Carspecken, 1996) were used to determine the consistency of results across strips of the primary record that described times when participants discussed and solved problems, as well as instances of interaction in the primary record that contradicted the overall categories about the patterns of communication and interaction that emerged during analysis.

Limitations

A limitation to the data collection and analysis procedures outlined in this chapter included the lapse in time from when the teachers engaged in the actual online sessions to the time when they were interviewed about their involvement in the course, a break of approximately 8 months. This lapse in time was addressed during the interviews by using interpersonal process recall (Kagan, 1984), where interviewees were asked to first reflect upon and then review salient sections of the recorded sessions.

During the online professional development, teachers completed written reflections regarding their experiences with implementing their learning in the classroom with students. These written reflections, though available, were not substantive enough to triangulate the nature of the teachers' experiences using virtual tools to discuss and solve problems because the questionnaires provided did not ask teachers to reflect on

those experiences. Only after the professional development ended was it determined that it would be a worthy focus of attention to investigate the nature of the participants' learning experiences related to their synchronous online interactions while discussing and solving problems using virtual tools. As such, the written reflections obtained from the teachers during the proceedings of the online sessions focused almost exclusively on their experiences with the content of the course and with their attempts to use the pedagogical practices taught during the course, and not about the nature of their learning experiences using virtual tools to discuss and solve problems. With this said, locating strong secondary written data to substantiate the findings from the primary data sources was a challenge since teachers were never explicitly asked to reflect upon these particular experiences. This limitation was specifically addressed during the participant interviews using the semi-structured interview protocol (see Appendix A) developed after analysis and coding of the primary record and interpersonal process recall (Kagan, 1984; see also Carspecken, 1996), whereby participants watched segments of their interactions using the video-recorded archives.

Another limitation in the methodology of this study was that peer reviewers were not used until after the initial coding and analysis of the primary record and creation of the interview protocol. This delay in using peer debriefers contradicted what Carspecken (1996) recommends, which is to have peer debriefing throughout the coding and analysis process and before generating dialogical data through the use of participant interviews. This limitation was mitigated by creating an interview protocol (Carspecken, 1996) under the direction of an expert qualitative researcher, who also served as the research methodologist for this study. No interviews with participants were conducted until the

interview protocol (see Appendix A) was thoroughly vetted by the research methodologist, and modifications to the phrasing and ordering of questions were made to limit the possibility of leading and bias during the interviews.

A limitation of potential researcher bias also existed in the analysis of the primary record, the creation of the interview protocol, and in how the participant interviews were conducted and analyzed. Because I, as the primary researcher, was also the course instructor, and I was heavily involved in each of the recorded sessions, a limitation of my personal biases encroaching upon the analysis of data could be considered likely. With this point articulated, however, the trustworthiness of the findings of this study were increased through the triangulation of multiple data sources, the use of peer debriefing and member checks, and the creation and vetting of the interview protocol following Carspecken's (1996) methods for dialogical data collection.

Conclusion

There is currently a lack of research regarding patterns of communication and interaction when individuals discuss and solve problems together while engaging synchronously online using virtual tools (see Francis & Jacobsen, 2013). This study investigated patterns of communication and interaction during three synchronous, online mathematics professional development sessions where participants used virtual tools to discuss and solve problems in real time (see Udy, 2016). Qualitative data analysis techniques, as outlined by Carspecken (1996), were used to analyze data in order to answer the research question of this study. Primary data sources analyzed in this study included the video-recorded archives of the synchronous online sessions, as well as audio-recorded and transcribed interviews with participating teachers and the assistant

course designer about their experiences using virtual tools to discuss and solve problems during these online sessions. Secondary data sources included an audio-recorded phone conversation and a video-recorded online practice session between the assistant course designer and the researcher of the study, as well as written reflections from participating teachers collected during and immediately after the online professional development sessions. Potential limitations to the research design of this study were provided, and techniques used to substantiate the trustworthiness of this study were described. The next chapter presents the findings from the analysis of these data sources and presents an answer to the research question of this study.

Chapter IV

Findings

The purpose of this research study was to investigate the patterns of communication and interaction that emerged as three fifth-grade mathematics teachers participated in a series of interactive, collaborative, synchronous online professional development sessions while solving and discussing fraction multiplication and division problems. This study explored data digitally archived in the online platform wherein these sessions took place, including specifically the three video-recorded archives that captured the collaborative interactions of participating teachers as they used virtual tools to discuss and solve problems. In addition, this study collected data through one-on-one interviews with the teachers who participated in the online sessions. These interviews were semi-structured and followed a protocol that was designed after analysis of the primary record had been conducted (see Appendix A). This primary record was created by describing and transcribing salient sections of the video-archived online sessions, specifically the three sessions that captured the interactions and communicative acts of participants while engaged in solving and discussing problems.

In order to substantiate the findings from the two primary sources of data—that is, the primary record of participants' interactions during the video-archived sessions and the transcriptions of the participant interviews—other data sources were also analyzed for triangulation purposes, as was explained in the previous chapter. The findings of this study are presented in this chapter. These findings have been organized into three categories that arose out of the analysis of the above-listed sources of data. These categories are structured according to patterns of communication and interaction that

emerged 1) among participants while solving and discussing problems, 2) among participants while using virtual tools, and 3) between the course instructor and participants when solving and discussing problems. Before the findings from the participant interviews and the final analysis of the primary record are described, a brief description of the findings from the preliminary analysis of the primary record, and how these findings informed the development of the interview protocol, has been provided in the following section.

Analysis of Primary Record and Development of Interview Protocol

Through the initial analysis of the primary record of the video-archived sessions, codes were identified that seemed to indicate that certain communicative and interactive patterns emerged among participants in connection to how they were asked to approach solving and discussing problems. In addition, these communications and interactions seemed to be influenced by a given problem's difficulty level. Through my analysis of the primary record, I noted that when the participants solved an easy problem independently, it seemed as though they generated multiple, different correct representations of the problem's solution using varied means to show their work (e.g., concrete fraction pieces, number lines, graph paper, pictorial sketches, etc.). As participants shared their different representations with the group, they asked questions of one another to understand better each other's work, and they seemed interested in learning what the other teachers in the group had thought about when independently solving each said problem.

On the other hand, when participants engaged collaboratively to solve problems, it seemed as though they did not elicit multiple representations for how a particular

problem could be modeled, but they were able to solve difficult problems that they might not have otherwise been able to solve if left to their own devices. During these times, the teachers talked and discussed with each other in the moment how they might go about representing a given problem. They worked together to model the solution to each said problem, but they did so in just one way. Through these discussions, the teachers offered ideas and asked questions of each other to come to understand how a problem could be represented and solved. This real-time collaboration helped the teachers to derive correct representations and solutions to even the more difficult problems. Conversely, when participants attempted to solve a difficult problem independently, they were unable to determine a correct solution, though they generated three, unique different representations in their attempt to find the answer. Thus, the communications and interactions that emerged as the participants engaged to solve difficult problems together were focused on problem solving, asking probing questions of one another, and sharing different ideas. Through these collaborative interactions, the participants were able to solve challenging problems.

These initial codes prompted me to create a semi-structured interview protocol (see Appendix A) that would allow participants to reflect upon times from the online sessions when they engaged collaboratively to solve both easy and difficult problems, as well as when they worked independently on both of these types of problems. Thus, as was explained in the previous chapter, the sections of the video-recorded archives that were selected for participants to view during the portion of the interview that used interpersonal process recall (Kagan, 1984; see also Carspecken, 1996) were classified by problem difficulty level, as well as by the approach in which participants solved them.

As a reminder, four scenarios of interactions from the video-recorded archives were presented to participants to view during the interviews. These segments of video included 1) an easy problem solved independently with subsequent sharing and discussion afterwards, 2) an easy problem solved collaboratively using virtual Cuisenaire rods, 3) a difficult problem attempted independently with subsequent sharing and discussion afterwards, and 4) a difficult problem solved collaboratively using virtual Cuisenaire rods. In addition, two participants recalled from memory solving a difficult problem collaboratively using annotations on the webinar's whiteboard, and, for these two interviews, those participants were also shown segments of that clip.

Moreover, because I had noticed during my analysis of the primary record particular patterns of interaction that seemed to have emerged among the teachers when they were asked to use virtual tools to discuss and solve problems, I asked the participants about these online tools during their interviews. In my initial analysis, I realized that one teacher was not engaging in solving the problems using the virtual manipulatives, though she did solve problems collaboratively when annotations were used to model problems on the webinar's whiteboard. I found evidence in the primary record that seemed to indicate that this teacher was not engaging with the other teachers to use the virtual manipulatives (specifically the virtual Cuisenaire rods) to represent and solve problems in the shared *Google Presentation*. I wanted to use the participant interviews to learn more about this particular discovery to confirm whether my preliminary analysis was correct. Thus, both before and after watching the aforementioned clips from the video-archived sessions, I asked participants to reflect upon what they remembered about the online tools we used during our sessions.

Finally, during the analysis of the primary record, I also noticed emergent patterns of communication and interaction between the participants and myself as they engaged online to solve and discuss problems. During this analysis, I found multiple instances where I had engaged in helping the participants represent and solve parts of some of the problems they were collaborating to solve. Often, I gave the teachers help by manipulating the virtual pieces for them or by drawing certain representations on the whiteboard for them. Because I wanted to learn more about the participants' views of this interaction, the final portion of the interview protocol included an additional fifth video clip of an archived interaction showing a particular instance when I interacted with the participants as they worked collaboratively to solve a problem. As was explained in the previous chapter, this clip was shown to one participant during her interview, but it was not shown to the other two participants because of a lack of time. However, all three teachers were asked to share their reflections about my interactions with them during the final moments of their respective interviews.

Communications and Interactions: Participants Solving and Discussing Problems

The first category that developed during analysis of the primary record focused on the communications and interactions that emerged as participants solved and discussed problems together. During each of the one-on-one interviews, participating teachers were asked first to share what they remembered about their work in solving problems during the online sessions. All three teachers recalled solving problems collaboratively, as well as observing different ways to solve problems. Teachers A and C remembered solving problems independently and then uploading pictures of their work for subsequent discussion by the group. Teacher A alone remembered that there was a change from the

first session to the second session in how the problems were solved. As was explained in the previous chapter, this change allowed the participants to solve problems collaboratively as opposed to independently. During the first session, teachers were only able to solve problems independently and then share about their solutions by uploading a picture of their work for others to view. During the participant interviews, and by using interpersonal process recall (Kagan, 1984), each teacher was shown segments of at least two problems that were solved independently (from session one) and two problems that were solved collaboratively (from sessions two and three).

The following sections have been structured to explain the interactions and communications that emerged among participants as they engaged in solving and discussing problems either independently or collaboratively. From the perspective of the participants themselves, these emergent interactions and communications have been framed through the lens of the advantages and disadvantages that resulted from the different ways in which they were asked to approach solving and discussing each given problem.

Solving Problems Independently

The patterns of communication and interaction that emerged as the teachers engaged to solve problems independently and thereafter shared their solution strategies have been categorized as being either advantageous or disadvantageous.

Advantageous communications and interactions. Advantageous communications and interactions that emerged when teachers worked independently included 1) how the teachers discussed the multiple representations and ways of thinking that arose from their different solution strategies, 2) how the teachers communicated their

thinking about the applications to their work in the classroom that they identified while sharing their individual ideas with the group, as well as the positive feelings that accompanied those interactions, and 3) how the teachers communicated different, but equally valuable, outcomes that resulted from their work when attempting both difficult and easy problems.

Multiple representations and ways of thinking. All three teachers reported that a definite benefit of solving problems independently and afterward discussing with others in the group various solution strategies was that this provided insight into other people's perspectives on the problems. This understanding provided the teachers with the opportunity to see different ways—or multiple representations—of solving problems, which they felt was an important component of their time together online. In mathematics education, the term *multiple representation* is often used to mean different solution strategies and includes different ways to represent a problem, be it by drawing a pictorial model, modeling with concrete objects, or using some sort of graphic organizer. Consider the following remarks made by Teacher A, who felt that follow-up discussions and sharing after solving a problem independently provided a good model to replicate in her own classroom:

> I don't have years of experience under my belt, so I haven't seen what all the kids are going to do, what their mistakes are going to be, that kind of thing. I can see them while they're happening, but it's nice to be able to predict, maybe, what some of the common mistakes are going to be. . . . I like the idea that we're all solving it differently, so we're seeing different solutions, and we're thinking about

> it in different ways, just so it wasn't a surprise. (personal communication, September 8, 2016)

Teacher B shared these remarks:

> I think [solving independently] was valuable, too, because you could see other people's perspective on the same problem. I think it's always good to see another perspective because then you'll have more tools, more mental tools . . . and you'll say, "Okay, there are different ways to solve this or different ways to look at it." (Teacher B, personal communication, September 9, 2016)

Teacher C commented about the benefits of seeing multiple representations and engaging in follow-up discussion, and mentioned liking the autonomy that she felt in choosing whatever manipulative she wanted to use to solve the problem:

> I love seeing the way other people solve problems and then us discussing it and really analyzing deep on how we were thinking. . . . I think [solving independently] was . . . valuable because you gave us the option to use any manipulatives we wanted. It still gives you an individuality. I solved the problem using these and then somebody [solved it a different way]—it still shows you other ways that people solve problems. (Teacher C, personal communication, August 31, 2016)

The assistant course designer also mentioned how rich conversation developed when the teachers were able to share their different solution strategies. In the following remarks, he mentioned how having the ability to talk about their own work possibly strengthened the teachers' confidence with the mathematics concepts being learned:

> You gave them an opportunity to show the way that they solved the problem, and I thought that was rich communication, and rich thought process behind why each teacher solved it the way that they solved it. [It] kinda helped them collaborate and come to like some sort of consensus so, . . . "You may have solved the problem, or Teacher A may have solved the problem one way, but Teacher B solved it the other way, both very similar, and neither one of them wrong." I thought that was rich . . . . The conversation was huge . . . . I think it just allowed the teachers to be more confident in whatever concept we were dealing with. (Assistant Course Designer, personal communication, March 31, 2016)

The above examples detailed how the teachers felt the time they spent solving problems independently was especially beneficial in helping to elicit multiple representations and multiple ways to think about and solve problems.

Application to classroom work and positive feelings. Similar to how Teacher A reported above that solving problems in different ways helped her not be surprised when she encountered different solution strategies in the classroom, Teacher C reported that seeing multiple representations also had implications for her work with students.

> To have that time to look at each other's work and to see ways that other people solved it, so that I would be ready then to address misconceptions or to be able to walk around my classroom to see how my kids are solving it [was] beneficial for me. (Teacher C, personal communication, August 31, 2016)

In addition to the benefit of multiple representations, Teacher A felt the structure we followed in solving independently and discussing as a group was a good pedagogical model: "It represented how you would want your groups to be working together in class,

and taking turns . . .sharing, and thinking, and using accountable talk—all of those kinds of things" (personal communication, September 8, 2016). On a different note, Teacher C expressed these sentiments when considering what it meant to her to have her work considered by the other teachers in the group: "My ideas are important, too. . . . I liked how we went through it and discussed my work. It made me feel like my work mattered . . ." (personal communication, August 31, 2016).

These comments revealed that the teachers felt that through discussion and exploration of multiple representations, they were aided in being better able to predict what misconceptions and struggles students would have when solving the same types of problems in the classroom. One teacher even expressed how being given the opportunity to share about her own work within the online course helped her feel as though her ideas were important and that her work was valued.

***Difficult Problems*.** In considering the difficulty level of some of the questions, the teachers also reflected upon the advantages that arose from solving problems independently. On the one hand, the teachers reflected that when given ample supports and the time necessary to really work through a problem, the struggle they experienced in attempting to solve challenging problems was beneficial. In referencing one such independently attempted and difficult problem, Teacher C remarked that she felt the teachers would have needed scaffolding or a suggestion by the instructor to get them started:

> We probably needed something to get us focused. . . . I think it [was] okay to let us struggle through that one . . . I think that it was beneficial that we were able to think on our own some of the time. (personal communication, August 31, 2016)

While referencing the same problem, which the teachers attempted independently but that none of them solved correctly, Teacher A made this comment:

> I remember feeling okay about the fact that I was struggling, because other people were struggling. That was nice. It was like—we're all in the same boat. . . . I remember feeling like, "Okay, I'm not just like a complete failure at math." I remember . . . being at school all day and feeling like I was a complete failure at math just because of what was happening at school, and the kids in front of me, and they weren't mastering things, and all that kind of stuff, so I really was starting to feel like, “I'm the worst math teacher ever.” I remember, when grown people are having struggles with it, I was like, "Okay, good. It's not just me." (personal communication, September 8, 2016)

In her reflections about a different problem that she classified as difficult, but that was also solved independently, Teacher B shared this perspective: “The valuable thing about the online experience is the language and the discussion that doesn’t necessarily happen in class [with students] . . . we had time to talk about things in class online” (personal communication, September 9, 2016).

These remarks showed that the struggle felt during the times when the teachers worked independently was considered beneficial. Teacher C considered that she might have been more successful if provided with a support or scaffold to help increase her focus, but that, in the end, she valued the time she was allowed to think on her own. Teacher A’s remarks highlighted how she felt like she was not alone in her struggle when she realized that the other teachers online also struggled when attempting to solve a particularly challenging problem. Last, Teacher B reflected on the time spent discussing

a particular problem after having solved it independently, and how the amount of time which had been allotted for group discussion during their time together online felt like more time than she was used to having with students in her own classroom.

***Easy Problems*.** While the above reflections were related to times when the teachers worked independently to solve difficult problems, Teacher C classified a particular problem that had been solved independently as easy and, after being asked what she would suggest for working to solve it, she replied: "I liked how you let us upload it and solve it on our own, and we discussed it. . . . I . . . think that [this] was beneficial, having some that we solved on our own" (personal communication, August 31, 2016). This quotation shows how Teacher C specifically identified the process we followed when solving problems independently—that is, solving a question alone, uploading a picture of it, and then discussing it as a group—as the preferred method when working to solve an easy problem.

Disadvantageous communications and interactions. In contrast to the aforementioned advantages, the teachers also identified disadvantages resulting from their communications and interactions while solving problems independently and then sharing afterward their individual solutions with the group. The teachers reported that the interactions and communications experienced during the times when they solved problems independently and shared their thinking afterward 1) did not always provide them with an avenue for receiving in-the-moment feedback, 2) caused one of them feelings of anxiety when sharing individual solution strategies, and 3) resulted in technical problems unique to the online environment. All teachers also mentioned that

the lack of certain interactions and communications while working on difficult problems independently made their work on those types of problems less effective.

Lack of feedback. Teacher B shared her belief that working independently does not provide a platform for getting feedback on ideas and, as a result, can require a lot of time and has the potential to result in frustration and fatigue. “If you’re just working by yourself, you don’t get ideas and you don’t get . . . feedback for your ideas. . . . So it takes a longer time to get things done. . . . It’s tiring and you might get frustrated” (Teacher B, personal communication, September 9, 2016). This comment revealed how one teacher felt working independently was disadvantageous because receiving ideas from others or feedback on one’s own ideas was an interaction that was not likely to happen. This teacher also mentioned that not having enough time to solve a problem and the possibility of becoming fatigued and/or frustrated while working independently was considered problematic. These examples highlight that this teacher felt that a lack of communication and interaction with others could have a negative effect on their ability to solve problems when they attempted to do so independently.

Negative feelings. On another note, and in talking about her feelings when having to present her own work to the group for discussion and analysis, Teacher C shared how the experience was anxiety provoking.

> I was nervous about me sharing. . . . It was scary; it was scary. . . . I knew I had the right answer, but even though I said I wasn't sure [I had the correct answer]—because I was thinking about reasonableness—just uploading it, having to upload it and show your work, it was nerve wracking. That was the first time I had ever done anything like that, so it was an experience. . . . It's like you're vulnerable

> because God forbid you have the wrong answer and then everybody's like, “Where do you teach?" That was scary. (Teacher C, personal communication, August 31, 2016)

These remarks highlighted the feelings of vulnerability that surfaced when Teacher C was asked to share her work with the group. For this teacher, interacting with the other teachers after working on a problem independently was anxiety provoking when considering the possible communications that could arise if the answer she shared was incorrect.

Technical problems. Teacher A commented on another disadvantage in her remarks regarding another participating teacher who had experienced technological problems when attempting to upload a picture of her work for the group to discuss. “To be very honest, I got annoyed, I think, with some of the technological difficulties. That felt tedious. . . . [One teacher] was struggling . . . she couldn't take the picture with her iPad, because the angle was wrong . . . ” (Teacher A, personal communication, September 8, 2016). Teacher A also talked about a logistical problem that she felt existed when participants solved problems independently and then uploaded their work for group discussion.

> I think it was just a little cumbersome. There's a lot of noise, and there's a lot of shuffling, just background kind of stuff. . . . I do sort of remember it being loud. I remember my husband being all the way on the other side of the house, and making comments about how loud we were, and how noisy it was. (Teacher A, personal communication, September 8, 2016)

These reflections highlighted technological problems that prevented a participant from successfully uploading and sharing a picture of her work. This inability to share her work limited one teacher in her ability to communicate her thinking with the group, and it caused Teacher A to feel annoyed at the tediousness of working with the technology in an attempt to communicate one's work. Other disadvantages shared were logistic in nature, such as how the use of concrete manipulatives and/or paper and pencil to represent a problem was noisy when listening to others work on their own through the online webinar and how the process of taking a picture of one's work in order to upload and share it with the group was tedious. These comments indicate that it could be possible that the noise and cumbersome sharing methods negatively affected the communications and interactions of the participants during the times when they were asked to share their respective solution strategies.

***Difficult problems*.** While each of the participants did not identify all of the aforementioned disadvantageous communications and interactions, the one hindrance they all identified as problematic was when they were asked to work independently on a challenging problem. In this instance, they were unable to represent accurately the solution to the problem using a pictorial or concrete model, which seems to have resulted from their inability to communicate and interact with one another. Teacher C commented about when participants were not prepared to share after having attempted to solve a problem independently: "Sometimes we weren't ready. Sometimes everyone wasn't ready. I think at one time, one person didn't share because they couldn't come up with anything, and I was so scared that that was going to be me" (personal communication, August 31, 2016). This comment seems to indicate that the teachers' communications

after attempting a difficult problem independently might have been affected by not feeling ready to share their results. Teacher C even reflected on a time when she was unable to come up with anything after attempting to solve independently a particularly challenging problem. This reflection connects to what was shared in the previous section by Teacher B about how students may require additional scaffolding to understand a problem when they are unable to continue: "We might have been stuck. . . . We might have needed some more clarifying to help us get going, because . . . I remember us being stuck for a while, and I never produced anything because we ran out of time" (Teacher C, personal communication, August 31, 2016).

For this particular problem, all three teachers commented, or in other ways demonstrated during the interview, that the correct representation for the problem was still giving them trouble or that they had needed additional help (beyond the time spent discussing it together online) to understand it fully. Said Teacher A, "I still struggle with the idea of trying to figure [it] out. . . . I feel like we left it hanging maybe, just a little bit" (personal communication, September 8, 2016). Teacher C expressed these thoughts, "That was hard. . . . We didn't have enough time. . . . I think [my district teacher development specialist] helped me with this one. I think this is the one she and I worked on together" (personal communication, August 31, 2016). Teacher B said this: "I think on this one in particular, it should have been a two-parter. . . . I just think we needed time. Well, I needed more time. I think everybody else probably got it" (personal communication, September 9, 2016).

These statements demonstrated that the teachers' attempt at solving this challenging problem through means of independent work was not sufficient for true

understanding to develop. Teacher A indicated during our conversation that she still struggled with how to represent the question conceptually. Teacher C mentioned she sought additional help from a district-level teacher development specialist in order to gain a clearer understanding of the problem. Teacher B and I actually engaged in solving the problem together during the interview as she attempted to work it out again.

On a related note, during the interviews, after participants viewed an archived video showing them collaborate to solve a problem, I asked them to speculate what they thought would have happened if they had solved the problem independently. The following exchange took place after Teacher A and I had viewed a particularly challenging problem that the group had solved collaboratively:

> Me: Had I given this one to you, hypothetically, as, "Go off on your own, take six minutes, and come back, and share your picture upload . . . ?"
>
> Teacher A: I think we would have gotten stuck.
>
> (personal communication, September 8, 2016)

For a different, yet challenging, problem that the group had solved collaboratively, I asked Teacher B and Teacher C to speculate about what they each thought would have happened if they had been asked to solve it independently. Consider the following dialogue I had with Teacher B.

> Me: What if I had done that one by saying, “Go off on your own and work and use graph paper and then send me a picture? What do you think would happen?
>
> Teacher B: That would not work for me.
>
> Me: No?

Teacher B: Because I was completely—I had nothing, no imagination for that at all.

(personal communication, September 9, 2016)

Teacher C shared something similar.

Me: If I had given that to you on your own and then said, “Share,” what do you think would have happened?

Teacher C: I probably wouldn’t have produced anything . . . .

(personal communication, August 31, 2016)

These speculations highlighted how the teachers felt that solving very difficult problems independently might not be the most effective means at helping someone arrive at a solution and/or develop an understanding of difficult concepts. Furthermore, these suppositions are supported with data from the primary record, which showed that each teacher was unsuccessful when attempting to solve a difficult problem independently. In addition, the teachers expressed that either they had to seek help from someone else to understand the problem after attempting to solve it online (as was the case with Teacher C), or they shared how they still struggled to understand the problem conceptually (as was the case with Teachers A and B).

Suggestions for increased effectiveness. The teachers each provided suggestions for what could have improved their time spent attempting to solve a difficult problem independently. All three teachers indicated that the difficult problem solved independently could have perhaps been better approached by using some sort of group collaboration. Teacher A described a hypothetical model for engaging cohorts of

teachers located at the same school or general physical location in solving a difficult problem as a collaborative group.

> You would want to send them off to solve the problem with their cohorts, then come back with 1, 2, or 3 solutions, so you would want to have a way to be able to pause for 6 minutes or whatever . . . if we're stuck, somehow there would need to be a way to throw up the red flag and be like, "I can't do it. I'm done." (Teacher A, personal communication, September 8, 2016)

In this reflection, Teacher A provided an idea for allowing interaction and communication to occur when solving a difficult problem. Implicit in her suggestion is that, first, collaborative interactions in a group could be helpful at generating multiple representations and that, second, a system for communicating that more help is needed could provide a way for participants to get assistance when unable to continue working.

Teacher B explained how, similar to the way that she was stuck when attempting to solve a difficult area problem, students need scaffolds to help them through difficult problems, which she asserted was only possible through collaboration. "Sometimes, you have to get kids unstuck, like give them some mental tools to look at stuff in different ways. That only happens through collaboration and allowing them to talk about it and see it different ways" (Teacher B, personal communication, September 9, 2016). Moreover, Teacher B reflected that additional scaffolding might be necessary when students engage in independent work to understand a concept, especially when they may not feel as if they know how to begin working on a given problem.

> It helps me to recognize that sometimes kids don't have a clue. They have no imagination for that concept, and that's why they are looking at you blank. You

> do have to let them see a part of it, maybe. Then, they can say "Oh," and discuss that little part. Then, they can say, "Oh, okay." Then something starts to click a little bit. Then, you show them a little bit more and then, you know. It's real: when people say they don't know, they don't know. (Teacher B, personal communication, September 9, 2016)

In these suggestions, Teacher B highlighted that allowing students to work on a question in chunks and discussing parts of the problem a little bit at a time are interactions and communications that can help learners develop understanding of an unfamiliar concept. Providing scaffolds was also something suggested by Teacher C when she reflected on what she might have done if in charge of the session when the teachers were stuck as they attempted to solve a difficult problem independently.

> Me: Had you done this session, if you were in charge, what would it have looked like for the teachers if you were leading this problem? How would you have done it?
>
> Teacher C: I would have scaffolded. I probably would have gave us one—like how you gave us grid paper—gave us one focus: "Okay, here's a string. Here's the actual string. Think about how you're going to show, whatever it was, three-fourths of the five." Maybe not leaving us so open.
>
> (personal communication, August 31, 2016).

In another instance, when reflecting upon her failed attempt at solving a challenging problem independently, Teacher B talked about simply needing more time. She expressed that the group's attempt at discussing the problem afterward was ineffective because there was not enough time.

Teacher B: I think I needed more time to get that concept right there, that I needed to work those out. I could have done it with the 60 like I was trying to do.

Me: Right, you were trying. You got close.

Teacher B: But, I had to understand another concept . . . .

Me: Do you think our collaboration online helped bring that—I mean, it didn't: we ran out of time.

Teacher B: Yeah, that's why it didn't.

(personal communication, September 9, 2016).

When considering the above conversation, I recognized during my analysis that my statement about online collaboration and running out of time could have been leading. In further analyzing the interview transcript, the following exchange revealed that Teacher B might have considered that having enough time to think completely through the problem was more important than collaboration with the group. The following set of exchanges occurred immediately following the above-quoted dialogue.

Me: What do you think about the way we went about solving this . . . ?

Teacher B: I think, for me, it could've been a two-parter. It could've been, okay, that day we did it, to come back the next time or another time or do another one.

Me: We needed more time to do it?

Teacher B: mm-hmm (affirmative)

Me: Notice this way was not the same as the graph-paper problem. What do you think about that? The way that we solved this one? Really, everybody went off on their own . . . .

> Teacher B: All of that is good, but I think on this one in particular, it should have been a two-parter.
>
> Me: Do you think it would've been any different . . . if I had somehow figured out how to put up [on the screen of the online webinar] a string where maybe we could draw. . . . Do you think that would've made a difference in the solving of this? If so, how? Or, if not, why not?
>
> Teacher B: I don't think it would have made a difference for me. . . . I just think we needed time.
>
> (personal communication, September 9, 2016)

These statements seem to indicate that Teacher B felt strongly that more time was requisite for gaining an understanding of the concept, though this does not mean she discredited group collaboration. When asked to reflect upon the different ways we solved problems together, Teacher B emphasized the importance of talking:

> [Our time spent solving fraction multiplication and division problems] just let me see that the talk time is very important, and teaching kids how to talk when they are working with math, that's really important. Because if they don't talk—so if you have a real quiet classroom and they're doing math, then they're not understanding something—they're not getting something. You have to have some talk time. (personal communication, September 9, 2016)

During a similar speculation, Teacher C expressed that she would have allowed for group collaboration on the difficult problem that the group attempted independently.

> Me: So, anything here you liked or disliked about the actual way we went about solving and discussing this?

> Teacher C: I probably would have added that to the collaborative one. (personal communication, August 31, 2016).

The above points indicate the teachers thought collaboration, scaffolds, and time could have helped them when they engaged to solve a difficult problem that had been attempted independently during the first online session.

Solving Problems Collaboratively

The patterns of communication and interaction that emerged as the teachers solved problems collaboratively have also been categorized as being either advantageous or disadvantageous.

Advantageous communications and interactions. As was indicated by each teacher during the participant interviews, collaboration was an important part of their online experience. Specific advantages with solving problems collaboratively included that the resulting communications and interactions allowed the teachers to share ideas, deepen their understanding of concepts by hearing ideas from others in the group, and receive immediate feedback on their contributions to the group's work. Further, the teachers felt that their communications and interactions while collaborating were more time efficient, resulted in positive feelings, helped them learn new tools for teaching fraction concepts, and had applications to their classroom work. Finally, the teachers felt that their communications and interactions while collaborating to solve problems improved the technological aspects of the online experience and, perhaps most importantly, were especially helpful when they worked together to solve difficult problems.

Sharing of ideas and deepening of understanding. The first clear advantage of solving problems collaboratively was that the teachers' communications and interactions allowed for them to add onto the ideas of others, build off each other's thinking, and bounce ideas off each other. In addition, through these collaborative interactions they were able to see different ways of thinking while engaging together to solve a problem. These findings were noted during the analysis of the primary record and later made explicit during the analysis of dialogue generated from each participant interview. When asked what she recalled about our time together online solving and discussing problems, Teacher A shared about the nature of the group's collaborative work:

> I remember there was one problem . . . where we were dividing things out, or we were chunking things, or we were drawing pictures and we were dissecting them or something, and somehow we all three ended up really building on each other, almost at the same time, and it felt more like we were in the same room together, which was really nice for that online sort of feeling. I've done lots of online classes where it's not collaborative at all, and that's okay with me. It doesn't bother me; but for that problem solving bit, I think it's better to have that sort of collaborative thing. (personal communication, September 8, 2016)

In a written reflection after the administration of the post-test assessment, Teacher B shared about the group's collaboration when solving problems: "It was very helpful that we were able to solve problems together. Solving problems with other [teachers] helped me to understand the math concepts we were studying" (personal communication, January 30, 2016). Teacher C also shared how she preferred the collaborative approach

to solving problems as opposed to working on the problems independently and sharing afterwards:

> I . . . remember having a program where we solved problems together at the same time collaboratively, where I could add something and the two other teachers were adding things and you were adding things, and so I thought that that was a lot more helpful because when someone added their idea, then that sparked an idea for me and I could add on to that. That was a little bit more helpful than me being nervous about solving myself and then uploading it and then everybody looking at me. (personal communication, August 31, 2016)

In this passage, Teacher C shared about how the collaborative interactions and online discussions while solving problems with the other teachers gave her additional ideas to build from, as well as allowed everyone to share their thinking and contribute to the solution of a problem at the same time. She also mentioned how she felt collaborating to solve a problem was more helpful than solving it independently and then experiencing anxiety when uploading it to share with the group.

In a similar way, Teacher B shared how seeing different perspectives helped her understand better the concepts addressed during the course: “I saw different ways, like somebody thinking different than I am. Just seeing it from a different perspective helped me to understand the concept more” (personal communication, September 9, 2016). Teacher A shared something similar: “Being in that group I think, was helpful, because it gave an avenue towards a collaborative feel to it, and getting somebody else's thoughts” (personal communication, September 8, 2016). Teacher B also shared about how she enjoyed the time spent collaborating to solve problems: “I like the collaboration . . . . If

you are in a group, . . . the group takes time too, but it's productive time. Because you get more ideas, you get feedback . . ." (personal communication, September 9, 2016). These quotations highlighted how the teachers appreciated the collaborative interactions that they experienced while solving problems together because such interactions allowed them to gather thoughts, ideas, and feedback from others.

Moreover, the teachers talked specifically about how they felt that solving problems collaboratively expanded their thinking and broadened their understanding. The result of this broadening of understanding was that some of the teachers reported experiencing specific moments of enlightenment while working with the others in the group. Teacher B initially reflected on their time spent collaborating while solving a challenging fraction multiplication area problem by sharing this thought:

> The one I remember kind of is we were on chart paper and when I first read the question, I thought of it one way, but then Teacher A and Teacher C thought of it another way, and their way was the correct way to think about it. I was like, "Oh!" (personal communication, September 9, 2016).

Later during her interview, after watching the archived video showing the group collaborating to solve this same problem, Teacher B responded: "I like the way we did that. It broadened my understanding, because I had no clue" (personal communication, September 9, 2016). Similarly, Teacher C shared the following remarks about two problems the group had collaborated to solve:

> I think the one question that kind of stumped me . . . was the one with the grid paper where we had to create a scale to solve it. I had to work with my students on that same . . . problem, and it was interesting how quickly they got it and

> understood it right away, and how long it took me to wrap my mind around it, but it helped with us working together, because we were talking to each other and we came up with a scale that could fit on the paper, on the screen in front of us. . . . Also, . . . we talked about a gas tank and displaying it vertically and that was filling it up and that was an "Aha!" moment for me, like, "Why didn't I think of that?" Just small things that I think help when you're collaborating with each other, to help you expand. (personal communication, August 31, 2016)

These comments showed how the teachers appreciated the opportunity to build off each other's thinking, receive ideas from one another, and get feedback on their work. The teachers also shared that seeing different ways of thinking about a problem helped them to understand better the concepts at hand.

Immediate feedback and time efficient. Another beneficial interaction that resulted as the teachers solved problems collaboratively was that they reported receiving in-the-moment guidance and help from the instructor or from the other teachers in the group. A quotation from Teacher C summarizes this benefit well:

> I liked working on it together because that kind of steered me in the right direction. . . . With the garden one, you gave us . . . a little nudge to think about the scale, or somebody [said] something about scale, and then that put us on the right track. (personal communication, August 31, 2016)

The teachers also mentioned that collaborating was more efficient and, that by collaborating, they saved time. Teacher B made the following remark:

> What I liked about the process is that most of the time it takes me a long time to think through math. Just having a team to help me solve it, it shortens my

> understanding time, because they would approach it differently than I would. I need that collaboration as a math teacher. I need that collaboration. . . . That worked out well for me, to be in a collaborative atmosphere. (personal communication, September 9, 2016)

When asked why she thought collaborating to solve problems was more helpful than uploading pictures of work completed independently, Teacher A shared these thoughts about how collaboration seemed to save time:

> I guess it was quicker. It was more efficient, I guess, to be able to do that, and it was much easier. . . . [It] felt very efficient, and productive . . . It felt like we were all on the same conversation at the same time. It felt like we were using our time well. (personal communication, September 8, 2016)

Teacher C also speculated that by collaborating, the group members might have gotten through a difficult problem that they were unable to get through on their own. In the following quotation, she mentions that as a group they might have been able to work successfully through the problem in the same limited amount of time as when they attempted solving it independently.

> We're actually talking out loud, so when I'm on my own thinking, I'm only, you know, I'm by myself, thinking by myself, but when we're working together, we may have gotten to it. Maybe in the same amount of time. (Teacher C, personal communication, August 31, 2016)

These statements summarized how the teachers appreciated the immediate feedback they received while engaged in collaborative work and how they felt collaboration was more efficient and helped to save time.

Positive feelings. Each teacher also shared positive feelings associated with working collaboratively to solve problems. The following statement, made by Teacher A after viewing a problem where the teachers collaborated using virtual Cuisenaire rods to solve a difficult fraction division problem, summarizes well the shared feelings the teachers expressed about their time spent collaborating: “That felt good. That felt like we were collaborating. . . . That felt good” (personal communication, September 8, 2016). While each teacher stated that she liked working together online, each also had her own individual reasons for why this was the case. Teacher C appreciated how working together meant that she could delay interaction and simply listen to others and not feel as though she had to share right away: “I was able to listen to what other people were saying . . . I didn't share right away” (personal communication, August 31, 2016). Teacher B liked the group collaboration because she didn’t feel pressure to get the answer correct:

> I thought it was good because everybody was trying to figure it out. There wasn’t the classroom pressure for people to get it right. You could just think about math. You had time to think about math. That was good. (personal communication, September 9, 2016)

In a written reflection after the post-test assessment, Teacher B also shared how she felt like collaborating helped to involve her equally in the group’s interactions:

> I liked the level of my engagement in the [professional development]. I was able to problem solve with my classmates. There was no "star pupil" who was the most actively involved student. I had more of a voice in an online environment. I felt that it would have been irresponsible for me not to participate in the activities of the course. (personal communication, January 30, 2016)

Teacher A shared these thoughts about why she liked the collaborative group:

> With Teacher C having a lot more experience, and Teacher B having a lot more experience than I do with that grade level, that was nice. I think that's one of the takeaways from just that collaborative process, is it sort of humanizes everybody a little bit. (personal communication, September 8, 2016)

In the above quotation, Teacher A shared that watching others struggle with difficult problems helped her not feel like she alone had a problem with math. She explained that, instead, the collaborative process helped to humanize everyone in the group.

In a similar way, the assistant course designer detailed benefits of the collaborative online environment and explained how, through their interactions, the teachers were able to form a community:

> I think that having that online environment where we could see, like the synchronous environment where I could actually see that person, made a huge difference . . . . I thought that piece was key because . . . they kinda developed their own little community. . . . They were able to like kinda trust and lean on each other. Whereas—I'm going back to the pre-test that they took— . . . you could just tell the difference. The vibe was a little different . . . and then like as we moved about through the course, you could kinda see them develop more relationship with one another and so they were able to bounce ideas back on each other. So, I thought the technology aspect—[At first], I was like, "Where are we going to go with this? Are they going to be comfortable with these people with whom they've only seen once? And now they're in the online environment?" I mean, essentially, they're like, not competitors, but—"I teach 5th grade, and so do

> you and so, I don't wanna expose some of my weaknesses, and . . . you probably don't want to either." But it was strange because they didn't really have that boundary, like they were very open with one another. (personal communication, March 31, 2016)

The above reports by the teachers highlighted how they experienced positive feelings while communicating and interacting with the others in the group to solve and discuss problems. In addition to the teachers describing different reasons why they appreciated their collaborations in the online platform, the assistant course designer shared thoughts about how the teachers' interactions seemed to change from the initial pre-test. He discussed how it seemed that the teachers formed a trusting kinship with one another because of their ability to see and communicate with each other.

Learn tool usage. In addition to the above-listed advantageous communications and interactions, the teachers all mentioned that the collaborative process helped them to learn how to use Cuisenaire rods.

> Teacher A: I think my big takeaway was I learned how to use Cuisenaire rods, which I like. . . . I felt very accomplished, being able to use Cuisenaire rods with the kids, just because I'd always seen them and been kind of fascinated with them, but never really got it.
>
> Me: Awesome. Do you think that came from a time when we were working and solving the problems together?
>
> Teacher A: Absolutely. That was absolutely us focusing on using the Cuisenaire rods as a part of the way to solve them.
>
> (personal communication, September 8, 2016)

Teacher C shared about a particularly challenging problem where Cuisenaire rods were used. Though she stated that the teachers struggled with the problem, she felt the struggle was necessary because they needed experience in learning how to use the rods.

> Me: Classify the value of our time together on this problem here.
>
> Teacher C: I would say still a ten, even though we struggled. I struggled; I would say that because I think that we needed that. We needed the experience with the Cuisenaire rods and the time to use them and see how to use them. Even though I said I was confused, I think it was still valuable.
>
> (personal communication, August 31, 2016)

In discussing the same problem, I asked Teacher B a similar question.

> Me: How would you rank the value of our time spent?
>
> Teacher B: It was a 10.
>
> Me: Yeah? And why would you rank it like that?
>
> Teacher B: Because I learned some more about Cuisenaire rods. Also, we had time to think. I think that's important in math. You can't get it like that.
>
> (personal communication, September 9, 2016)

These quotations illustrated how the teachers appreciated how the collaborative online environment aided them in learning how to use Cuisenaire rods, a tool for learning fraction concepts that they had previously been unsure of how to use.

Application to classroom work. Still another advantage reported by some of the teachers was that solving problems collaboratively provided them with a pedagogical model that had implications for their work in the classroom with students. Still reflecting

upon the aforementioned area problem, Teacher B and I engaged in the following discussion:

> Me: What did you think about the way that you and the other teachers—and really me too, because I was always jumping in and saying something—what did you think about the way we worked—the actual way we worked in solving and discussing this problem?
>
> Teacher B: That was beneficial, because in the classroom, you want the same kind of collaboration in the classroom. I'm having to teach these students how to talk to one another in class and not judge one another and not make fun of people when they have a response, whether they think it's right or wrong. (personal communication, September 9, 2016)

These remarks highlighted how Teacher B felt the communications and interactions among the teachers during the online professional development provided her with a good model to follow for considering how she wanted her own students to talk and act in the classroom. Teacher A shared something similar about how the group's collaboration helped her prepare for her work in the classroom.

> The idea that when four of us sit down and have a conversation—even though we all four are coming from a different place—when we put it together, the solution's going to be better than it was with us four individually. There were lots of times when I really felt that, even as far as just predicting what the kids were going to do, right or wrong, or . . . just being able to sort of pre-think. (personal communication, September 8, 2016)

These remarks revealed that the teachers appreciated how their communications and interactions while solving and discussing problems online directly related to their work in the classroom with students.

Improvements in logistics of technology. An additional benefit that Teacher A remarked on was related back to the noise that was often created when the teachers engaged in modeling a problem using concrete materials instead of virtual ones. After watching an archived video clip showing a time when all of the teachers were using virtual tools to solve a problem collaboratively, Teacher A shared this thought: “There wasn't a lot of fumbling. There wasn't a lot of noise going on. It didn't feel like we were waiting” (personal communication, September 8, 2016). These remarks seem to suggest that Teacher A felt communications and interactions were improved and wait time was decreased during the times when problems were solved collaboratively versus independently.

***Difficult problems*.** Over and above all of the aforementioned advantages, some of the teachers shared why they felt that solving problems collaboratively was particularly beneficial when solving difficult problems. Teachers A and C both indicated that collaboration was helpful when solving difficult problems, especially because it helped them to work through the struggle of understanding challenging concepts. Consider the following remarks made by Teacher C after reflecting about how the group worked together to solve a difficult fraction multiplication area problem:

> When I first heard Teacher B and Teacher A talking, . . . it still didn't click on me at that point. I don't think it clicked on me how to solve the problem until she said, "It's got to be bigger," or something. And then I said, “Oh!” I was thinking,

> “How can we show thirds? Three squares could equal one yard.” But yeah, that was tricky. I mean, you really stumped us on that one: because you never think about how to display multiplying fractions on grid paper. I wouldn't even think about how to do that with manipulatives . . . . I think us working together and talking through it—and your clarifying questions and being able to see it—really, really was beneficial. (personal communication, August 31, 2016)

In a similar way, after watching a video archive that showed how the group solved a challenging multiplicative-comparison fraction division problem, Teacher A shared these thoughts about how the group worked through the struggle of understanding the concept at hand:

> This, I think, is what I remember about us working better together, and maybe it was working through the struggle . . . . This is what I remember about things working really well. . . . You let us struggle, but then it was like you threw a little bit of something in, once we sort of realized, "No, we're really stuck. We're not struggling, we're like drowning." Yeah, it helped. (personal communication, September 8, 2016)

Relatedly, Teacher C discussed that she preferred to work collaboratively on difficult problems: “Something more like the garden where we have to think, or a ruler, I think there was a ruler problem and that was, oh . . . serious. Something like that, I think I would be open to collaborating with . . .” (Teacher C, personal communication, August 31, 2016).

After showing Teacher C the video archive of the ruler problem mentioned in the above quotation, which had been attempted independently, we engaged in the following dialogue:

> Me: You remembered this was hard; you would have said this one was hard?
>
> Teacher C: Yep, that's the one we should have worked on together.
>
> (personal communication, August 31, 2016)

During his interview, the assistant course designer recalled the phone conversation he and I had wherein we brainstormed how to allow for synchronous interactions in a shared space. In the following remarks, he commented about how an interactive space was a more appropriate venue for when the participants were asked to solve difficult problems:

> Someway you got them all to collaborate in one workspace . . . . And I think it came from that conversation that we had about, “Okay, so they were able to do it in isolation, but when the problems get a little more challenging—and I want them to be able to use that same space simultaneously, then ‘How do we do that?’’’ (personal communication, March 31, 2016)

These statements indicated how the teachers found collaborating in a group particularly helpful when solving difficult problems. Specifically, the teachers expressed how the group collaboration aided them in working through the struggle of learning to understand some of the more complicated concepts that were discussed during the sessions. Moreover, the assistant course designer referenced how the collaborative workspace was designed to foster synchronous interactions, and he indicated how such a platform is appropriate to use when solving harder problems.

Disadvantageous collaborations and interactions. Disadvantageous communications and interactions that emerged while working collaboratively included 1) how the teachers felt concerns with technology affected their ability to communicate effectively with each other, 2) how feelings of vulnerability were experienced by one teacher while interacting with virtual pieces in the shared space, 3) how a lack of multiple representations resulted from the teachers interacting to solve the problems together, and 4) how it was predicted by one teacher that problems in communicating might arise if the group collaborated to solve easy problems.

***Technical problems*.** Teacher A commented that, occasionally, the problem the group was working to solve was not always displayed in the same screen, which made it difficult for the teachers to see the wording of the problem while working to solve it: "There were some times that we didn't have the problem visually in front of us . . ." (personal communication, September 8, 2016). This comment was substantiated through analysis of the primary record:

> Me: So the problem we are going to solve is an area problem. I don't know if you guys can see this. I'm looking at the nearpod. I wonder how I can get this on the same screen. [Observer Comment (OC): *"This" meaning the text of the problem.*] Hmmm…Can you guys see the problem?
>
> Teachers A and C [in unison]: No.
>
> Teacher B: I can't see it.
>
> Me: I'm afraid that if I paste it in, it's gonna move the graph paper, let's try it anyway . . . . Okay, so what I'm gonna do is, I'm gonna—you might need to write it down, you might need to write something down because I am going to go

> to the graph paper and you will be unable to see the [problem]. [OC: *I have forgotten that during the one-hour recorded practice session with the assistant course designer, I had determined a solution to this problem. I had planned to paste the text of the problem in the webinar's chat box in order to enable participants to see the text of the problem and annotate on the screen at the same time*.]
>
> (personal communication, December 2, 2015)

Another technological difficulty mentioned by some of the teachers was that they, on occasion, experienced a slight lag or drop in internet connectivity while solving problems together, though this seemed to be considered a minor inconvenience. Teacher A made this comment:

> I remember all three of us could move [the Cuisenaire rods] at the same time, or could write on [the webinar screen] at the same time. There was a little bit of lag, but compared to the way it was previously . . . that was better. (personal communication, September 8, 2016)

Moreover, Teacher C shared this thought:

> Sometimes [our interaction online] was frustrating. I think with the internet connection too, there was a lag or a delay or I would get dropped. And so, I would miss little pieces of the discussion. . . . I think it worked because you had the tools, like I said, that you would have face to face. With the exception of the lags, which happens. (personal communication, August 31, 2016)

These reflections show how technological problems occasionally impeded the participants' abilities to communicate and interact effectively with one another.

Yet another disadvantage was mentioned by Teacher A, who commented that, because the participants did not always know how to use the virtual tools, there was occasionally dead time or longer periods of wait time. Teacher A's thoughts are captured in this section of interview transcript, which took place immediately after we viewed a clip that showed the teachers working with virtual Cuisenaire rods to solve a fraction division problem:

> Me: What do you think about the way we worked on this, or how we did this problem?
>
> Teacher A: I think that was awful, now that I watch the video . . . .
>
> Me: What was awful? . . . What do you think was the awful part?
>
> Teacher A: I don't know. I don't know what we were waiting for. . . . I don't know why it was so much dead time . . . . It was just cumbersome. . . . I personally don't remember why that was cumbersome. I have this weird hunch that all the fumbling around was us trying to fix something, or find something. (personal communication, September 8, 2016)

Near the end of the above-quoted section of transcript, Teacher A alludes to "fumbling around." She had explained her thinking about this more completely in a preceding section of our interview. After expressing that she thought all of the fumbling and noise (that she remembered as being problematic during the times when problems were solved independently) had been fixed when we engaged to solve problems collaboratively, Teacher A made this remark: "It wasn't better. It was still shuffling in the background, and just dead time" (personal communication, September 8, 2016). After reviewing Teacher A's written reflection after the post-test assessment, it might be concluded that

these technological problems were considered to be too much of a problem for her to be willing to engage in another online professional development. Consider these comments Teacher A made after indicating that she would prefer to, in the future, attend a face-to-face training over an online training: “Watching people struggle with the online learning platform makes me crazy enough that I would opt for in-person” (personal communication, February, 11, 2016).

These statements revealed that there were technological concerns felt by some of the teachers when collaborating online to solve problems, and that these problems seemed to affect the group’s ability to communicate and interact in ways that felt efficient and productive.

***Vulnerability*.** Beyond these technical problems, Teacher C mentioned feeling vulnerable when moving the virtual manipulatives in the shared *Google Presentation* document. After analyzing a section of interview transcript wherein Teacher C had shared her experience using the virtual Cuisenaire rods to solve problems, I found this statement: “You’re just vulnerable out there” (personal communication, August 31, 2016). I asked Teacher C in a follow-up exchange after her interview to clarify what she had meant by this phrase. This was her reply:

> I was referring to using the Cuisenaire rods, tools that I wasn't used to using, in the shared space where everyone could see me working with them and I was feeling vulnerable about my work. At that point in the sessions and how we were working together, I wasn't too worried about getting the wrong answer. Like I stated, often times I worked backwards and knew the answer already. It was just using the tools to show the answer, let alone tools I had never really used before,

> that made me a little nervous. (Teacher C, personal communication, September 25, 2016)

The statement above related to Teacher C's feelings of vulnerability when manipulating objects in the shared platform online. Teacher C recalled that her feeling of vulnerability seemed to be less acute as the sessions progressed because she felt more comfortable with the other teachers in the group.

Lack of multiple representations. Another disadvantage to solving problems collaboratively surfaced during the analysis of the primary record, though complete substantive data from the participant interviews was lacking to verify fully if all participants felt this disadvantage existed. As was reported previously, some participants felt that collaboration allowed them to see different ways of thinking about a problem. Without negating that differences in thinking were shared among the teachers as they worked collaboratively, a disadvantage related to a lack of multiple representations seemed to exist when participants collaborated to solve problems. Essentially, when the teachers worked together to solve a problem, they produced their solution, and its corresponding representation, in just one way. Generating just one representation when solving problems collaboratively contrasted times when the teachers solved problems independently. For example, while attempting to solve a difficult problem independently, Teacher A represented the problem using number lines, while Teacher B used fraction tiles, and Teacher C used graph paper. Such multiple representations were not generated when the teachers collaborated to solve problems. Consider the following excerpt of interview dialogue between Teacher A and myself as we reflected on a problem that had been solved collaboratively using Cuisenaire rods:

Me: Do you think you needed the Cuisenaire rods to solve this one, or would it have been different if we had used some other way to solve it?

Teacher A: I don't think you needed the Cuisenaire rods. I do think that problem lends itself to Cuisenaire rods. I don't think you need it. I could easily have drawn boxes that were basically the Cuisenaire rods, but without the color, and the different steps of it, and all that kind of stuff. I think—my memory is that our focus was literally you training us to use Cuisenaire rods, so you would say, "This is how you do it with Cuisenaire rods." I think. We had kind of gotten away from, "Just randomly, how would you solve this problem?" We had gotten into, "Okay, how would you solve this problem," and, "Oh by the way, here's the way you do it with Cuisenaire rods."

Me: Right. If you think back to the first clip we just watched, where you had done it this way, Teacher B had done it slightly different. Actually, all three of your pictures were very similar, but a little bit different, and the conversation was a little bit different. You're saying, on this one, I kind of said, "Here's the problem, and do it with this item, or this tool."

Teacher A: Or, "This is how you do it with this tool." Because we had requested it. It wasn't like you were forcing that on us as a strategy, but you were just explaining that strategy because we needed it.

Me: Right, but it didn't really elicit anyone else's—It wasn't like different ways—we weren't talking about different ways on this one.

Teacher A: Exactly.

Me: I didn't put words in your mouth, did I?

Teacher A: No. . . . No, no. You're just paraphrasing what I was saying, but yeah, absolutely.

(personal communication, September 8, 2016)

In this particular section of my interview with Teacher A, she talked about how the group had requested learning how to use Cuisenaire rods, how she could have shown the solution to the given problem using a pictorial model, and how I explicitly showed them how to use the rods. In addition, she mentioned how the group had gotten away from randomly showing how they might solve a particular problem and that they were more focused on just learning how to represent a problem using, in this specific instance, Cuisenaire rods. I suggested that this was evidence that we had stopped talking about different ways to represent the problem, and Teacher A agreed with my analysis. I was afraid that I had made a leading statement when making this suggestion, but Teacher A confirmed that I had correctly paraphrased what she had said. In addition, Teacher A completed a member check whereby she reviewed the entire transcription of the interview and my coding analysis and she responded to me in writing: “I went through both documents . . . . I didn't see any comments or inferences that I didn't agree with” (personal communication, October 1, 2016).

These findings seem to suggest that when the participants engaged collaboratively to solve problems, multiple representations were not elicited. This finding could be considered a disadvantage in mathematics, as one of the primary goals of collaborative group work among students is in the elicitation of multiple representations (NCTM, 2014; Smith et al., 2009).

***Easy problems*.** In addition to the above-listed disadvantages, Teacher C referenced a specific drawback to solving problems collaboratively that was related to a problem's difficulty level. Teacher C felt that solving easy problems collaboratively could make otherwise simple solutions seem complicated. Consider the following exchange during my interview with Teacher C after we viewed an easy problem that had been solved independently:

> Me: If I had given that problem to you, hypothetically . . .to work on and solve together, what do you think of that? Would that have made a difference?
>
> Teacher C: Maybe. Maybe. Because, like I said, I had already solved it; I thought it was an easier problem, and then maybe I would have thought that people were making it more complicated. . . . Because to me, a really easy problem that somebody goes all out making it real complicated, I don't know if I could have followed along with that on something that I felt like was simple. (personal communication, August 31, 2016)

While this passage was the only one of its nature, and a speculation on the part of Teacher C, it revealed that easy problems were possibly not best solved using group collaboration.

Suggestions for increased effectiveness. The sole suggestion for increasing the effectiveness of our time collaborating to solve a problem using Cuisenaire rods was mentioned by Teacher A. She shared her belief that it may have simply been more efficient for me, as the course instructor, to show participants how to solve the problem: "It just would have been better if you had said, 'This is how you do the problem,' and we all listened and said, "Okay, fine" (Teacher A, September 8, 2016).

Purposes and Effects of the Approach to Engagement

Through the analysis of the data related to the communications and interactions that emerged as participants solved problems either independently or collaboratively, as well as the resulting effects—either advantageous or not—of solving a problem using either approach, considerations arose related to the purposes behind why one might use a particular mode of engagement to enact participants in solving problems during a synchronous online mathematics professional development course.

While the teachers in this study reflected that allowing for more time, group collaboration, or scaffolding could have effectively helped them solve a difficult problem attempted independently, they did not fully discredit the value of the struggle they felt while doing so. Along these lines, Teacher A expressed a critical point when suggesting the value of using cohorts of teachers to engage in collaborative problem solving versus just solving problems independently. As she reflected on a challenging problem that was solved collaboratively, Teacher A shared that perhaps the mode of engagement—that is, either solving a problem independently or solving a problem collaboratively—should depend upon the instructional goals of the facilitator. Consider the following remarks:

> Huddling around the screen, trying to put our heads in the same place, that just wouldn't work with the cohort thing. But individually, this last example worked so much better. I guess that would just depend on the point, or the delivery, or whatever the goal was. (Teacher A, personal communication, September 8, 2016)

Later during analysis of this interview, I contacted Teacher A and asked her to explain what she meant by the word "that" in the last sentence of the above passage. She responded: "'That' referred to the way you would choose to manage the group, either a)

going away and solving a problem or b) solving it collaboratively on the screen together" (Teacher A, personal communication, September 28, 2016). In a very similar manner, Teacher C expressed a view regarding the purposes behind either working independently to solve a problem or engaging collaboratively to do so:

> I don't know if you were strategic in this, but how you planned the problems that we would solve alone and together based on how challenging they were, it does matter. . . . I like how you mix up the problems. . . . You seem very strategic on how you picked what we would solve together, and what we wouldn't . . . . (personal communication, August 31, 2016)

These statements are interesting to me as both the course instructor and as the researcher in this study. In truth, I was not at all strategic in the way I planned for teachers to engage in solving the problems online. As was explained in Chapter 3, after the first online session—where the structure of the session allowed only for independent work followed by group discussion—I felt dissatisfied that the teachers were unable to collaborate to solve problems. As is documented by secondary data sources, I engaged in a phone conversation and a subsequent online practice session with the assistant course designer with the sole intent being to determine a way that the participants could solve problems together. Through those efforts, we figured out a way to allow for group collaboration. Having successfully determined a way for the participants to collaborate to solve problems, I never reverted to asking them to solve problems independently. This means that, during the second and third sessions of the course, the participating teachers were asked to engage in solving all of the remaining problems collaboratively.

As I analyzed the primary record, I noted the benefit of multiple representations that emerged out of the independent work of the first session. This finding was supported during the participant interviews when each teacher mentioned how seeing multiple representations was beneficial to her when discussing problems that had been solved independently. In addition, during my analysis of the primary record, I noted that for easy problems attempted during the second and third sessions, I kept leading the participants back to multiple representations. It was as though I wanted them to see multiple representations, even though by putting them together to collaborate to solve each problem, I had, in effect, limited them to solving each such problem in just one way. This action on my part was noted specifically by Teacher A during her interview. Consider the following conversation between Teacher A and myself as she reflected upon some of the pedagogical decisions I made as the instructor of the course:

> Teacher A: I think there were a couple of times—and I don't know why—I think there were a couple of times where you did most of the recap, just reiterating what we had done, I guess.
>
> Me: Like re-explaining some things?
>
> Teacher A: I wouldn’t necessarily change that, because I don’t know why that was.
>
> Me: Right. Interesting.
>
> Teacher A: I know there were times when you were doing that, but you were actually trying to lead us towards—I can't even remember what it was now.
>
> Me: Like a different representation?

> Teacher A: I just remember you saying, “We all solved it like this, but this is how we did it in the planning guide, or this is how we—” I can't remember. I'm frustrated that I can't remember.
>
> Me: That’s okay.
>
> Teacher A: It was something to the effect of, "Yes, this is how we all solved the problem,” and “Yes, this is perfectly fine," but here's another . . . .
>
> (personal communication, September 9, 2016)

The remainder of this conversation turned to Teacher A and me discussing a specific, easy problem the participants had solved collaboratively. In the example we discussed, the participants all solved the problem in the exact same way and in very little time. After they arrived at their solution, I then spent more time asking them if they could represent the problem in a different way.

That the difficulty level of the problem corresponded with my instructional goals was revealing: I found through analysis of the sessions that I spent time encouraging the participants to consider multiple representations when they were collaborating to solve an easier problem, while I did not do this at all with the difficult problems solved collaboratively. For example, during the second session, after the participants collaborated to solve an easy problem using virtual fraction circles, I proceeded to engage them in further conversation where the assistant course designer and I showed the problem using a pictorial representation with rectangles as well as with virtual Cuisenaire rods. We spent 28 minutes discussing this easy problem, though the teachers themselves had derived the solution to the problem in less than five minutes. In a similar manner, after the teachers solved an easy problem during the third session using a pictorial model,

I spent additional time showing them how to use Cuisenaire rods to model the problem. It seems that an understanding of the benefits of multiple representations was an underlying, though perhaps, at the time, implicit goal of mine when presenting easy problems for the group to solve.

For the difficult problems solved through collaboration, no such attempts were made to elicit multiple representations. It was as though the collaborative struggle in determining one way to solve the problem was sufficient: it seemed that my tacit instructional goal for the teachers while solving difficult problems was simply to get them to the solution and have them understand at least one way to represent it. During the second session, the group spent only 17 minutes collaborating to solve the difficult area problem—10 minutes less time than what was spent discussing the easy problem during that same session. During session three, the final three questions attempted were each difficult. We spent 11 minutes, 16 minutes, and 13 minutes, respectively, solving each of these difficult problems. In each instance, we collaborated to solve the problems and discussed only one way that each could be solved.

Negative case analysis revealed it was not just difficulty level that made collaborating an important approach when solving a problem. In one instance, the teachers engaged to solve an easy problem using Cuisenaire rods. Though the problem was one that the teachers all knew how to represent pictorially (as was established by their pre-test assessment results), they valued the time spent collaborating on the problem because they did not know how to represent it using Cuisenaire rods. In this way, the difficulty of the tool itself made collaborating on the problem a worthwhile endeavor.

This finding was supported during participant interviews with each teacher when asked about this specific problem. Consider Teacher A's remarks:

> Me: Classify this problem, easy or hard?
>
> Teacher A: Hard. Well, harder. This is one that you definitely have to draw out a picture for. . . . especially because we were trying to use the Cuisenaire rods . . . . (personal communication, September 9, 2016)

Teacher C described why she felt group collaboration was useful even though she had classified the problem as easy:

> Me: What do you think was difficult, or—you said this was an easy problem or a difficult problem?
>
> Teacher C: I said it was easy, but it was difficult to show with the rods, because I had never used them before.
>
> Me: . . . If it was an easy problem, but it was hard to work through, what would you recommend if you were to do this? Would you have put people like, "Okay, just work on it for five minutes and come back and share?"
>
> Teacher C: No, . . . because this was the first problem where we used the Cuisenaire rods, so I think I liked how we modeled it together as a group. I wouldn't have changed that. I wouldn't have changed [the] way that . . . it was presented.
>
> Me: Okay. Even though it was easy?
>
> Teacher C: Yes.
>
> Me: We were collaborating on an easy problem, but the collaboration was helpful?

> Teacher C: Yes, the collaboration was helpful with the rods for the first time. (personal communication, August 31, 2016)

Teacher B responded in a similar manner:

> Me: What do you think if this had been, "Go on your own and solve it?" Any thoughts about that?
>
> Teacher B: I didn't have any experience with Cuisenaire rods, so then I would've been like, "I don't know."
>
> Me: Right. Do you think you could've gotten the answer, but maybe not using Cuisenaire rods?
>
> Teacher B: Yeah, I could've done it with paper and pencil.
>
> (personal communication, September 9, 2016)

These statements revealed a dual purpose in collaborating to solve a problem. While the teachers felt collaborating was especially beneficial when engaging to solve a difficult problem, they also felt like collaboration was useful when learning to use virtual Cuisenaire rods to represent a problem, even though the problem itself was one they felt confident they could solve using a pictorial model.

Thus, through analysis of the dialogical interview data, and in conjunction with the analysis of the primary record, different purposes emerged for using a particular approach to engage participants in solving problems. Findings revealed that these purposes were connected to, and perhaps even influenced by, the difficulty level of the specific problems participants solved. On the one hand, engaging participants to solve an easy problem independently generated multiple representations. The teachers appreciated this approach for several reasons, not the least of which was that they felt better prepared

to work in the classroom with students and felt as though their time discussing different solution strategies helped prepare them to address student misconceptions and diverse ways of thinking. Conversely, one teacher speculated that solving an easy problem collaboratively might have been disadvantageous to her learning in that it might have brought about unnecessary confusion if a member of the group were to suggest a possible solution strategy that was overly complicated.

On the other hand, engaging participants to solve a challenging problem independently resulted in an overall inability to produce a correct representation of the problem's solution, though some of the teachers commented that the difficulties they experienced while attempting to solve such a problem independently was valuable. One teacher even commented how she felt better about her own struggle when she realized that other teachers also found a certain problem especially arduous. When the teachers solved difficult problems collaboratively, however, they felt great satisfaction about their work. The teachers described how, by talking with one another and building off each other's ideas, they were able to grasp the concepts required to understand the challenging problems. The teachers also reported how working together on difficult problems felt more efficient and saved time. In addition, the teachers reported that through group collaboration they were able to learn how to use Cuisenaire rods. In this way, the teachers reported that it was beneficial that they worked together both to solve hard problems and use tools with which they were previously unfamiliar.

Summary

This section presented a description of findings from the analysis of data concerning the patterns of communication and interaction that emerged as participating

teachers engaged in two different approaches to discussing and solving problems during an online mathematics professional development. Findings suggest that there were advantages and disadvantages to both approaches to solving a problem—that is, either solving a problem independently with subsequent sharing of individual solution strategies or solving a problem collaboratively using interactive virtual tools. Figure 8 provides a summary of the advantages, disadvantages, and suggestions for improvement that were shared by participating teachers during their interviews when they remarked on the communications and interactions that resulted from the times when they solved and discussed problems during the online professional development.

Type of Problem	How Problem Was Attempted and/or Solved	Emergent Patterns of Communication and Interaction		
		Advantages	**Disadvantages**	**Suggestions for Improvement**
All	**Independently**	• Multiple representations generated naturally • Connections made to classroom practice of sharing in groups • Positive feelings generated when sharing ideas	• Feelings of anxiety when sharing own solution strategies • Technical problems prevented sharing of work; noisy to listen to through webinar	
	Collaboratively	• Fostered sharing of ideas, deepening of understanding • Immediate feedback received; time spent together felt more efficient • Resulted in positive feelings about work • Supported learning of virtual tools, Cuisenaire rods • Interactions correlated to desired structure for student engagement in classroom • Improvements in technology made interactions less noisy and sharing of ideas easier	• Technical problems resulted in drops in connectivity; navigation problems • Manipulations of virtual pieces in shared space caused feelings of vulnerability • Problems were represented in only one way; lack of multiple representations	• Instructor to explicitly show how virtual Cuisenaire rods are used to represent problems
Easy	**Independently**	• Beneficial component of course design; opportunity to share own work appreciated		
	Collaboratively		• Potential for over complication of otherwise easy-to-understand concept	
Hard	**Independently**	• Productive struggle appreciated; felt better about personal struggle when struggle of others was realized	• Lack of feedback has potential to frustrate • Unable to correctly represent problem's solution	• Allow for more time to think about problem • Allow group collaboration • Provide scaffolds to support thinking of group members
	Collaboratively	• Difficult problems solved successfully		

Figure 8. Summary chart of advantages, disadvantages, and suggestions for improvement shared by teachers about the times when they communicated and interacted to solve and discuss problems.

Participating teachers noted specific advantages and disadvantages for times when they solved problems independently and then communicated and interacted afterward while discussing their solution strategies. The advantages mentioned for solving problems independently and then sharing with the group included 1) how multiple representations and ways of thinking were highlighted through the participants' conversations, 2) how participants' interactions and work online connected to their efforts at engaging students to share ideas in the classroom and how the teachers felt positive about their interactions with the other teachers online, 3) how some of the teachers appreciated the struggle felt when attempting to solve a difficult problem and how, for one of these teachers, her interactions with the others helped her feel okay about her personal toil with understanding a challenging problem, and 4) how one teacher considered that solving easy problems independently and then sharing with the group was a beneficial component of the design of the online course.

The disadvantages mentioned for solving problems independently and then sharing afterward included 1) how a lack of feedback had the potential to lead to feelings of frustration, 2) how feelings of anxiety existed for one teacher when sharing her solution strategies to the group for discussion, 3) how technical problems resulted in a teacher being unable to share her picture upload and how the online interactions were noisy to listen to through the webinar platform, and 4) how for a difficult problem, the teachers were unable to produce a correct representation of its solution. The teachers reported that their time spent solving a difficult problem independently could have been improved if 1) more time had been provided to think about the problem, 2) collaboration had been allowed, and 3) scaffolds had been provided to support them in their thinking.

The advantages reported about the collaborative work in which participants engaged while solving and discussing problems included 1) how the sharing of ideas and the deepening of understanding were fostered through communication, 2) how immediate feedback was received during the problem-solving process and how the time spent together felt more efficient, 3) how their collaborative interactions resulted in positive feelings among participants about their work, 4) how the teachers were supported in learning to use virtual tools, including Cuisenaire rods, 5) how their interactions correlated to the structure of engagement they wanted to enact in the classroom, 6) how the interactive platform improved the logistics of the technology, in that it reduced the noise level and made sharing one's ideas easier, and 7) how difficult problems were solved successfully.

The disadvantages mentioned about solving problems collaboratively included 1) how technical problems related to the online programs used during the sessions meant participants were occasionally unable to see and/or interact with one another or navigate easily through all of the components of said programs, 2) how manipulating virtual pieces in the shared space caused one participant to feel vulnerable because her actions were visible by others, 3) how participants only represented problems in one way, which resulted in a lack of multiple representations from being highlighted, and 4) how one participant speculated that solving easy problems collaboratively could potentially be cause for over complication of otherwise easy-to-understand material. One teacher suggested that the time spent collaborating to solve a problem using virtual Cuisenaire rods could have been improved had I simply showed them explicitly how to model the problem's solution.

Through analysis of the primary record, four different scenarios emerged regarding the ways in which problems were represented and solved during the online professional development. These scenarios were 1) easy problems solved independently with subsequent sharing and discussion afterwards, 2) easy problems solved collaboratively using interactive online tools, 3) a difficult problem attempted independently with subsequent sharing and discussion afterwards, and 4) difficult problems solved collaboratively using interactive online tools. Qualitative findings suggest differences existed in the effects of using online tools to discuss, represent, and solve problems collaboratively versus when using such tools to represent and solve problems individually. Data analyzed qualitatively in this study seem to support the notion that for problems where prior knowledge was strong and participants could represent and derive solutions independently of one another (i.e., as when solving easy problems), individualized work highlighted multiple representations better than did collaborative work. Conversely, when prior knowledge and ability to independently, and correctly, solve a problem was low (as was determined by teachers' pre-test assessment results), evidence suggests that collaboration in real time using virtual tools supported successful task completion. Teachers reported that collaborative problem solving was especially advantageous when solving difficult problems, as well as when using an unfamiliar virtual tool to represent problems, as was the case when the teachers used Cuisenaire rods for the first time.

Communications and Interactions: Participants Using Virtual Tools

Before viewing any segments of the recorded archives to stimulate recall, each participating teacher was asked during her one-on-one interview to relate from memory

what virtual tools she remembered using during the online course. All three teachers remembered having colored pens and drawing on the webinar's whiteboard screen to solve problems collaboratively. While all three teachers recalled being able to draw at the same time to solve problems, only Teachers A and C went into detail about the various technological tools that were used during the sessions. These two teachers described in detail how the online platform was used to support their interactive collaborations to solve and discuss problems at the same time. They both recalled being able to add to and build upon one another's work, and they both recalled being able to manipulate pieces, literally "moving them around," to solve problems (Teacher A, personal communication, September 8, 2016). Teacher A alone recalled that, initially, participants were only able to take pictures of their work and share their solution strategies by uploading pictures for others to see. In comparison to taking pictures of their independent work and uploading it to share with others, Teacher A stated how the shared space, which allowed the teachers to collaborate together, "was a lot easier" (personal communication, September 8, 2016).

The similarities and differences among each teacher's response regarding the advantages and disadvantages of using virtual tools during the online professional development were revealing: the findings showed a difference in beliefs about the usefulness/nature of virtual tools. These core beliefs connect to communications and interactions that surfaced during the times when participants were engaged in solving and discussing problems.

Advantageous Communications and Interactions

When asked to share her thinking about the virtual tools, Teacher B shared how she felt that the ability to see everyone writing and drawing with a different color on the webinar's whiteboard was effective because "everybody had a voice then" (personal communication, September 9, 2016). When asked to describe their thoughts about the virtual tools, Teachers A and C shared in detail about the various advantages of the tools. Both of these teachers stated that by using the online tools, the group was able to model and solve problems together. Teacher A shared how the tools aided the group in their efforts to interact collaboratively:

> The nice thing about the shared presentation was just that we could collaborate. There was one point where we literally were all talking, and we were saying, "Okay, I'm going to use a blue pen, and I'm going to draw this in, and you're going to use a yellow pen, and shade that in." That felt very efficient, and productive, and collaborative, and . . . satisfying . . . . (personal communication, September 8, 2016)

Teacher C shared how the virtual tools allowed the group to engage in the same type of work that one could expect to experience in a face-to-face training:

> I think the advantage is you still get that hands on practice. You're sitting in front of each other at a PD, and so having online tools to manipulate and move around, as you would concrete objects, was the same. I think that's beneficial for time's sake. Being able to meet in that way and still being able to do the same things you would do face to face, I think it's beneficial. (personal communication, August 31, 2016)

In addition to providing a way for the teachers to work together to solve problems, Teacher C mentioned how the online tools provided me, as the course instructor, with the ability to help her learn and understand the concepts presented during the online sessions:

> I was on the wrong track, but because everybody can see each other and you were watching and guiding and helping clarify. . . . We could all see it at the same time. . . . I wouldn't have done it correctly, but because we could stop and see—right then and there, and fix—I think that was helpful. (personal communication, August 31, 2016)

Teacher A reflected that the actual movements of the pieces were beneficial: "I like [the online tools] better because you can see you moving it, and there's something helpful, when you're talking . . . there's something sort of kinesthetic about it, that watching it move is actually helpful" (personal communication, September 8, 2016).

Moreover, Teacher A described how these virtual manipulatives were not only useful to her as an adult learner, but positively affected the interactions of the group:

> I think for adult learners, the virtual manipulatives are really good, because we are trying to model what are we going to do with the kids, but actually, I think having the actual manipulatives was tedious. . . . I felt like once we were all focused on the same screen at the same time, it was working better. (personal communication, September 8, 2016)

In the above quotation, Teacher A alludes to a time when not everyone was looking at the same screen at the same time. During this portion of our interview together, Teacher A specifically referenced a difficulty that Teacher B had experienced when attempting to use concrete fraction manipulative pieces to solve a challenging problem. This

disconnection referenced by Teacher A was made explicit when Teacher B explained to me, during her own interview, how she felt about using the virtual manipulatives to solve problems. This disconnection is explained more fully in the next section.

Disadvantageous Communications and Interactions

While each teacher mentioned certain disadvantages with the online tools, Teachers A and C expressed concerns that were connected with logistical and technical problems subsequently affecting the group's ability to interact and communicate in the most efficient way possible, while Teacher B expressed a certain disengagement with the online manipulatives in general.

Logistical and technical problems. After stating that three short sessions were not long enough to see a change in their familiarity with all of the online tools, Teacher C expressed to me in her interview that, "We weren't as familiar with the tools as you were . . . [and] . . . our familiarity with working the tools maybe wasn't as extensive . . ." (personal communication, August 31, 2016). Teacher C then indicated that, because of this unfamiliarity, she and the other teachers needed guidance from me, as the course instructor, to use the tools effectively to represent and solve problems. The assistant course designer shared similar thoughts about how the collaborative sessions might have been better had the teachers practiced with the interactive tools before the session:

> I think maybe we needed a little more time . . . to get the teachers where we needed them to be with the technology. . . . I just felt like maybe if we could have . . . developed that whole virtual piece prior to Day 1 . . . where we got the whiteboard to be like interactive all at the same time synchronously. . . it would

> have been nice to have gotten that piece prior to, like have them practice that . . . . (personal communication, March 31, 2016).

On another note, Teacher A expressed that, beyond just not having used them again since the online sessions, she had started to get annoyed with "some of the technological difficulties" experienced by Teacher B, stating that the struggles had started to feel "tedious" (personal communication, September 8, 2016).

Disengagement with virtual manipulatives. While Teacher A reported these technological difficulties, Teacher B expressed no such frustration about her use of the online tools. Instead, Teacher B expressed how she did not consider the virtual tools to be the same as tangible, concrete manipulatives:

> The disadvantage is that it's not a 3-D tool. If I'm using manipulatives, that's another type of experience. You use more of your senses. Then, if you use the online tools, you're not using that many of your senses . . . . The disconnect for me is that it's not real to me unless I see it, I mean, unless I touch it, and then I can manipulate it and say, "Okay," but if it's moving pieces like that, I get an understanding, kind of, but it doesn't click unless I am touching it. (personal communication, September 9, 2016)

This disconnection experienced by Teacher B did not seem to mean that she discredited virtual tools completely, however. After re-watching the archived video that showed how the teachers and I effectively used virtual Cuisenaire rods to model a challenging fraction division problem, I asked Teacher B to share her thoughts about the online tools that we used. She replied, "Yeah, I like those. Cuisenaire online is good for me, yeah, but I still want to do it, you know?" (Teacher B, personal communication, September 9, 2016).

During each of the times when the teachers engaged in representing and solving a problem using virtual Cuisenaire rods, Teacher B would use real, concrete Cuisenaire rods that she had with her at her home, as opposed to working with the others online to show the problem using the virtual rods.

Often, as Teacher B worked out problems using concrete materials, her conversation seemed disconnected from the rest of us. This was noted by observer comments I recorded in my thick notes as the teachers were engaged in solving a challenging fraction division problem using the virtual Cuisenaire rods.

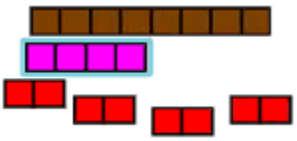
Me: So I guess what I might prompt you to think about is, where do you see a half a mile because we know she ran a half a mile this week, so we need to first identify where a half a mile is (4 second pause)

Teacher B: (To self, looking down at rods by her) Okay, 1, 2, 3, 4 ,5 [OC: *Again, this makes me wonder if having her work with actual rods perhaps disconnected her from working with the group*.]

Teacher C: So, maybe the 4? Maybe the purple?

Me: OK, so why don't you compare that? (I move the red pieces; Teacher C puts a purple piece by the brown). (About a 12 second pause in talking while pieces are moved)

Me: So what do you see now? (8 second pause)



Teacher C: Well now I see 4 and that's half of 8

> Teacher B: Oh, okay [OC: *I'm not sure, but I think this is a comment to herself*]. She ran 8 times as many miles as she did last week. But last week she only ran half, so this, so we wanna know how many miles she ran last week? She ran uh, did she run 3 and ½ miles last week? (Her question was directed to the group) (To self) This is the other half. [OC: *I just realized, Teacher B is talking but working off screen by herself. Her comments are to herself as she is looking down and away from the screen. She does not seem to be looking at what is on the screen and she is not really talking to the others in the group; she is not saying what she is doing with the pieces. Her communication here is quite different from when the teachers were drawing together on the screen*.]
>
> (personal communication, December 16, 2015)

This disconnection from the group seemed to be what Teacher A was referencing when she described her feelings of frustration about the technological problems experienced by Teacher B:

> For me, as a participant, it was painful to watch her struggle with it, but also, I just kept thinking, "What is she getting out of this if she can't see what we're doing, or we can't see what she is doing?" And, because there were only 3 of us, it was magnified every time one of us had a problem with something, because literally a third of us were off. (personal communication, September 8, 2016)

These quotations seem to indicate that Teacher B struggled to engage fully with the representations in the online platform, and this disconnection possibly affected her engagement with the other participants during the times when they had been asked to collaborate and use virtual tools to solve challenging problems. Teacher A questioned

how Teacher B was benefiting from the online sessions if she was unable to see what the group was doing or if the group was unable to see what she was doing.

Suggestions for Increased Effectiveness

Considering the virtual tools that were used during the times when problems were solved, some of the teachers made suggestions about how to increase the effectiveness of the technology in order to make the online experience more seamless. One teacher also reflected about how she felt I could have shown the participants more explicitly how to use some of the virtual manipulatives. These points of consideration are addressed in the next two subsections.

Technical improvements. Teacher A made suggestions for how to increase the effectiveness of our synchronous online collaborations while solving problems. While considering how to make the collaborations in the online professional development better, Teacher A and I engaged in the following conversation.

> Teacher A: The only thing I could say, going forward, if you were going to "can" this, and make this a [district] thing, maybe there's a different technology, or something, where you could almost control some of that to where it's just being sort of spoon fed maybe a little bit easier.
>
> Me: What do you mean, spoon fed . . . of the what?
>
> Teacher A: I'm just thinking about [how] Teacher B was having so much trouble, and I was having trouble . . . . Maybe there's a way to make it so that basically any school laptop, it's going to run, it's going to be . . .
>
> Me: The functionality of the program is not going to be an issue?

> Teacher A: It's going to work. You're going to go to the [district's online platform]—because we all have access to [it]—you're going to double click on the [platform], and this thing is going to open. Then everything's going to be within that thing, just to sort of normalize the experience, then maybe cut down on some of that potential for technical disaster.
>
> (personal communication, September 8, 2016)

The thoughts shared above by Teacher A revealed some of her thinking about possible ways to make the online experience better regarding, specifically, the participants' ability to access all of the components of the course in one place. As it was, participants were asked to log in to up to four different systems during our time together. They had to log in to the webinar platform to have audio, speech, chat, and screen sharing capabilities. They had to log in to *nearpod* to view the presentation slides that were being advanced for them as we went from question to question. They had to log in to the district's digital platform to access the link to *nearpod* and other course documents and materials, and they had to log in to *Google Presentation* in order to manipulate the virtual fraction pieces. Teacher A expressed how having to log in to so many different systems increased the potential for technical problems to arise.

In addition to mentioning that Teacher B had experienced technical difficulties, Teacher A related that she ran into trouble herself when accessing the multiple components of technology required for the course. She and I discussed the problems she experienced when attempting to access all of the components of the course.

> Me: I recall, I think you had two computers going sometimes?

> Teacher A: I always did, . . .that's because my laptop is so old that I don't have a microphone in my home laptop, and my school laptop wouldn't run some things, so I always had my phone for audio, then my computer for visual.
>
> Me: I remember you had a setup, but you had it always working.
>
> Teacher A: I was on a couch, but I felt like I was surrounded by technology. It worked. It was fine, but it was not quite as seamless as it could have been. (personal communication, September 8, 2016)

These thoughts indicate how Teacher A had to use multiple pieces of technology to log in to all components of the system, and that feeling as though she was surrounded by technology made the process of engaging in the online programs less than ideal.

Teacher C shared about how she was impeded in her efforts to see everyone during our online conversations because of the multiple different programs she had to have open on her computer. Consider the following discussion she and I engaged in about her experience interacting online with all of the aforementioned computer programs.

> Me: Anything . . . with these online tools with regards to the tools when we were solving problems together, is there something you would have liked to have been different?
>
> Teacher C: Maybe have—I think we talked about that—have two screens up or something like that, because I like to be able to see everyone, but then when we would have to switch to n*earpod* or something else, I wouldn't see anybody. Being able to see those two things simultaneously I think would have been

> helpful, and I think you mentioned that we could do something like that with a VGA cord or something like that.
>
> Me: Yeah, by connecting it to a monitor.
>
> Teacher C: I was going to try it, but . . . I didn't want to mess up anything. I wanted to make sure I was on there on time.
>
> (personal communication, August 31, 2016)

In the above conversation, Teacher C revealed how she was dissatisfied with not being able to see the other participants when she switched between programs. Although we had discussed before the sessions began how the participants could connect an additional monitor to their laptop computers, Teacher C expressed a fear that if she attempted this action, then she might inadvertently cause a problem for herself in accessing the online sessions in time for our synchronous meetings.

These findings suggest that the teachers would likely have had a better experience during their engagements with one another had the technology been more seamless and allowed them to visually see one another at all times, regardless of whatever component of the online course they were engaged in using.

Pedagogical considerations. On a different note, Teacher A made a suggestion related to pedagogical actions on my part as the course instructor related to the participants using virtual tools. When considering the times when we used the virtual Cuisenaire rods to solve problems, Teacher A reflected that it might have been more efficient for me to just show the participants how to use the rods, as was related in a previous section of this paper. Her specific suggestions are listed again here for reference.

> Me: What do you think about the way we worked on this, or how we did this problem?
>
> Teacher A: I think that was awful, now that I watch the video . . . .
>
> Me: What was awful?
>
> Teacher A: It just would have been better if you had said, "This is how you do the problem," and we all listened and said, "Okay, fine."
>
> (personal communication, September 8, 2016)

These remarks indicate that the teachers struggled in using the virtual Cuisenaire rods and that Teacher A thought it might have been better for me to just show them how to use the rods. Later on during this same conversation, Teacher A explained how she specifically felt that it was awful how the participants were waiting and how there was dead time while they were engaged in modeling the problem using Cuisenaire rods. Teacher A's reflection above indicates that she might have believed that the online tools were best learned, at first, through direct modeling by the teacher. More information about this topic is included in the upcoming section of this paper as it relates to the communications and interactions that occurred between the course instructor and participants.

Summary

This section presented information about the communications and interactions that emerged among participants as they used virtual tools to solve and discuss problems while participating in the online professional development. Figure 9 provides a summary of the advantages, disadvantages, and suggestions for improvement that were shared by participating teachers during their interviews when they remarked on the communications

and interactions that resulted from the times when they used virtual tools during the online professional development.

	Emergent Patterns of Communication and Interaction		
	Advantages	**Disadvantages**	**Suggestions for Improvement**

Using Virtual Tools	• Everyone had a voice to participate
	• Shared presentation allowed for helpful, efficient, and productive interactions
	• Same as face-to-face training
	• In-the-moment feedback on virtual manipulations
	• More efficient than concrete, physical manipulatives
		• Logistical and technical problems
		  ○ Impeded from seeing or hearing others
		  ○ Difficult navigation of multiple programs
		• Disconnection to virtual manipulatives
		  ○ Effects of disconnection on others
			• Logistical and technical
			  ○ All-in-one technology platform
			  ○ Two computer monitors
			• Pedagogy
			  ○ Direct instruction of tool usage

Figure 9. Summary chart of advantages, disadvantages, and suggestions for improvement shared by teachers about the times when they used virtual tools.

Several reasons the teachers felt that using virtual tools was advantageous and disadvantageous were provided. Advantages to using virtual tools included how the tools allowed everyone to have a voice to participate equally in the discussions, how the shared presentation allowed for helpful, efficient, and productive interactive collaborations, and how interacting and obtaining the same hands-on practice as is often received during face-to-face trainings was considered by the teachers as beneficial. Other advantages listed included some teachers feeling that it was helpful to receive in-the-moment feedback on their virtual manipulations and, for one of these teachers, how the interactions with the virtual manipulatives were considered more efficient than using concrete, physical manipulatives.

Disadvantages included logistical and technical problems connected to a lack of familiarity with the tools and technological difficulties with running all of the components of the applications used during the online sessions. These problems led to a

need for the participants to receive help from the course instructor in how to use the online tools to represent and solve problems. Moreover, a disadvantage for one teacher was a disconnection she felt with the virtual manipulatives. Another teacher commented about how the technical problems faced by the teacher who felt disconnected from the virtual manipulatives was difficult to watch and caused her to wonder what this other teacher was gaining from the training when she was unable to be fully present with the others online. Last, logistical and technical problems impeded the teachers from seeing each other at all times when using multiple online applications and in other ways complicated their interactions when attempting to communicate their thinking with one another. Some teachers suggested remedies for these technical problems and provided pedagogical suggestions for how to increase the effectiveness of their experience interacting with the virtual tools. The pedagogical suggestion provided by one teacher referred to the communications and interactions that occurred between participants and myself as the course instructor, which is the topic explored in the next, and final, section of this chapter.

Communications and Interactions between Course Instructor and Participants

This section of text provides details about the patterns of communication and interaction that emerged between the participating teachers and myself, as the course instructor, during the times when they collaborated to represent and solve problems during the online professional development. Contradictory findings from the preliminary analysis of the primary record and the participant interviews are described in the subsections to follow.

Preliminary Analysis of the Primary Record

During my initial analysis of the primary record, I found what seemed like evidence that communications and interactions between the teachers and myself were not in alignment with my beliefs about how a mathematics educator should lead participants through a training. Specific segments of the primary record showed me directly moving the virtual pieces and/or drawing on the screen to model certain problems for the teachers during times when they had been asked to collaborate to solve problems. These instances of me contributing to solving the problem by drawing or manipulating pieces were especially prevalent when the teachers had been asked to use Cuisenaire rods or comparative models to represent and solve certain of the more difficult fraction division problems. A peer debriefer, familiar with both qualitative research and mathematics education, commented about these interactions in her feedback to me, indicating that these behaviors were not consistent with what the literature recommends for allowing participants to struggle when solving mathematical problems. I agreed with the peer reviewer, and several of my observer comments while creating the primary record of the teachers' interactions while watching the archived videos commented on this point.

Analysis of Participant Interviews

As was explained in the previous chapter, I designed the final portion of my interview protocol to elicit feedback from the participants about my interactions with them (see Appendix A). In an effort to determine whether the teachers thought I had provided too much help or whether I had completed work for them, I asked them to consider the roles that I played during the times when they solved and discussed

problems. The teachers' responses during these segments of their interviews contradicted my presupposed beliefs: they felt all of my interactions were helpful and appropriate.

Consider the following remarks made by Teacher C after watching a clip where the teachers collaborated using graph paper to solve a challenging fraction multiplication problem: "You gave us wait time and we were just sitting there like, 'I don't know.' That's when you gave us some questions to talk through it. I liked how you let us struggle like you would in your classroom" (personal communication, August 31, 2016). Teacher C commented more about my interactions near the end of her interview as we engaged in the following discussion:

> Me: What role do you think [I] played . . . as the course instructor during the times when you were solving problems together?
>
> Teacher C: More like a facilitator. More you, per se, asking questions, facilitating, guiding through, helping, never validating what the right answer was, but encouraging us to think deeper and to share.
>
> (personal communication, August 31, 2016)

This comment made by Teacher C indicates that she thought I took on the role of facilitator and encourager. She recalled that I helped the participants, asked them questions, and declined to validate whether their answers were correct.

After viewing the section of video that explicitly showed me doing work for the teachers, Teacher C and I engaged in the conversation recorded below. Teacher C begins this segment of dialogue by referring to the way in which the teachers were asked to engage in solving the problem.

Teacher C: It was another one where you left it open, right? With what we could use, or what we were going to draw . . . you said think about how to show it?

Me: I started this one by saying, "It's on you guys." And then I think I put the rectangles in.

Teacher C: Yeah, to get us going.

Me: Yeah, maybe. . . . So do you think this confirms what you shared before, about my role?

Teacher C: Yeah, more as a facilitator and guiding and helping us through, yeah. It confirms it, I think.

Me: I was a little worried when I watched this myself, because I was like, "I'm doing all the work," right? Like what I said right there—

Teacher C: No, but I think you came—

Me: —"I'm doing the work. I'm doing the work for them."

Teacher C: —It came based on what you were seeing. You were seeing us kind of talk through, and so then you started putting up what we were talking through. It was helpful, because we weren't as familiar with the tools as you were, so I think that helped, you putting those up and kind of get us, "Oh, okay. We can add because we can write, too." . . . I think you weren't doing too much because we needed help guiding through, "Okay, so what should we write? Should we write on here?" and, "He's going to start it for us so we can write."

(personal communication, August 31, 2016)

In this reflection, Teacher C agrees in part that I put the rectangles on the screen for the participants, but she indicated that this, to her, did not mean that I was doing too much of

the work for them. Rather, she explained that she thought my assistance was warranted based on the participants' unfamiliarity with the online tools. She explained her perception that my assistance was helpful because it allowed the participants to know how they could go about representing their thinking and solving the problem.

In the interviews with Teachers A and B there was not enough time to view the clip that Teacher C viewed that showed me drawing the boxes on the screen for the participants. However, there were sections of other clips these teachers viewed during their respective interviews that showed me contributing in various ways to help them solve problems. Immediately following the viewing of a clip where I had moved Cuisenaire rods on the screen to model the fraction division problem the teachers were attempting to solve, Teacher A made a comment about my actions. In the following comment, the word "that" refers to how I was manipulating the pieces on the screen for the teachers: "I think that's you doing that, while we're waiting for them to have a conversation" (personal communication, September 8, 2016).

Later, when discussing more about Cuisenaire rods, Teacher A shared these thoughts about my interactions with the teachers: "I think my memory is that our focus was literally you training us to use Cuisenaire rods, so you would say, 'This is how you do it with Cuisenaire rods'" (personal communication, September 8, 2016). After viewing a challenging problem that the teachers had worked to solve using the rods, it seemed as if Teacher A confirmed that I jumped in to help when the participants were struggling when she made this comment:

> It didn't feel like we were waiting. You let us struggle, but then it was like you threw a little bit of something in, once we sort of realized, "No, we're really stuck.

> We're not struggling; we're like drowning." Yeah, it helped. (personal communication, September 8, 2016)

These comments seem to suggest Teacher A recognized that I was literally doing the work for the participants. When asked about this more specifically near the end of her interview, Teacher A and I engaged in the following conversation. In this conversation, I recall from memory a section of the archived video where Teacher A had specifically commented about me moving the pieces for the teachers.

> Me: On some of these clips that I watched, I thought I did too much of the work . . . like . . . for instance, where I'm moving all the pieces. Then you're like, "I'm just watching you move all the pieces. You're doing a really good job." And I was like, "Oh, you know, I'm just trying to go fast." There's another clip where I'm doing something, and I'm like, "Okay, I'm going to stop." I put up, "Go ahead and just solve this however you want." I say, then—like 30 seconds later—who's the one putting on? Me! It was very concerning to me to watch that.
>
> I wanted to get your . . . perception, especially after you watched that . . .did you think that that happened where I kept jumping in, and kind of taking?—I don't know if it was pushing . . . I keep telling you guys, "Let the people struggle. Let them struggle." Then here I come: I kept chirping in.
>
> Teacher A: I didn't feel like you were being pushy, or pushing a certain strategy, or pushing an anything. I think I thought of you almost as another participant, so I almost feel like maybe—for one thing, I may not have been looking at the screen—but then, another thing is, maybe I was just noticing what you were doing. I think at the end it was really focusing on the Cuisenaire rods and, “How

> do you solve that problem using the Cuisenaire rods?" I really was watching you do it, because I was processing, in my head, "Oh, right, so it's 16, because that's the halves, and that's the . . . ." You know what I mean? I was really watching you do it.
>
> (personal communication, September 8, 2016)

In the above passage, I presented my thinking to Teacher A for analysis. She explained her view that she considered that I was almost like another participant in the group, how she might not have even been looking at the webinar screen at the times when I was manipulating pieces, and that she believed that, when she did watch me manipulate the pieces, it helped her to learn how to use Cuisenaire rods. These comments seem to confirm that Teacher A agreed I was doing the work of manipulating the pieces, but she did not consider this to be problematic.

Teacher B shared something very similar about my interactions with the participants during the time when they were learning how to use Cuisenaire rods. Consider the following exchange that took place between Teacher B and myself during her interview after watching a clip where Cuisenaire rods were used to represent the problem's solution:

> Me: Anything that you liked or disliked about the way we went about solving this problem?
>
> Teacher B: It was good. Yeah. Because you have more experience with Cuisenaire rods you could tell us how to use them more . . . .
>
> (personal communication, September 9, 2016)

The above statement seems to indicate that Teacher B considered my knowledge of Cuisenaire rods important, in that I could tell the participants how to use them. I asked her to share her thoughts about my interactions with her and the other participants when they were working on problems. In response, Teacher B shared the following thoughts:

> Without you, we wouldn't have made it through. Because when we get stuck or you ask the question, I'm like, "Really?" It gets you to thinking. We needed you to be a part of the group. Like, I think everybody's experience or lack of experience is important to the group. You have to have everybody in the group, because we have to have a leader, because if we don't have a leader then we won't get through the hard stuff—that was all hard to me. We wouldn't get through. We need a leader to ask the questions. So then you say, "Okay, oh, I didn't think about that." (personal communication, September 9, 2016)

In the above quotation, Teacher B expressed how she felt that she and the other participants would not have made it through the challenging problem without the questions that I asked to stimulate their thinking. Teacher B expressed that there was a need for me to be part of the group, and according to her analysis, the participants needed a leader to ask them questions and bring to their attention certain concepts so that they could successfully complete difficult tasks.

After the above response, I asked Teacher B a more direct question about whether or not she believed I had assisted the teachers too much during the times when they solved difficult problems. Consider the following section of our conversation together:

> Me: Some of these videos that I've watched . . .I've been kind of critical on myself . . . . On . . . some of these, I'm doing all the moving of the pieces. I tell

> you guys, "Go ahead, work on the problem," and then I'm the one . . . I start to go in and draw the boxes and insert the boxes. Did you think that there was a lot of that with this course where I was coming in and doing a lot of the work and the lifting, as opposed to allowing that to be on you guys?
>
> Teacher B: No. I didn't come thinking about a perfect situation, so I don't think education is about perfection. Like a perfect teacher or perfect whatever. I don't think it's about that. I think it was a good situation, an effective situation, because we all learned and we grew. That was good.
>
> (personal communication, September 9, 2016)

In the above conversation, I very explicitly brought up the fact that I felt like I was doing all the moving of the pieces. I recognized that when I asked Teacher B if she thought I was doing a lot of the work for the participants that my question could be considered leading. In her response during the interview, she replied that she did not think I did too much of the work, and she commented about how she does not think there is such a thing as a perfect situation in education. She shared that she felt the course was good and effective because all of the participants learned and grew. During member checks, after receiving the coding analysis from her interview, Teacher B confirmed she had read my analysis and that she had no desire to clarify further any of her comments from the interview. This seemed to indicate to me that she agreed with the analysis and the coding as I had prepared it.

Summary

This section of the chapter presented different interpretations of the communications and interactions that emerged between the course participants and

myself, as the instructor, during the times when problems were solved and discussed collaboratively using virtual tools. In this section it was related how, during my preliminary analysis of the primary record, I found segments of the archived sessions that showed me drawing on the whiteboard or manipulating the fractions pieces for the participants. A peer reviewer commented that she felt like these actions on my part were contradictory to effective practices in mathematics education, and I agreed with her.

During the interviews with participating teachers, I asked them if they thought I had done too much of the work for them. In their responses, the teachers recognized that I had moved pieces or added to the drawings when they were solving problems, but they did not report thinking that there was anything wrong with these actions. The teachers commented that they felt like my contributions during the times when they were collaborating to solve problems were merited: they felt they needed additional help in learning how to use the online tools and manipulatives, and they speculated that they might not have been successful in solving some of the more difficult problems without my help.

Conclusion

This chapter presented findings from the qualitative analysis of the sources of data listed in the previous chapter in an effort to answer the research question of this study regarding the patterns of communication and interaction that emerged during synchronous, online professional development sessions where three fifth-grade math teachers learned about fraction multiplication and division using virtual tools. During these online sessions, participating teachers used virtual tools to either solve problems collaboratively or discuss problems attempted independently. The analysis of the

primary record of the video-recorded archives of these sessions led to the development of an interview protocol (see Appendix A), which was used to conduct semi-structured participant interviews. This preliminary analysis revealed that differences existed regarding the resulting effects of the participants solving a problem either independently or collaboratively. These effects resulted in different patterns of communication and interaction that seemed influenced by a given problem's difficulty level. The findings from the analysis of the dialogical data of this study were organized into three broad categories.

The first category related to the communications and interactions that emerged as participants solved problems either independently or collaboratively. A description of what the participants reported as the advantages and disadvantages to solving problems either independently or collaboratively was provided. Qualitative analysis of data obtained through participant interviews supported the preliminary analysis described above. Findings further revealed different effects from using virtual tools to discuss, represent, and solve problems collaboratively versus using such tools to represent and solve problems individually. These results demonstrated that for problems where prior knowledge was strong and participants could represent and derive solutions on their own, solving problems independently naturally resulted in multiple representations of solutions. Conversely, solving problems collaboratively resulted in a single representation of a problem's solution. Moreover, when prior knowledge and ability to independently and correctly solve and represent a problem was low, findings revealed that collaboration in real time using virtual tools supported successful task completion. Further, participating teachers reported that collaborative problem solving was

advantageous when solving difficult problems, as well as when learning to use unfamiliar virtual tools to represent otherwise easy problems.

The second category of communication and interaction analyzed in this study focused on data related specifically to the times when participants used virtual tools to solve and discuss problems. A list of both the advantages and the disadvantages participants shared regarding their use of virtual tools while solving and discussing problems was provided. While advantages included the teachers being able to engage collaboratively to solve problems, a disadvantage emerged for one teacher in that she felt disconnected from the virtual manipulatives. This disconnection seemed to affect this teacher's interactions with the other members of the group during times when they had been asked to collaborate and use the virtual manipulatives to represent and solve fraction problems.

Contradictory results became evident in the analysis of the final category of communication and interaction examined in this study. This category looked specifically at the exchanges that took place between participants and the course instructor. While the course instructor felt his actions in manipulating pieces and drawing on the whiteboard screen during the times when the participants were collaborating to solve problems was not in line with effective practices reported in mathematics education literature, the participating teachers did not share the same belief. The teachers reported that the actions taken by the course instructor were helpful as they learned to use the new virtual tools of the course, and they speculated that they might not have been as successful in representing and solving certain of the more difficult problems without the assistance of the course instructor.

Chapter V

Conclusions

This chapter presents a brief review of the literature related to the need for this study, as well as a detailed discussion of the findings from this study. The forthcoming discussion is related to how problem difficulty and mode of engagement, virtual tools, and facilitator contributions affected the communications and interactions that emerged as teachers solved and discussed fraction multiplication and division problems while interacting online during three professional development sessions. For each of these discussion points, findings from the previous chapter have been connected to the literature discussed in Chapter 2, and implications for practice and future research are proposed.

Relevant Literature

Advances in technological communication systems now allow classroom teachers to learn content and pedagogy through the convenience of online trainings (Dede et al., 2009). Researchers in the field of online learning have called for additional opportunities for classroom teachers to collaborate online while engaging in experiences that allow them to solve difficult problems connected to practice-based tasks (Herrington et al., 2003; Herrington et al., 2004; Reeves et al., 2004). However, these same authors report that most developers of online learning have not yet created such opportunities for classroom teachers (Reeves et al., 2004).

Cady and Rearden (2009) expressed concern about the integrity of online learning modules, specifically those that address practices designed to help teachers engage students in learning a particular concept, when limitations in the online learning platform

make it seemingly impossible to model effectively the practices being taught. In particular, these researchers felt dissatisfied that they were unable to see participants' manipulations while solving problems online using virtual manipulatives (Cady & Rearden, 2009). Professional development literature clearly establishes that teachers should experience learning that mirrors the types of practices they are expected to implement in their own classrooms (Darling-Hammond & McLaughlin, 1995; Desimone, 2009). This expectation is particularly true of mathematics professional development sessions focused on helping teachers learn conceptual models for involving students in lessons about representing and solving fraction multiplication and division problems (Rhine & Bennett, 1998), as this topic is challenging for both students and teachers to understand (Ball, 1990; Li, 2008; Lo & Luo, 2012).

Overview of Study's Significance

Little research exists regarding the nature of communications and interactions that emerge when mathematics educators participate in synchronous, online professional development sessions that employ virtual tools to assist them in solving and discussing problems collaboratively (Francis & Jacobsen, 2013). The current study investigated the patterns of communication and interaction that surfaced as three fifth-grade mathematics teachers interacted with one another and their course instructor while solving and discussing fraction multiplication and division problems. During their time together, these teachers worked both independently and collaboratively using online tools, including virtual fraction pieces that could be seen and manipulated by all. Understanding the nature of these teachers' experiences is important because researchers in the field of synchronous online collaborations have highlighted the fact that genuine

collaboration among participants, as well as immersed involvement with representations in an online platform, are important components, among others, of effective online task engagement (Herrington et al., 2003). Further, this type of collaboration is especially important in teacher professional development sessions where an expectation exists that learning experiences for educators include tasks that are practical and related directly to the work they perform each day in the classroom with their students (Darling-Hammond & McLaughlin, 1995).

The findings from this current study contribute to the body of research related to synchronous, online mathematics professional development and course design, specifically research related to how virtual tools can be used to involve participants in collaborative problem solving (Francis & Jacobsen, 2013). The content of this chapter provides a discussion of the findings of this study, which were detailed in the previous chapter. In addition, recommendations for the implications of these findings as related to practice and future research are provided.

Discussion of Findings

This section includes a discussion about how problem difficulty and mode of engagement, virtual tools, and facilitator contributions affected the communications and interactions that developed as participating teachers solved and discussed fraction multiplication and division problems while interacting online during three professional development sessions. Within each of the following subsections, findings from this study are considered in light of the literature presented in Chapter 2. Thereafter, implications for practice and future research are also provided.

Problem Difficulty and Mode of Engagement

While analyzing data from the primary sources of this study for emergent patterns of communication and interaction, information surfaced pertaining to the advantages and disadvantages of both the independent and collaborative work in which the participants engaged. Two of the teachers in this study alluded to different purposes for their being asked to solve problems either independently or collaboratively. Teacher C shared how she felt I had been strategic in designing how the teachers were asked to approach solving problems, and Teacher A remarked that I had likely asked the teachers to solve problems independently or collaboratively dependent on my instructional goals for each problem. These findings indicate that developers of online mathematics professional development sessions, where opportunities exist for participants to represent, solve, and discuss problems, ought to consider carefully their underlying objectives when deciding how participants will be asked to approach problem-solving tasks.

In relating the above comments from the teachers to the outcomes of the analysis of data from this study, findings indicated that different communications and interactions resulted when participants solved problems collaboratively or independently. Further, it was found that the difficulty level of a problem and the teachers' familiarity with the virtual tools used to represent and solve it further influenced these communications and interactions. This finding substantiates what Rhine and Bennett (1998) described about how developers of professional learning sessions need to design carefully the problem-solving tasks that they present to teachers. NCTM (2000) explained that professional development should provide teachers with opportunities to "engage in meaningful learning through individual and collaborative experiences" (p. 7). Connecting

specifically to teacher learning through online mathematics professional development, Francis and Jacobsen (2013) expressed that their most important finding was in understanding how the selection of the task used to engage participants in solving a problem collaboratively using virtual tools resulted in the success or failure of their time spent together online. In much the same way, the conclusions drawn from this current study indicate that, when designing problem-solving experiences for teachers, it is important to consider problem difficulty as well as whether participants are asked to approach solving a given problem independently or collaboratively.

Moreover, Hiltz (1998) discussed differences between engaging in online learning experiences in isolation or through collaborative efforts and sought to determine if one approach was more effective than the other. In the end, Hiltz (1998) expressed that online instruction should consider community formation as its primary objective, reporting that such community formation is established through "the exchange of ideas, information and feelings" (p. 7). In a similar way, Ellis and Kisling (2009) expressed that online professional development should offer exciting and dynamic content to stimulate and motivate participants, including tasks that require problem solving. The study described in the previous chapters of this paper offered a series of problems for participants to solve either independently or collaboratively. As they represented and solved these problems, participants were able to communicate their ideas and feelings, which is likely what led to their becoming more comfortable with each other as the sessions progressed. A discussion about the communications and interactions that emerged while the participants in this study solved and discussed problems both

independently and collaboratively is presented in the following two sub-sections of this paper.

Solving problems independently. An important finding from the analysis of data from the current study, and one that was categorized by the teachers as being an advantage, was that multiple representations emerged naturally when participants worked independently to represent and solve problems. Considering that multiple pathways to derive the solution to a problem did not surface when participants solved problems collaboratively, independent work seemed to have unique value to the teachers' learning experiences. The teachers in this study specifically expressed how the time spent discussing their respective solution strategies was beneficial to their work in the classroom and helped them to develop a deeper understanding of the concepts being taught. Furthermore, the teachers expressed positive feelings associated with their opportunities to share their thinking and described how doing so while engaged together online emulated the types of experiences they wanted their students to experience while in the classroom.

Darling-Hammond and McLaughlin (1995) stated that professional development must deepen teachers' understanding of how their students learn, as well as provide effective methods to support that learning. These statements relate to the importance of eliciting different representations during mathematics professional development sessions focused on problem solving because multiple ways of seeing how a problem can be solved helps one to understand students' thinking in a deeper way (NCTM, 2014). That the teachers appreciated the various ways in which problems were represented and solved as the group talked about their differing solution strategies also reflects research by Smith

et al. (2009), who presented an effective structure to design problem tasks in a way that brings forth diverse perspectives and multiple representations.

Considering the disadvantages shared by the teachers about times when they solved problems independently, recall how Teacher C felt particularly anxious when sharing her own solution strategies with the group. Teacher C expressed that she was afraid that if she made a mistake or misrepresented a concept then the other teachers in the group might ask her probing questions that could be perceived as implying a certain level of concern about her ability to teach effectively. Because the first two questions presented to the teachers for them to solve independently were easy, it is possible that these feelings of anxiety were reduced because all of the teachers were successful in solving the problems on their own. Similar outcomes might not have resulted had the initial problems teachers attempted been difficult. Later in her interview, Teacher C shared specifically that solving problems both independently and collaboratively was helpful and that she appreciated the opportunity to share her own work because it made her feel that her ideas were valued. These remarks demonstrate the ideas set forth by Hebert (2007) about how discourse can help to develop communities of practice among teachers participating in online learning sessions.

In much the same way, Teacher A expressed her view that the struggle felt while solving a hard problem independently also contributed to online community formation. Recall that Teacher A shared how she felt better about her own struggle when she saw that other teachers were also struggling with a difficult problem. Being able to relate to the other teachers in the group helped the teachers feel like they were human, too. This humanization of the teachers as they struggled when attempting to solve a difficult

problem independently was important in that it contributed to their forming a real community of practice.

The above statements indicate the importance of carefully designing the sequence of problems that participants in an online training are to experience as they work independently, while recognizing the benefits of presenting both easy and difficult problems. Careful consideration to the sequencing of such problems is likely to affect whether participants develop trust with the other members of the group. Developing trust among other participants as well as the session facilitator is something that Hebert (2007) indicated is a necessary component of engagement when seeking to form authentic online communities.

Solving problems collaboratively. The teachers who participated in this study viewed their ability to work together collaboratively to solve fraction multiplication and division problems as effective because it helped them learn the concepts taught during the online sessions, fostered the genuine sharing of ideas, saved them time when determining the solution to a problem, and allowed them to receive immediate, in-the-moment feedback on their work. In addition, all of the teachers expressed positive feelings about their collaborative interactions and stated that collaboration was especially beneficial when solving difficult problems. These statements reflect research by Desimone (2009) and Signer (2008), who both stated that collaboration is an essential component of teacher professional development. They also demonstrate recommendations offered by NCTM (2000), where it was asserted that teachers should participate in important learning experiences that are collaborative in nature.

Desimone's (2009) call for authentic opportunities for collective participant engagement in online professional development trainings was illustrated when the teachers in this study reported how their learning connected specifically to the content they were teaching. In the current study, teachers reported they were able to work together to solve problems that they later used with their own students in the classroom. Participants expressed that their ability to interact with one another and receive immediate feedback during the online sessions helped them reflect on their work with students and apply their learning from the online training to their classroom practice. Experiencing content and practices in advance of when it was to be taught in the classroom aligned with recommendations outlined by Ostashewski et al. (2011). These authors proposed that experimenting with concepts and exploring ideas with peers online before introducing said concepts and ideas to students in the classroom is an important component of effective online teacher professional development.

On another note, a disadvantage related to the uniformity of the teachers' work as they represented and solved problems emerged when they worked collaboratively. When the teachers in this study solved problems together, multiple solution strategies did not surface. This lack of multiple representations is problematic because in mathematics education, varied ways to arrive at the solution to a problem are highly valued when representing concepts in general (NCTM, 2014), but are especially important when representing fraction multiplication and division concepts (Ball, 1990; Li, 2008; Lo & Luo, 2012). Reflecting more on the disadvantages of working collaboratively, some teachers expressed frustrations with technical problems, such as drops in connectivity, which would occasionally result in them being unable to hear or see others in the group.

Finding solutions to problems regarding technology was an important component of Hebert's (2007) recommendations, where it was emphasized that technical support should always be available to help teachers navigate successfully through applications being used in online teacher professional development sessions.

Virtual Tools

The teachers in this study reported learning how to represent certain concepts using pictorial models and fraction manipulatives, specifically Cuisenaire rods. During the professional development sessions analyzed in this study, participating teachers were able to draw representations, see and move virtual manipulatives, and discuss their thinking in shared spaces online while solving problems. Being able to interactively manipulate fraction pieces and draw representations to problems on the webinar's whiteboard provided a solution to my initial concern about participants' inability to collaborate with each other while solving problems.

In addition, this solution helped participants and myself avoid the problems that Cady and Rearden (2009) described in their study. These researchers reported that online mathematics instructors felt dissatisfied when they were unable to model pedagogical practices effectively, as they were unable to see their participants' interactions with virtual manipulatives in the online environment. In the current study, the teachers reported that seeing me demonstrate how to use the virtual pieces helped them to understand better how certain of the more difficult problems could be represented using Cuisenaire rods. These comments are representative of research by Signer (2008), who stated that online facilitators should guide participants through direct interaction and active monitoring.

Teacher A indicated that a disadvantage to solving problems collaboratively with virtual tools was that there was shuffling and noise, though she stated that during some of our times together while solving problems, this hindrance was not apparent. After reviewing the data, it was discovered that Teacher B was causing most of the noises about which Teacher A complained. In these instances, Teacher B was engaged in working with concrete manipulatives near her computer to solve the problems the others in the group were attempting to solve collaboratively with the virtual Cuisenaire rods. In attempting to understand possible reasons for Teacher B's use of concrete manipulatives instead of virtual ones, it is important to understand what information teachers received during their initial orientation to the professional development in which they were participants.

Before the online sessions began, teachers were asked to have concrete fraction manipulatives available for use during the times when they would be working to solve problems. Because teachers were initially only going to solve problems independently, take a picture of their work, upload it to *nearpod*, and then share their thinking with others, having concrete fraction pieces was necessary in that they were expected to model solutions to the problems presented. Considering this initial structure for solving and sharing about their thinking, Teacher B's use of concrete manipulatives to represent and solve the fraction problems presented was in alignment with the original design of the professional development sessions. The noise created during the second and third sessions, when participants were asked to engage collaboratively to solve problems using virtual tools, was not created during the first online session because during the first

session all participants were put on mute while they took time to solve problems independently and upload a picture of their work.

Because Teacher B was unable or uninterested in entering the *Google Presentation*, she simply continued to use concrete fraction manipulatives while the other two teachers and I began to use the virtual ones. The primary record and Teacher B's own reports during her interview revealed that she continued to, and preferred to, use concrete fraction pieces instead of the virtual ones. This fact being considered, it is recognized that the disconnection to the virtual pieces that Teacher B experienced might have resulted, in part, because an unclear expectation was communicated at the start of the second session when virtual manipulations were first used to engage participants in collaborative problem solving.

Teacher B's inability to enter the *Google Presentation* seemed to affect her subsequent interactions with the group, in that she continued to focus her efforts on using concrete fraction pieces that could be neither seen nor manipulated by the others in the group. Teacher A specifically commented about this point, expressing that she was unsure what Teacher B was gaining from the training, as she was unable to access the shared space and the other teachers were unable to see her manipulations of the concrete pieces. Even though her interactions were disconnected from the group, Teacher B expressed during the interview how fond she is of Cuisenaire rods and how she attributes her continued use of Cuisenaire rods to the learning that took place during the online professional development sessions. In fact, during her interview, Teacher B provided many reasons for why she likes the rods and how the rods are beneficial in helping students develop conceptual understanding of fraction concepts. Teacher B seemed to

benefit from her use of the concrete rods during our online trainings—and perhaps even the virtual rods that she may have viewed through the shared screen. However, the other teachers appeared unable to benefit from the learning and thinking of Teacher B because they were unable to see what she was doing with her concrete materials. In this way, the irritation experienced by Teacher A relates directly to what Cady and Rearden (2009) shared regarding the instructors in their online training who felt frustrated by the limitation of the online platform in that it was not able to support them in seeing the real-time movements of other people's manipulatives.

This finding seems to indicate why it may be important for a course instructor to engage participants in the same shared space and ensure that all feel comfortable with and have access to the online tools being used (Francis & Jacobsen, 2013). Teacher C commented that the input she received from me when she was incorrectly modeling a problem with the virtual Cuisenaire rods in the shared space was helpful because she was able to correct her mistake in the moment. Because I, as the instructor, could see what she was doing with the rods when she was incorrectly modeling a problem, I was able to ask her questions and help her reconsider the meaning of her representation in connection to the concept of the problem at hand.

This communication between Teacher C and myself was quite different from the communications expressed by Teacher B when she was attempting to use the concrete rods to solve problems on her own. During those instances of communication, the ideas that Teacher B expressed seemed to be more interruptive to the thinking of the group than connected to it. This disconnection is likely what led Teacher A to feel annoyed that Teacher B could not contribute effectively to the thinking of the group in the shared

space. Had Teacher B been more involved in the virtual manipulations in the shared space, it is likely her thinking and ideas could have helped the other teachers learn how to use Cuisenaire rods without needing as much help from me, as the course instructor.

Teacher B's disconnection with the virtual fraction pieces is similar to the problem that Herrington et al. (2003) described that must be overcome in order for participants engaged in an online learning environment to become "immersed in the learning context that has been created for them" (p. 63). According to these researchers, participants involved in a collaborative online experience must be willing to suspend their disbelief in the components of the task that are merely representations of other things. Thus, it could be reasoned that the disconnection Teacher B felt with the virtual fraction pieces might have resulted from a disbelief that these virtual pieces could engage her senses in meaningful ways as she worked to understand how to use them to represent and solve problems. In contrast to these feelings of disconnection experienced by Teacher B, Teachers A and C both commented that the virtual fraction pieces were effective at helping them learn concepts. These beliefs about the usefulness of the virtual fraction pieces mirror findings from Reimer and Moyer (2005), who stated that virtual manipulatives are equally as effective as concrete manipulatives in helping individuals learn fraction concepts.

On another note, Teacher C expressed feeling vulnerable when moving virtual pieces in the shared space where everyone else in the group could see her actions. She explained that these feelings of vulnerability were lessened because the group had been working together for some time. Because she felt comfortable enough with the other teachers and me, she was willing to risk moving the pieces, even if she made a mistake.

The feeling Teacher C had that she could make herself vulnerable in front of the group is important because it relates to research by Hebert (2007), who stated that participants involved in synchronous, online professional developments need to feel a sense of trust with each other. The importance of creating a trusting environment echoes findings from Francis and Jacobsen (2013), who found that supporting participants in establishing individual identities in the online environment helped the participants know it was okay for them to make mistakes in their attempts to solve problems collaboratively.

Finally, Teachers A and C mentioned they occasionally experienced technological problems while working with the virtual tools. At times, they were unable to see the question on the screen of the shared workspace, they had difficulties connecting to and navigating through all of the applications being used, and they did not feel as confident in using the tools in the online environment as they would have liked to have been. All of these considerations demonstrate how continuous technical support should be available to participants in an online training, and relate to what Hebert (2007) wrote about designing effective online professional development.

Facilitator Contributions

The teachers and I had a difference of opinion about the appropriateness of the assistance I provided them as they engaged collaboratively to represent and solve certain of the more challenging problems. On the one hand, there were a few instances when I feel my communications facilitated and guided without being overly helpful, as was the case when the teachers solved a challenging fraction-multiplication area problem and a fraction-division gas-tank problem. In these instances, I agree with the teachers that the assistance I offered was appropriate, in that it aligned with recommendations by Signer

(2008), who stated that facilitators of online professional learning sessions should guide participants through direct interaction and active monitoring. On the other hand, when the participants used the virtual Cuisenaire rods, I ended up doing many of the manipulations on the screen for them. This was the case during the very first attempt at using the rods, where only Teacher A was initially successful at entering the *Google Presentation* document. For that problem, the other two teachers viewed the interactions through the shared screen, but they were initially unable to manipulate the pieces themselves. Eventually, Teacher C was able to interact with the rods, but in every case of using the virtual Cuisenaire rods, I would manipulate the pieces in some way in an effort to help the teachers be successful. Teacher B never engaged with the virtual rods and, in most instances of using the rods, Teacher A watched the manipulations on the screen, without moving pieces herself.

In her reflections, Teacher A presented some statements that seemed contradictory. First, she stated that she appreciated how I let her and the other teachers struggle to learn how to use the rods. However, she also stated that it was helpful when I engaged in manipulating the rods to show the teachers how to solve certain problems when they were really struggling. Second, she stated that she felt it would have been better had I just shown the participants how to use the rods directly, especially at first when there was a lot of wait time as the participants struggled to understand how the rods could be used to represent a given problem. On the other hand, Teacher A stated that I showed the participants directly how to use the rods to solve certain problems because they had requested to learn from me how to use them. Though these statements seem contradictory, they accurately detail what happened during the sessions. I did allow wait

time for the participants to struggle as they attempted to represent problems using Cuisenaire rods, but eventually I started moving the pieces to show how each of these problems could be represented, especially when the participants seemed unsure of how to proceed.

The above comments by Teacher A highlight how actions on my part as the course instructor had an effect on the teachers' interactions and communications. When I allowed them to struggle and waited for them to consider how to approach solving a difficult problem or representing a certain problem using the rods, I effectively provided them with time to think through the problem and struggle. Being allowed additional time to consider their work and struggle was something that all three teachers mentioned as beneficial when they were asked to solve difficult problems independently. When I engaged to help the teachers understand how to represent problems using the Cuisenaire rods, I effectively provided them with direct instruction and the result, according to the teachers, was that the online sessions helped them to learn how to use Cuisenaire rods.

Along these lines, Teacher C said she felt the actions I took in representing the problems were helpful because the teachers were still unfamiliar with the online tools. Teacher B remarked that she believes there is no such thing as a perfect situation in teaching and expressed that she did not think my actions were inappropriate. Rather, she felt satisfied that our interactions online were beneficial in helping the other teachers and her learn and grow. My actions in manipulating pieces and, at times, drawing on the screen for the participants, contradicted recommendations by Smith et al. (2009), who described a process for how an instructor can facilitate effective problem solving and subsequent discussion in a mathematics classroom without directly helping learners solve

a problem they are attempting. Although I often prompted the teachers with guiding questions to facilitate their work in representing certain of the problems, at other times, I represented certain problems for them. While I found these actions on my part to contradict the practices outlined in mathematics education research (NCTM, 2000, 2014), the teachers seemed to have differing opinions than me.

Implications for Practice

The following subsections discuss implications for practice related to discussion points presented in the previous three sections.

Problem Difficulty and Mode of Engagement. An implication for practice related to problem difficulty and mode of engagement suggests that those who design a professional development replete with problem solving consider carefully the difficulty level of various problems used. After determining problem difficulty, it is recommended that consideration be given to the different outcomes that might result as learners engage to derive solutions either independently or collaboratively. If asking participants to solve a difficult problem independently, the designer should be aware that additional time and specific scaffolds might be needed in order for them to complete the task successfully. All three teachers in this study expressed how they needed more time when working to solve a difficult problem independently. Thus, thought might be given to having learners receive difficult problems ahead of time and asking that they arrive for the online session with already-completed pictures of their work.

Another alternative also mentioned by each of the teachers in this study would be to allow for group collaboration on difficult problems. Depending on whether the instructional goal is for learners to produce multiple representations, experience

productive struggle, or collaborate to arrive at a correct solution, different approaches in how they are asked to approach solving a particular problem might be considered (for a more complete discussion of teaching practices that support these goals, see NCTM, 2014). In the event that all three of the aforementioned goals are expected outcomes, one might contemplate how to allow small groups of learners to experience productive struggle as they work collaboratively to represent the solution to a problem. Thereafter, each small group could be asked to share their different ways of thinking about the problem with the other groups at large, which would likely elicit multiple, different representations of the problem's solution.

Another implication for practice related to this topic includes considering how to design an online professional development training for a much larger group of participants. While it may have been nearly impossible to engage the teachers from the current study in multiple, small-group discussions, as there were only three of them, proposing a way to expand the technology used in this study to a larger group of teachers could be advantageous. It could be reasoned that, if a large group of participants were allowed to solve problems collaboratively in small, synchronous online groups, then multiple representations for how a problem can be solved would be generated. In addition, participants working in small groups would likely benefit from collaborating to solve difficult problems, which the teachers in this study reported was especially advantageous.

Hiltz (1998) postulated that synchronous online interactions were not conducive to large groups of learners, asserting that the size of the group could potentially impede the session facilitators from effectively interacting with all learners. Chauvot and Lee

(2014) might disagree with Hiltz (1998), however, in that they asserted that ever-changing technologies are able to facilitate the types of interactions online that are most valued in face-to-face trainings. Because collaborative problem solving with conceptual models and manipulatives is an important component of mathematics education (NCTM, 2000, 2014), it might thus be advantageous to determine an efficient and effective means to allow for multiple group collaborations in online learning sessions wherein participants interactively engage with virtual manipulatives to solve problems together.

One way to facilitate these types of interactions with a larger group of participants could be to use an online management system that allows for small groups of participants to access their own whiteboard screens, annotation tools, and screen-sharing applications, as available through the *Blackboard Collaborate* program. A facilitator using a learning platform that allows for groups of participants to be separated easily into small groups could follow a protocol where each participant in a group would be assigned a role. Role assignments could help with the management of small-group activities, and could better ensure that video archives are created that highlight each group's interactions. For example, one person could be assigned to share her screen to display interactive manipulations from a *Google Presentation* slide; another person could be asked to archive the small-group interactions to record the group's work in real time; and another person could be directed to display the question of the problem on the whiteboard screen or in the chat box. Additionally, a unique advantage to hosting group collaborations online rather than in a face-to-face training exists: if done correctly online, all small group interactions can be video recorded and archived. These recordings of interactions could include the "name tags" that are generated when participants manipulate pieces in

Google Presentation. These archives would allow the session facilitator to view later all groups' interactions, even those he was unable to view during the time of the actual online session.

Virtual Tools. A recommendation to practice linked to the discussion about virtual tools repeats what many other researchers in online instructional design have stated: it is important to allow participants to practice using all components of an online platform in advance (Ellis & Kisling, 2009; Hebert, 2007; Ostashewski et al., 2011; Signer, 2008). Considering the problems that Teacher B experienced with accessing the shared space during the professional development described in this study, implications exist for other practitioners hoping to use virtual manipulatives in a collaborative online environment. One such suggestion includes considering the ramifications of not having a way for participants to practice using virtual manipulatives before their time together online.

Various technical problems can arise when participants attempt to access and use virtual tools in the online environment, and these problems are likely to affect the subsequent communications and interactions of the online group. This repercussion connects to research by Hebert (2007), who stated that those who develop online training sessions need to be flexible in their use of technologies, as modifications and adaptations are likely to occur after the start of such trainings. For the current study, a disruption in the way in which the group members communicated with each other seemed to have occurred because a clear expectation was not established regarding the use of the interactive virtual manipulatives. Because participants were not allowed to practice with

these online tools before the beginning of the second session, they may have felt less comfortable or confident with their ability to use the tools to solve problems.

Beyond the difficulties faced by the teachers in accessing the shared space and practicing using the virtual tools, other specific implications to practice surfaced in the review of the multiple primary and secondary sources of data analyzed during this study. There were multiple instances when a certain feature of an online application we were using was discovered and then later forgotten. For example, I learned while practicing online with the assistant course designer that I could copy and paste the text of a problem into the chat box to allow participants to use grid paper on the white board screen and read the text of the problem at the same time. Although I had determined this solution in advance of our time together online, I forgot about it during the actual session with the teachers. In much the same way, I had many other "discoveries" which were later forgotten and then only sometimes remembered. These discoveries included learning 1) how teachers who were unable to access *Google Presentation* could view the shared desktop screen through the webinar to see the group's interactions, 2) how "name tags" in *Google Presentation* were archived through the screen-share application, allowing me to identify later who was moving a certain virtual manipulative, and 3) how the ability to annotate in the shared screen of the webinar allowed for all participants to draw over the top of the representations created with the virtual manipulatives in *Google Presentation*.

Because certain features of the technological applications used during this study were discovered and then later forgotten, it would have been advantageous to keep a record of such discoveries. Because my expertise in the online system surpassed the expertise of the teachers who participated in the training, as was mentioned by Teacher C

during her interview, it would have been prudent for me to record and then demonstrate important features of the online applications as they were discovered. Doing so might have improved communications and interactions while participants and I worked together online. Further, because the teachers were at varied levels of expertise with the technology being used, implications to practice include considering what Yamagata-Lynch (2014) shared about how instructors in synchronous online courses should be deliberate about the way participants engage online, as well as conscientious about what structures are put in place to moderate their learning.

Facilitator Contributions. An implication to practice stemming from the discussion about facilitator contributions includes how those who develop and lead synchronous online professional learning sessions should be aware of the ease by which they can interfere with learners' work. During the current study, my interference in the work of the participants surfaced when they were asked to collaborate in the shared space to represent problems using virtual manipulatives or drawings. In much the same way that classroom teachers can inadvertently help students who are collaborating during a face-to-face lesson, having the ability to interact with the participants in my study allowed me to help them manipulate pieces and draw representations as they worked to solve problems.

There were instances during this study when participants simply needed help understanding how to use a particular tool because of their unfamiliarity with the online environment. In these instances, I considered that the guidance I offered the teachers was appropriate and helpful because I merely assisted them in understanding how to navigate the features of the online applications. However, when I began to do the actual work of

problem solving for the participants, I considered my actions contrary to the effective practices outlined in mathematics education research (NCTM, 2000 2014). Thus, the capacity for unintentional interference indicates how it might be important for online course instructors to establish and communicate ahead of time what their role will be when participants work together to solve problems. This communication could look like an instructor explaining, perhaps, that she is willing to help participants understand how a tool is used, but that she is unwilling to help them use the tool to solve a particular problem.

Implications for Future Research

The next three subsections discuss implications for future research related to discussion points presented previously concerning problem difficulty and mode of engagement, virtual tools, and facilitator contributions.

Problem Difficulty and Mode of Engagement. The findings discussed about mode of engagement and problem difficulty could prompt future research studies designed to investigate the effects of involving teachers in a synchronous, online professional development using either independent or collaborative means of engagement. Such a study could investigate the differences that arise when groups of participants are asked to solve problems collaboratively versus where individual participants are asked to solve problems individually and then share their findings. A study that seeks to determine the effects of collaborative group work on successful task completion versus independent work (with subsequent sharing of ideas) could have implications for practice in the way that future online professional development sessions are structured for practicing teachers. More specifically, a future study as is described

above could further investigate the effects on participants of either mode of engagement as they solve easy and/or difficult mathematical problems. Specific effects that could be analyzed through such a research study might include 1) time on task, 2) success rate at solving given problems, 3) number of, variations in, and types of representations generated through individual or collaborative efforts, 4) the effectiveness of either approach in developing conceptual knowledge of the content in question, and/or 5) the nature of emotions generated (e.g., stress, nervousness, trust, etc.).

One finding that emerged from the analysis of the participant interviews from the current study, which was not found during the analysis of the primary record of observations, was how the teachers reported that they felt like their online collaborations while solving problems were more efficient than when working independently. In addition to the teachers expressing how they saved time by working collaboratively, they also expressed that solving problems together helped them to acquire an understanding of new concepts more effectively. As such, a future research study that investigates the effects of independent work versus collaborative work in solving either easy or difficult problems might be able to find evidence that supports or expands on the findings from the current study. This additional evidence could more effectively generalize said findings to a broader population. Because this study investigated the communications and interactions of only three online sessions wherein problems were solved by only three participants, there was not enough data available to make any certain claim regarding generalizable effects of independent or collaborative group work on participants' problem-solving efforts.

Moreover, a research study building off the ideas presented above could investigate the implications of using multiple groups of participants who collaborate with their respective group members in order to solve mathematical problems using virtual tools. Research by Smith et al. (2009) highlights that multiple representations can be generated when multiple groups work collaboratively. The idea of using multiple groups of participants who collaborate online to solve problems using virtual tools is a field of research in need of additional exploration. Additionally, an online professional development designed with multiple groups of participants could provide more information about the advantages and disadvantages of collaborating to solve problems versus solving problems independently. Such a study could also investigate the effects that either mode of engagement has on participants' experiences as they attempt to solve easy and/or difficult problems.

Virtual Tools. A future research study could look at the effects of participants' independent or collaborative work when using a certain manipulative, like Cuisenaire rods, in three or four different conditions. For example, participants might collaborate in groups or work independently in either a face-to-face or an online environment. Such a study could attempt to measure the results of each condition using some sort of skill-based test in an attempt to evaluate if face-to-face or online interactions are more effective at developing conceptual understanding of fraction concepts. Such a study could extrapolate on Reimer and Moyer's (2005) findings regarding the effects of using virtual manipulatives to teach fraction concepts in the classroom. Considering the specific disconnect that Teacher B felt from virtual manipulatives, a study to investigate the effectiveness of using such online tools in a professional development could have

implications regarding the design of future online professional learning sessions for classroom teachers.

Facilitator Contributions. A research study designed to investigate the effects of the level of facilitator interaction with participants while they solve problems together during an online professional development could bring clarity to the difference of opinions that existed between the teachers who participated in this study and me, as their course instructor. Such a research study could provide information about the level of conceptual understanding developed, and subsequently retained, when participants either do or do not receive assistance from their course instructor while representing and solving problems using virtual manipulatives during a synchronous online training. The teachers in this study felt as though the help I rendered them was appropriate and, at times, necessary. However, I felt, as did a peer reviewer, that the help I provided the teachers was contradictory to best practices outlined in mathematics research (NCTM, 2014). Thus, a study that investigates the long-term effects of receiving such help versus experiencing productive struggle without receiving help from a facilitator could provide information about the appropriate level of facilitator interaction when considering the best way to determine mastery of a concept being learned.

Summary

The first discussion point presented in this chapter related to how different patterns of communication and interaction emerged when participants were asked to solve problems either independently, with the sharing of their solution strategies afterward, or collaboratively, by using virtual tools. Moreover, it was shared how these communications and interactions were influenced by the difficulty level of a given

problem and the tools utilized to solve it. Through this discussion, it was suggested that those who design online professional learning sessions for teachers consider the possible outcomes of asking participants to approach solving a problem in a certain way, as well as the effects of having them use certain virtual tools to represent their work. More specifically, it was proposed that designers of online learning consider in advance the benefits that come from teachers solving problems independently (e.g., that multiple representations are likely to surface) or collaboratively (e.g., that challenging problems can be solved successfully).

Furthermore, it was advised that careful attention be given to the various advantages and disadvantages of presenting both easy and hard problems to a group of learners, as well as to the order in which such problems are displayed. Supplementary discussion highlighted how consideration of these details is important because such attention is likely to influence the level of trust developed among group members and the likelihood in which participants feel as though they belong to a true community of practice. Other points discussed include the importance of developing an awareness of the feelings participants might experience when asked to display their work and recognizing that technological problems can impede group communications and interactions. Implications for practice related to mode of engagement and problem difficulty were also shared. These implications include considering how to design online, mathematics professional learning sessions while considering the effects of using both independent and collaborative approaches when solving easy and hard problems, in addition to being mindful of modifications that might be required to engage large groups of participants in collaborating virtually.

A second topic was also presented about how virtual tools can affect participants' communications and interactions. During this discussion, it was suggested that those who facilitate real-time problem solving using virtual tools consider the importance of ensuring that all participants have access to—and have practiced using and navigating between—the tools in the shared online platform in advance of the first learning session. Evidence from the current study showed that when participants were not engaged successfully in the interactive components of the online platform, their ability to interact successfully with others was greatly affected. In the discussion presented, it was advised that session facilitators communicate clear expectations about participants' expected level(s) of participation in these shared spaces, recognize the possible feelings participants might experience when displaying their thinking visually for others in the group to see, and consider how certain individuals might believe, at least initially, that virtual tools are unable to engage their senses in meaningful ways.

Other items discussed include how continuous technical support is likely to be needed if expecting groups to collaborate using only virtual tools and how a willingness to take risks in displaying one's thinking using online manipulatives is only likely to develop after participants have worked together for a sufficient amount of time. Beyond these recommendations, additional suggestions included how session organizers can minimize technological problems by keeping a more accurate record of the features of the different online applications being used and then clearly communicating the implications of those records with all participants. Finally, a future research project was described wherein one might evaluate the effects of using virtual manipulatives to engage

participants in learning fraction concepts while solving problems collaboratively online versus the effects of using concrete manipulatives in a face-to-face training.

The final topic discussed in this chapter related to the influence of my actions, as the facilitator, on the communications and interactions of the participants in this study. In the discussion presented, it was shared how my actions in representing components of the problems being solved by participants contradicted recommendations from mathematics literature. While the participants in this study felt that my actions were appropriate, I did not. A suggestion provided for others interested in facilitating online learning, who hope to avoid unintentional interference during the time allotted for participants to collaborate, is to establish and communicate in advance what they, as facilitators, are and are not willing to do during times when participants solve problems together. Because the teachers in this study had a different opinion about the appropriateness of my interactions, a possible research study was also suggested. Such a study could investigate differences in measures of conceptual understanding between different groups of participants based on whether they receive or do not receive help from an online facilitator as they collaborate to solve problems using virtual tools.

Conclusion

This study investigated the patterns of communication and interaction that emerged during three, synchronous, online mathematics professional development sessions where three fifth-grade teachers used virtual tools to solve and discuss fraction multiplication and division problems. The findings from this study addressed a gap in the literature about how virtual tools can be used to support real-time collaborations among those engaged in solving and discussing mathematical problems synchronously online

(Francis & Jacobsen, 2013). This chapter presented discussion points related to how the difficulty level of a given problem and the approach by which participants engaged to solve it affected subsequent interactions and communications during the three aforementioned sessions. Additionally, a discussion was presented related to how the virtual tools used during these online sessions and the contributions of the session facilitator affected participants' interactions and communications.

This dissertation presented findings that contribute significantly to the field of online teacher learning, most expressly to the literature concerning the use of virtual tools to enact practices that are most valued in face-to-face mathematics trainings. When considering the online learning sessions described in this paper, coming to understand how patterns of communication and interaction developed and changed according to problem difficulty, mode of engagement, virtual tool usage, and facilitator contribution is significant because synchronous online learning is an ever-growing field of interest for educational researchers and practitioners (Dede et al., 2009). Moreover, the conclusions drawn through analysis of the data sources of this research project are especially important to the field of online mathematics professional development, as limited research is available to inform practice about how virtual tools can be used to support interactive, synchronous online collaborations while solving problems (Francis & Jacobsen, 2013).

Because best practices in mathematics pedagogy require collaboration with mathematical models—and demonstrating such practices has hitherto been difficult to do using synchronous online platforms (Cady & Rearden, 2009)—the detailed description of participating teachers' experiences supplied in this dissertation, though not generalizable,

are highly transferable to practice and future research related to online mathematics professional learning. The outcomes described in this report demonstrate how ever-changing online technologies can effectively support the types of interactions that are often valued and considered crucial to the success of face-to-face professional learning sessions (Chauvot & Lee, 2014). As such, future research projects and online professional development courses would benefit from considering the discussion points presented in this chapter and expanding upon the findings that were heretofore described

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Appendix A

Interview Protocol used with Teacher Participants

Topic Domain: *Participant's Impression of Discussing/Solving Problems Online*
[Covert Categories: What the participant remembers about the times we worked on or discussed problems online. Does she identify scenarios where discussing/solving together (vs. solving independently) was effective at helping her understand difficult problems? Does she mention anything about multiple representations highlighted? Anything about the time spent solving easy problems that she already knew per her pre-test score? Thoughts on the different types of online technology used to conduct the problem-solving aspects of the sessions.]

1. Can you walk me through what you remember about the times when we were together online and we discussed or worked on math problems?
    a. We only met together a total of 5 times online and only during three of our online sessions did we work on math problems. Do you think it is possible that a change in how problems were solved/discussed could take place over the course of these three sessions?
        i. (If interviewee responds yes) Do you think any sort of change occurred for us over the course of the three sessions where we worked on math problems together?
            1. (If interviewee responds yes) Can you explain what you think changed with regards to when we worked on math problems together?
            2. (If interviewee responds no) How did things remain the same?
        ii. (If interviewee responds no) What do you think it would take for a change to occur with regards to how problems are solved/discussed during an online professional development where people work together to solve math problems?
    b. Was there anything specific that you liked or didn't like about when we discussed or worked on math problems?
        i. Would you have done anything differently regarding the time when we focused on solving/discussing problems if you were in charge of these sessions?
            1. (If interviewee responds yes) Can you please share?
            2. (If interviewee responds no) Why do you think that is the case?
    c. What can you tell me about the different types of online tools that we used during this course while solving problems?
        i. What did you like and/or dislike about the online tools we used during our time together?
        ii. What advantageous and/or disadvantages do you think there are in using the different tools we used in these online sessions?
        iii. With regards to these tools we used while working on problems, is there something you would have liked to have been different?
            1. If the answer is yes, can you please share?

2. If the answer is no, why do you think that is the case?

2. I would now like to discuss more specifically some instances when we solved different problems together online and what this experience was like for you as a participant. I have selected a few different video clips of our time together solving different problems, and I would like to share at least one or two of these with you to get your insight into what it was like for us to solve problems together online. Before I do that, though, I wonder if there are specific instances that you remember specifically that you would like to discuss. Is there a specific problem that we worked on that you recall and would like to share your thoughts about?
    a. (If interviewee responds yes) What problem do you remember and what would you like to share about that experience? [USE FOLLOW-UP QUESTIONS FROM #3 BELOW AFTER INTERVIEWEE SHARES HER PERSPECTIVE ON THE QUESTIONS SHE RECALLS – IF POSSIBLE, suggest that we view a portion of the segment she has recalled, and then ask follow-up questions or elaborate on the questions using prompts from #3 below after viewing the clip(s).]
    b. (If interviewee responds no, or after they are done sharing) Would it be all right with you if we spent a couple of minutes together viewing a portion of one of the videos I've selected?
        i. (If interviewee responds yes) Please know that you can stop me at any time as we view this clip together. If anything stands out to you that further helps explain what you have previously shared, I would greatly appreciate you elaborating on your thoughts.
        ii. (If interviewee responds no) Would you mind if I provided you with a few video clips to watch at a time that might be more convenient for you and, in addition, provide you with a few prompts that you can simply respond to in writing and email back to me?

3. [Refer to or show a clip of an <u>easy problem</u> that was solved <u>ON THEIR OWN</u> and then the picture was uploaded—one that they all knew the answer to from the pre-test (Session 1: Question 2—Teacher A: 1:27:48 – 1:29:39; Teacher B: 1:29:39 – I say, "I'm going to take the privileges back"; Teacher C: 1:21:51 – Claude talks)]
    - Would you classify this problem as an easy or hard problem?
        - Why do you classify it as _____? Can you please explain your classification?
    - What did you think about**:**
        - Our work together in solving/<u>discussing</u> this problem?
        - How we worked on this question?
        - The way we solved/<u>discussed</u> this problem?
        - The technology or online tools we used to facilitate our solving and/or <u>discussion</u> about this problem?

- What did you perhaps like or dislike about the way we approached solving and/or discussing this problem?
- How would you classify or rank the value of our time spent together solving/discussing this question?
    - Why would you classify/rank it as such?
        - Can you please explain your thinking?

4. [Refer to or show a clip of an easy problem that was solved TOGETHER ONLINE—one that they all knew the answer to from the pre-test (Session 3: Question 1. All Teachers Start at 32:15 to where I say Google Document reference, then jump to 37:00 until 41:30, when Teacher A says 28 loaves]
    - Would you classify this problem as an easy or hard problem?
        - Why do you classify it as _____? Can you please explain your classification?
    - What did you think about**:**
        - Our work together in solving/discussing this problem?
        - How we worked on this question?
        - The way we solved/discussed this problem?
        - The technology or online tools we used to facilitate our solving and/or discussion about this problem?
    - What did you perhaps like or dislike about the way we approached solving and/or discussing this problem?
    - How would you classify or rank the value of our time spent together solving/discussing this question?
        - Why would you classify/rank it as such?
            - Can you please explain your thinking?
5. [Refer to or show a clip of a difficult problem that was solved ON THEIR OWN—one that they did NOT know the answer to from the pre-test (Session 1: Question 3. Show question at 1:41:00, then jump to 1:45:34 go until I say, "If it was multiple choice probably could have guessed it"]
    - Would you classify this problem as an easy or hard problem?
        - Why do you classify it as _____? Can you please explain your classification?
    - What did you think about**:**
        - Our work together in solving/discussing this problem?
        - How we worked on this question?
        - The way we solved/discussed this problem?
        - The technology or online tools we used to facilitate our solving and/or discussion about this problem?
    - What did you perhaps like or dislike about the way we approached solving and/or discussing this problem?
    - How would you classify or rank the value of our time spent together solving/discussing this question?
        - Why would you classify/rank it as such?

        - Can you please explain your thinking?

6. [Refer to or show a clip of a difficult problem that was solved TOGETHER ONLINE—one that they did NOT know the answer to from the pre-test (Session 3: Question 6. Start at 1:36:28 until about 1:45:00, after Teacher C says "I was totally confused.")
    - Would you classify this problem as an easy or hard problem?
        - Why do you classify it as _____? Can you please explain your classification?
    - What did you think about**:**
        - Our work together in solving/discussing this problem?
        - How we worked on this question?
        - The way we solved/discussed this problem?
        - The technology or online tools we used to facilitate our solving and/or discussion about this problem?
    - What did you perhaps like or dislike about the way we approached solving and/or discussing this problem?
    - How would you classify or rank the value of our time spent together solving/discussing this question?
        - Why would you classify/rank it as such?
            - Can you please explain your thinking?

    [*NOTE: If short on time, view the clips related to questions #4 and #6 above regarding the easy and difficult problems that were solved together online. If time is short, ask question 2.B.ii above about whether they would mind watching the video clips at home and responding in writing to the questions outlined underneath question #3 above.]
7. Is there anything more you would like to share regarding our time together online solving these fraction multiplication and division problems?

Topic Domain: Teacher's impression of the role that I (and/or Claude) played as instructors in the online course.
[Covert categories: Does the participant mention that the instructors interfered too much or provided too much support with the solving of the hard problems? Identification that the instructors spent a lot of time helping teachers learn online tools in order to work through the word problems]

8. Throughout the clips we have looked at or discussed today, myself and/or Claude are certainly present during discussions and sharing. What are your thoughts about my or Claude's interactions with you and the other participants in the course, specifically during the times when we worked on questions?
    a. Was there anything you liked or disliked? Would you feel comfortable sharing?
        i. (If interviewee responds yes) Please do.
        ii. (If interviewee responds no) That's not a problem.

b. What role do you think that Claude and I played as instructors during the times when we solved/discussed questions together?
    i. Can you please explain how you think the roles that we played affected your experience as participant?

9. I would now like to show you a specific clip where the instructors are involved with you and the other participants during a time when we were solving/discussing a problem online. As we are going through this clip, feel free to ask me to stop the recording if there is anything additional that you think of regarding me and/or Claude's interactions with participants during the problem-solving portions of the sessions. [Refer to or show a clip of ME doing a lot of the work and heavy lifting on a particular problem (Session 3: Question 5. Start at 1:21:58 til about 1:27:00 after Teacher A has said "Twelve")
    a. Do you have any additional thoughts to share after viewing this clip?
        i. (If interviewee responds yes) Please do.
        ii. (If interviewee responds no, or after they answer "yes" to 8.a.i) Does this clip confirm what you shared about the role that you think Claude and I played as instructors during the times when we solved/discussed questions together?
            1. If yes, can you please explain?
            2. If no, can you please explain?
10. Is there anything more you would like to share regarding our time together online solving these fraction multiplication and division problems and my or Claude's interactions with you and the other participants?

Appendix B

Interview Protocol used with Assistant Course Designer

Topic Domain: *Views on Participation in an Online Professional Development Course on Fraction Multiplication and Division*

1) Can you walk me through what a typical session in the fraction online course was like?
2) Did anything change over the course of the sessions where we solved problems together as a result of the phone conversation that we had after the first session?
3) Can you compare and contrast the 1st session to the 2nd session?
    a. [If differences mentioned] Do you think those differences change the experience for the teachers?
        i. [If yes] In what ways?
        ii. [If no} Why do you think that is?
    b. [If differences mentioned] How would you describe the teachers' interactions during these sessions?
4) Was there anything specific about the course that you liked or didn't like?
    a. Do you mind sharing?
5) There was one session where the teachers worked together on the screen to solve a problem using graph paper. Do you remember this? If the answer is yes, can you talk me through what you remember from that experience?
   [Show part of graph paper problem after initial recall, Session 2, Question 2, and then re-ask the above questions for #5]
6) What are your thought about the online tools that were used in the course?
7) If another math training was offered about a topic you wanted to learn more about and the choice was for you to either attend the training face-to-face or online (in a similar fashion to this course), which training do you think you would enroll for, and why?

Appendix C

University of Houston Division of Research

Protection of Human Subjects Approval Letters

UNIVERSITY of HOUSTON

DIVISION OF RESEARCH

October 2, 2015

Mr. Joshua Udy
c/o Dr. Jennifer Chauvot
Curriculum and Instruction

Dear Mr. Joshua Udy,

The University of Houston's Institutional Review Board, Committee for the Protection of Human Subjects (1) reviewed your research proposal entitled "Online Mathematics Professional Development: Impact on Teacher Practice and Student Achievement" on July 24, 2015, according to federal regulations and institutional policies and procedures.

At that time, your project was granted approval contingent upon your agreement to modify your protocol as stipulated by the Committee. The changes you have made adequately fulfill the requested contingencies, and your project is now APPROVED.

- **Approval Date: October 2, 2015**
- **Expiration Date: October 1, 2016**

As required by federal regulations governing research in human subjects, research procedures (including recruitment, informed consent, intervention, data collection or data analysis) may not be conducted after the expiration date.

To ensure that no lapse in approval or ongoing research occurs, please ensure that your protocol is resubmitted in RAMP for renewal by the **deadline for the September, 2016** CPHS meeting. Deadlines for submission are located on the CPHS website.

During the course of the research, the following must also be submitted to the CPHS:

- Any proposed changes to the approved protocol, prior to initiation; AND
- Any unanticipated events (including adverse events, injuries, or outcomes) involving possible risk to subjects or others, within 10 working days.

If you have any questions, please contact Samoya Copeland at (713) 743-9534.

Sincerely yours,

Lorraine R. Reitzel

Dr. Lorraine Reitzel, Chair
Committee for the Protection of Human Subjects (1)

PLEASE NOTE: All subjects must receive a copy of the informed consent document, if one is approved for use. All research data, including signed consent documents, must be retained according to the University of Houston Data Retention Policy (found on the CPHS website) as well as requirements of the FDA and external sponsor(s), if applicable. Faculty sponsors are responsible for retaining data for student projects on the UH campus for the required period of record retention.

Protocol Number: 15611-01 Full Review: X Expedited Review: ___

316 E. Cullen Building Houston, TX 77204-2015 (713) 743-9204 Fax: (713) 743-9577

COMMITTEES FOR THE PROTECTION OF HUMAN SUBJECTS.

UNIVERSITY of HOUSTON

DIVISION OF RESEARCH

March 23, 2016

Mr. Joshua Udy
c/o Dr. Jennifer Chauvot
Curriculum and Instruction

Dear Mr. Joshua Udy,

The University of Houston's Institutional Review Board, Committee for the Protection of Human Subjects (1) reviewed your Request for Revision of the research proposal entitled "Online Mathematics Professional Development: Impact on Teacher Practice and Student Achievement" on February 19, 2016, according to federal regulations and institutional policies and procedures.

At that time, your project was granted approval contingent upon your agreement to modify your protocol as stipulated by the Committee. The changes you have made adequately fulfill the requested contingencies, and your project is now **APPROVED**.

- **Approval Date: March 23, 2016**
- **Expiration Date: October 1, 2016**

As required by federal regulations governing research in human subjects, research procedures (including recruitment, informed consent, intervention, data collection or data analysis) may not be conducted after the expiration date.

To ensure that no lapse in approval or ongoing research occurs, please ensure that your protocol is resubmitted in RAMP for renewal by the **deadline for the September, 2017** CPHS meeting. Deadlines for submission are located on the CPHS website.

During the course of the research, the following must also be submitted to the CPHS:

- Any proposed changes to the approved protocol, prior to initiation; AND
- Any unanticipated events (including adverse events, injuries, or outcomes) involving possible risk to subjects or others, within 10 working days.

If you have any questions, please contact Samoya Copeland at (713) 743-9534.

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